International Olympiad on Astronomy and Astrophysics

Problem Book

(2007–2021)

Introduction

The International Olympiad on Astronomy and Astrophysics is an international competition for high-school students along the lines of the other *International Science Olympiads*, covering the fields of Astronomy and Astrophysics.

Participating students, who must not be attending higher education and must be under the age of 20 on 30th June of the year of the event, solve theoretical, data analysis, observational problems individually and in teams under controlled and timed conditions over a period of 10 days.

Problem tasks (conforming to the Statutes and Syllabus) are set by the local organisers, verified by the team leaders of the participating countries, and presented to the participating students in their own languages. Problems are designed to reflect current astronomical knowledge as well as test the students' abilities. Medals and other prizes are awards to the most successful participants as specified in the Statutes.

Additionally each event includes cultural and social occasions and excursions designed to widen the students' experience.

The host country covers all costs (accommodation, food, excursions) of the primary team from each participating country (consisting of up to 5 students and up to 2 adult team leaders) during the event, except for travel and visa costs to the host country. In addition, the host country may at its discretion accept Observers and (secondary) Guest Teams, specifying a fee for each to cover costs.

List of Past Events

The IOAA has been successfully organised every year since the first event (until 2020), held in Thailand in 2007. Below is a brief summary of these events.

1st IOAA (2007)	Chiang Mai, Thailand
2nd IOAA (2008)	Bandung, Indonesia
3rd IOAA (2009)	Tehran, Iran
4th IOAA (2010)	Beijing, China
5th IOAA (2011)	Katowice, Chorzow & Krakow, Poland
6th IOAA (2012)	Rio de Janeiro, Brazil
7th IOAA (2013)	Volos, Greece
8th IOAA (2014)	Suceava, Romania
9th IOAA (2015)	Magelang, Indonesia
10th IOAA (2016)	Bhubaneswar, India
11th IOAA (2017)	Phuket, Thailand
12th IOAA (2018)	Beijing, China
13th IOAA (2019)	Keszthely, Hungary
[*]1st GeCAA (2020)	Estonia (online)
14th IOAA (2021)	Bogota, Columbia (online)

[*] GeCAA is an online astronomy and astrophysics competition held instead of the cancelled IOAA 2020 by the IOAA international board and Estonian Astronomy Olympiad Committee.

Problems of IOAA

IOAA 2007 – Chiang Mai, Thailand

The first IOAA was held from 30^{th} November to 9^{th} December 2007, to commemorate the 80^{th} birth anniversary of the King Bhumibol Adulyadej of Thailand and the 84^{th} birth anniversary of the Princess Galyani Vadhana of Thailand. Azerbaijan, Bangladesh, Belarus, Bolivia, Brazil, China, Greece, India, Indonesia, Iran, South Korea, Laos, Lithuania, Myanmar, Poland, Romania, Singapore, Slovakia, Sri Lanka, Thailand and Ukraine participated in the first IOAA. The International Board, composed by the leaders of every attending country, formally adopted statutes of IOAA. The international board also elected, for a five-year term, a President (Dr. Boonrucksar Soonthornthum, Thailand) and a General Secretary (Dr. Chatief Kunjaya, Indonesia).

<u>QUESTION 1.</u> (30 points for 15 short questions, 2 points for each short question)

Show, in a few steps in the writing sheets, your method of solution. Write your final answers in the answer sheets provided. Partial credits will be given for answers without showing method of solution.

1.1 For an observer at latitude 42.5^o N and longitude 71^o W, estimate the time of sun rise on 21 December if the observer's civil time is -5 hours from GMT. Ignore refraction of the atmosphere and the size of the solar disc.

1.2 The largest angular separation between Venus and the Sun, when viewed from the Earth, is 46^o. Calculate the radius of Venus's circular orbit in A.U.

1.3 The time interval between noon on 1 July and noon on 31 December is 183 solar days. What is this interval in sidereal days?

1.4 One night during a full Moon, the Moon subtends an angle of 0.46 degree to an observer. What is the observer's distance to the Moon on that night?

1.5 An observer was able to measure the difference in the directions, due to the Earth's motion around the Sun, to a star as distant as 100 parsecs away. What was the minimum angular difference in arc seconds this observer could measure?

1.6 A Sun-orbiting periodic comet is the farthest at 31.5 A.U. and the closest at 0.5 A.U.. What is the orbital period of this comet?

1.7 For the comet in question 1.6, what is the area (in square A.U. per year) swept by the line joining the comet and the Sun?

1.8 At what wavelength does a star with the surface temperature of 4000 K emit most intensely?

1.9 Calculate the total luminosity of a star whose surface temperature is 7500 K, and whose radius is 2.5 times that of our Sun. Give your answer in units of the solar luminosity, assuming the surface temperature of the Sun to be 5800 K.

1.10 A K star on the Main Sequence has a luminosity of $0.4\,L_{\odot}$. This star is observed to have a flux of 6.23×10^{-14} W.m^{-2}. What is the distance to this star? You may neglect the atmospheric effect.

1.11 A supernova shines with a luminosity 10^{10} times that of the Sun. If such a supernova appears in our sky as bright as the Sun, how far away from us must it be located?

1.12 The (spin-flip) transition of atomic hydrogen at rest generates the electromagnetic wave of the frequency $\nu_0 =$ 1420.406 MHz. Such an emission from a gas cloud near the galactic center is observed to have a frequency $\nu = 1421.65$ MHz. Calculate the velocity of the gas cloud. Is it moving towards or away from the Earth?

1.13 A crater on the surface of the Moon has a diameter of 80 km. Is it possible to resolve this crater with naked eyes, assuming the eye pupil aperture is 5 mm ?

1.14 If the Sun were to collapse gravitationally to form a non-rotating black hole, what would be its event horizon (its Schwarzschild radius)?

1.15 The magnitude of the faintest star you can see with naked eyes is $m = 6$, whereas that of the brightest star in the sky is $m = -1.5$. What is the energy-flux ratio of the faintest to that of the brightest?

QUESTION 2 **A PLANET & ITS SURFACE TEMPERATURE** **(10 points)**

A fast rotating planet of radius R with surface albedo α is orbiting a star of luminosity L. The orbital radius is D. It is assumed here that, at equilibrium, all of the energy absorbed by the planet is re-emitted as a blackbody radiation.

a.) What is the radiation flux from the star at the planet's surface? (1.5 points)

b.) What is the total rate of energy absorbed by the planet? (1.5 points)

c.) What is the reflected luminosity of the planet? (2 points)

d.) What is the average blackbody temperature of the planet's surface? (2 points)

e.) If we were to assume that one side of the planet is always facing the star, what would be the average surface temperature of that side? (2 points)

f.) For the planet in problem d:

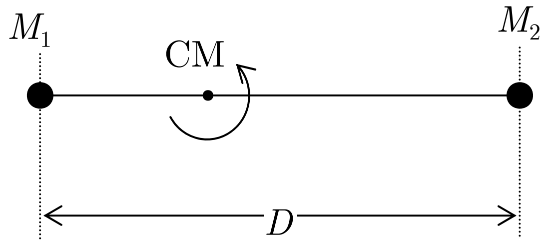
$\alpha = 0.25$,

$D = 1.523$ A.U.,

calculate its surface temperature in kelvins for the value of

$L = 3.826 \times 10^{26}$ W. (1 point)

QUESTION 3 BINARY SYSTEM (10 points)



A binary star system consists of M_1 and M_2 separated by a distance D. M_1 and M_2 are revolving with an angular velocity ω in circular orbits about their common centre of mass. Mass is continuously being transferred from one star to the other. This transfer of mass causes their orbital period and their separation to change slowly with time.

In order to simplify the analysis, we will assume that the stars are like point particles and that the effects of the rotation about their own axes are negligible.

a) What is the total angular momentum and kinetic energy of the system? (2 points)

b) Find the relation between the angular velocity ω and the distance D between the stars. (2 points)

c) In a time duration Δt, a mass transfer between the two stars results in a change of mass ΔM_1 in star M_1, find the quantity $\Delta\omega$ in terms of ω, M_1, M_2 and ΔM_1. (3 points)

d) In a certain binary system, $M_1 = 2.9\ \mathrm{M}_\odot$, $M_2 = 1.4\ \mathrm{M}_\odot$ and the orbital period, $T = 2.49$ days. After 100 years, the period T has increased by 20 s. Find the value of $\dfrac{\Delta M_1}{M_1 \Delta t}$ (in the unit "per year"). (1.5 points)

e) In which direction is mass flowing, from M_1 to M_2, or M_2 to M_1? (0.5 point)

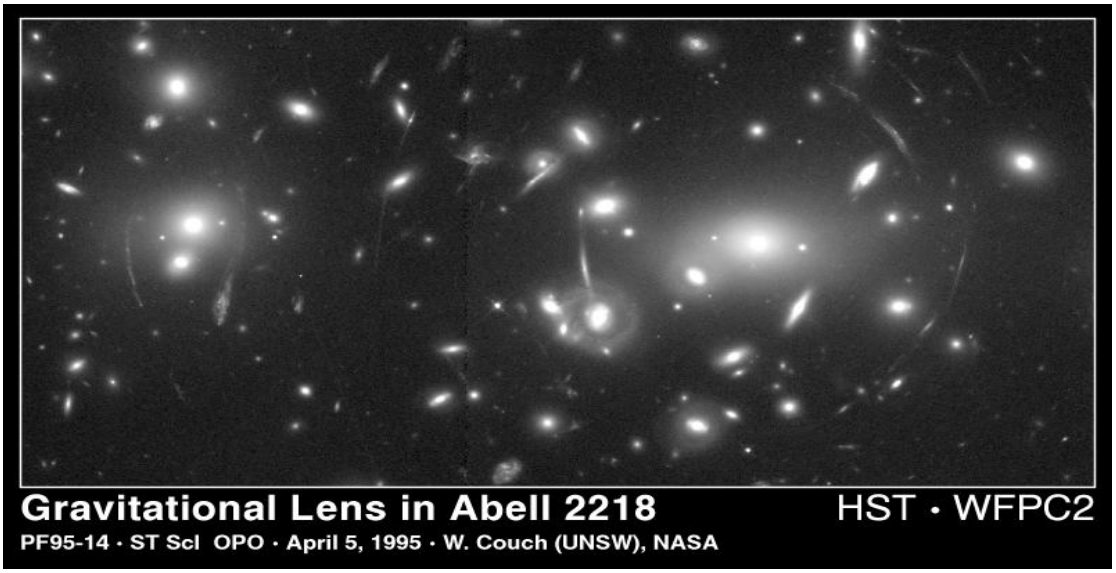
f) Find also the value of $\dfrac{\Delta D}{D\Delta t}$ (in the unit "per year"). (1 point)

You may use these approximations:

$(1+x)^n \sim 1+nx$, when $x \ll 1$;

$(1+x)\left(1+y\right) \sim 1+x+y$, when $x \ll 1$, $y \ll 1$.

QUESTION 4 GRAVITATIONAL LENSING (10 points)



The deflection of light by a gravitational field was first predicted by Einstein in 1912 a few years before the publication of the General Relativity in 1916.
A massive object that causes a light deflection behaves like a classical lens.
This prediction was confirmed by Sir Arthur Stanley Eddington in 1919.

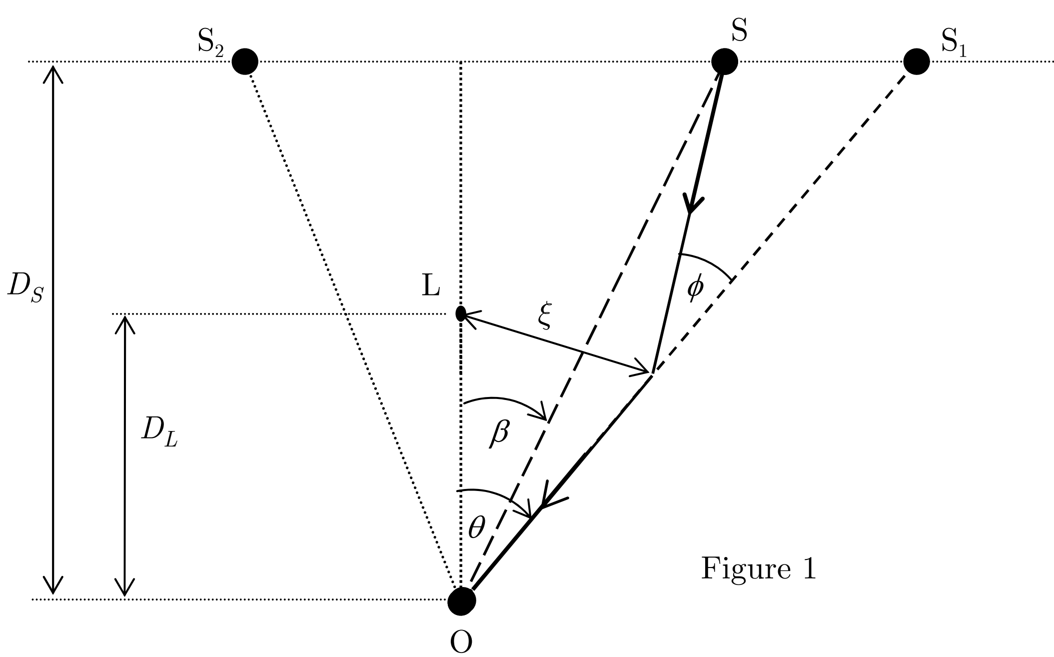


Figure 1

Consider a spherically symmetric lens, with a mass M with an impact parameter ξ from the centre. The deflection equation in this case is given by:

$$\phi = \frac{4GM}{\xi c^2} \quad \text{, a very small angle}$$

In figure 1, the massive object which behaves like a lens is at L. Light rays emitted from the source S being deflected by the lens are observed by observer O as images S_1 and S_2. Here, ϕ, β, and θ are very very small angles.

a) For a special case in which the source is perfectly aligned with the lens such that $\beta = 0$, show that a ring-like image will occur with the angular radius, called Einstein radius θ_E , given by:

$$\theta_E = \sqrt{\left(\frac{4GM}{c^2}\right)\left(\frac{D_S - D_L}{D_L D_S}\right)} \qquad \text{(2 points)}$$

b) The distance (from Earth) to a source star is about 50 kpc. A solar-mass lens is about 10 kpc from the star. Calculate the angular radius of the Einstein ring formed by this solar-mass lens with the perfect alignment. (1 point)

c). What is the resolution of the Hubble space telescope with 2.4 m diameter mirror? Could the Hubble telescope resolve the Einstein ring in b)? (2 points)

d). In figure 1, for an isolated point source S, there will be two images $\left(S_1 \text{ and } S_2\right)$ formed by the gravitational lens. Find the positions (θ_1 and θ_2)of the two images. Answer in terms of β and θ_E. (2 points)

e). Find the ratio $\frac{\theta_{1,2}}{\beta}$ ($\frac{\theta_1}{\beta}$ or $\frac{\theta_2}{\beta}$) in terms of η. Here $\theta_{1,2}$ represents each of the image positions in d.) and η stands for the ratio $\frac{\beta}{\theta_E}$. (2 points) f). Find also the values of magnifications $\dfrac{\Delta\theta}{\Delta\beta}$ in terms of η for $\theta = \theta_{1,2}$ ($\theta = \theta_1$ or $\theta = \theta_2$), when $\Delta\beta \ll \beta$, and $\Delta\theta \ll \theta$. (1 point)

The 1st International Olympiad on Astronomy and Astrophysics

Chiang Mai, Thailand

Experimental Competition (Data Analysis)

Monday, 3 December, 2007

Please read this first:

1. The time available is 3 hours for the experimental competition (Data analysis). There are three questions (and a set of data table).
2. Use only the pen provided.
3. Use only the front side of ***writing sheets***. <u>Write only inside the boxed area.</u>
4. Begin each question on a separate sheet.
5. For each question, in addition to the ***blank writing sheets***, there are the ***Answer Sheets*** where you *must* summarize the results you have obtained. Numerical results should be written with as many digits as are appropriate.
6. Write on the blank ***writing sheets*** whatever you consider is required for the solution of the question. Please use *as little text as possible*; express yourself primarily in equations, numbers, figures, and plots.
7. Fill the boxes at the top of each sheet of paper with your country code, your student code, the question number, for <u>each question</u> the consecutive number of each sheet (Page Number), and the total number of ***writing sheets*** used. If you use some blank ***writing sheets*** for notes that you do not wish to be marked, put a large X across the entire sheet and do not include it in your numbering.
8. Students given questions, writing sheet and answer sheets in English and national language can answer in any one sheet but must return both to the marker (examiner).

9. At the end of the exam, arrange all sheets for each problem *in the following order:*
   - *Answer Sheet(s)*
   - used *writing sheets* in order
   - the sheets you do not wish to be marked
   - unused sheets and the printed question

   Place the papers inside the envelope and leave everything on your desk. You are not allowed to take *any* sheets of paper out of the room.

Some useful information for calculation

Astronomical unit (A.U.)	149,597,870 km
Mean distance, Earth to Moon	384,399 km
Obliquity of the ecliptic	23° 26'
Earth's mean radius	6,371.0 km
Earth's mean velocity in orbit	29.783 km/s
Sidereal year	365.2564 days
Tropical year	365.2422 days
Sidereal month	27.3217 days
Synodic month	29.5306 days
Mean sidereal day	23h56m4s.091 of mean solar time
Mean solar day	24h3m56s.555 of sidereal time

<u>Question 1</u> **Galilean moons (4 points)**

Computer simulation of the planet Jupiter and its 4 Galilean moons is shown on the screen similar to the view you may see through a small telescope. After observing the movement of the moons, please identify the names of the moons that appear at the end of the simulation. (Simulation will be played on screen during the first fifteen minutes and the last fifteen minutes of the exam)

<u>Question 2</u> **The Moon’s age (8 points)**

The 60th anniversary celebrations of King Bhumibol Adulyadej's accession to the throne of Thailand (GMT +07) were held on the 8th to the 13th June, 2006. Photographs of the Moon taken at the same hour each night are shown below:

8th June, 2006

9th June, 2006

10th June, 2006

11th June, 2006

12th June, 2006

13th June, 2006

Assuming that Albert Einstein’s birth was at noon on 14th March, 1879, use the data provided above to find the Moon’s age (number of days after the new moon) on his birth date in Germany (GMT +01). Please show the method used for the calculation in detail. Estimate the errors in your calculation.

Question 3 **Solar System objects (8 points)**

A set of data containing the apparent positions of 4 Solar System objects over a period of 1 calendar year is given in Table 1. Show your method of data analysis carefully and answer the following questions.

Location of observer	Latitude :	N 18° 47' 00.0"
	Longitude :	E 98° 59' 00.0"

3.1 Put the letters A, B, C and D beside the appropriate objects on the answer sheet. **(2 points)**

3.2 During the period of observation, which object could be observed for the longest duration at night time? **(1 point)**

3.3 What was the date corresponding to the situation in 3.2? **(1 point)**

3.4 Assuming the orbits are coplanar (lie on the same plane) and circular, indicate the positions of the four objects and the Earth on the date in 3.3, in the orbit diagram provided in your answer sheet. The answer (sheet) must show one of the objects as the Sun at the centre of the Solar System. Other objects including the Earth must be specified together with the correct values of elongation on that date. **(4 points)**

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The 1st International Olympiad on Astronomy and Astrophysics

Chiang Mai, Thailand

Experimental Competition (Observation)

Sunday, 2 December, 2007

Please read this first:

1. There are 2 parts to the questions. You have to use the provided equipment to point, observe and answer where appropriate.
2. The time available is **40 minutes** in total for the experimental competition (Observation), **20 minutes for each part**.
3. In Part I, use only the provided celestial object pointer to point at the target specified in the questions. **No other pointer is allowed**. The marker (examiner) at each observation station will then mark the answers directly in the question sheet. You must not write anything in the sheet apart from your country code and student code.

4. In Part II, use the provided binoculars to observe the objects and then answer the questions by writing or drawing directly in the question sheets.
5. At the end of both parts, leave the question sheets with the marker (examiner) at each observation station. You are not allowed to take any sheets of paper out of the observation station.
6. Use only the provided pen or pencil.
7. Students given questions in English and national language can answer in any one sheet but must return both to the marker (examiner).
8. Fill the boxes at the top of each sheet of paper with your country code, your student code.

Country Code	Student Code

PART I: Use the provided celestial object pointer (total 10 points)

1.1 Move the pointer along the celestial equator. **(1 point)**

✓ ✗

1.2 Aim the pointer at the vernal equinox. **(1 point)**

✓ ✗

1.3 In the constellation of Pegasus and its vicinity there is an obvious square of bright stars (*Great Square of Pegasus*), aim the pointer at the brightest star of the square. **(2 points)**

✓ ✗

1.4 Aim the pointer at the star named alpha-Arietis (α-Ari). **(2 points)**

✓ ✗

1.5 Start from the star named Aldebaran (α-Tauri) in the constellation Taurus, turn the pointer 35 degrees northward followed by 6 degrees westward (in equatorial coordinate). Then, aim the pointer at the brightest star in the field of view. **(4 points)**

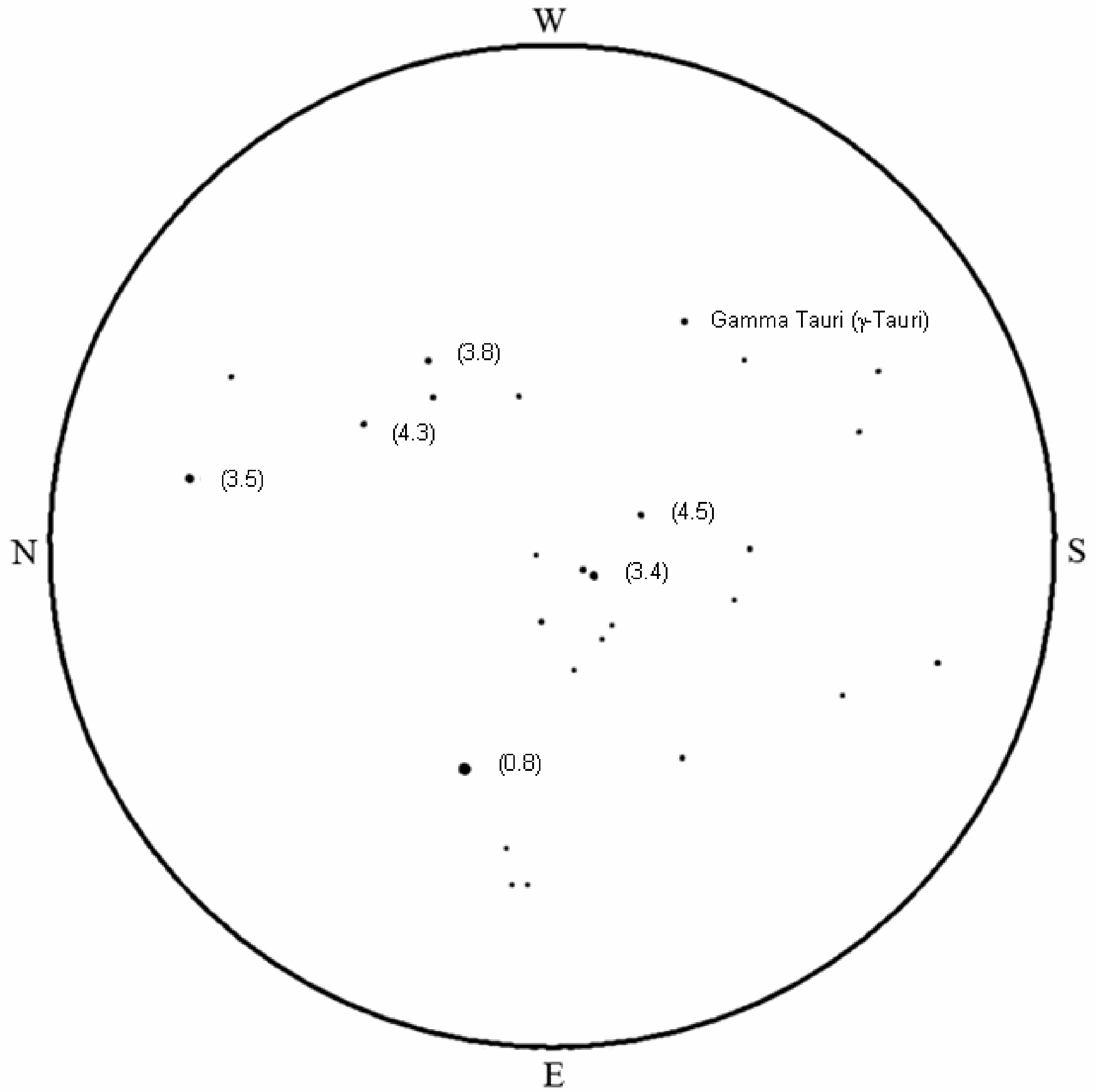
✓ ✗

Signature of Marker______________________________

Country Code	Student Code

PART II: Use the provided binoculars (total 10 points)

2.1 The open star cluster "Hyades" in constellation Taurus is one of the nearest clusters to us, being only 151 light years away. From the provided chart with brightness of some stars indicated by the apparent magnitude in parentheses, please estimate the apparent magnitude of the star Gamma-Tauri (γ-Tauri) to the nearest first decimal digit. **(5 points)**



7 X 50 Binocular FOV

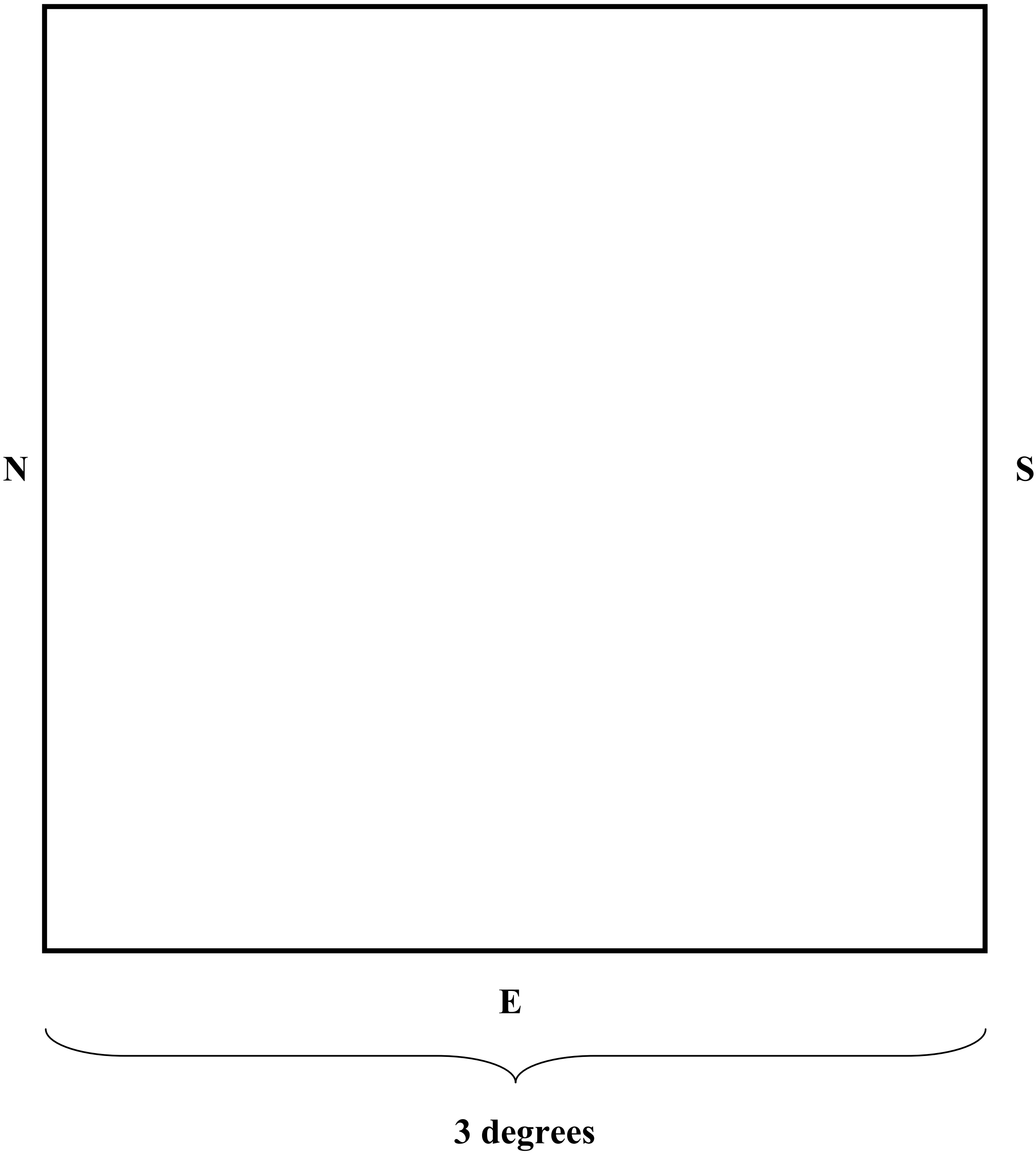
Answer: The apparent magnitude of γ-Tauri = _____________

Marker (examiner) comments on sky condition: _______________________________________

Country Code	Student Code

2.2 Observe the Andromeda Galaxy (M31) then draw the approximate **<u>shape and size</u>** of the galaxy that you see through the binoculars in the frame below with correct orientation (in equatorial coordinates). The field of view of the binoculars is 6.8 degrees. **(5 points)**

W



Marker (examiner) comments on sky condition: ____________________________________

IOAA 2008 – Bandung, Indonesia

The second IOAA was from 19$^{\text{th}}$ to 28$^{\text{th}}$ August 2008, with teams from 22 countries including a team from Cambodia and an observer from Malaysia.

The 2nd International Olympiad on Astronomy and Astrophysics

Bandung, Indonesia

Saturday, 23 August 2008

Theoretical Competition

Please read this carefully:

1. *Every student receives problem sheets in English and/or in native language, an answer book and a scratch book.*
2. *The time available is 5 hours for the theoretical competition. There are fifteen short questions (Theoretical Part 1), and three long questions (Theoretical Part 2).*
3. *Use only Black or dark blue pen*
4. *Use only the front side of* ***answer sheets****. Write only inside the boxed area.*
5. ***Begin answering each question on a separate sheet.***
6. *Numerical results should be written with as many digits as are appropriate.*
7. *Write on the blank* ***answer sheets*** *whatever you consider is required for the solution of each question. Please express your answer primarily in term of equations, numbers, figures, and plots. If necessary provide your answers with concise text. Full credit will be given to correct answer with detailed steps for each question. Underline your final result.*
8. *Fill in the boxes at the top of each sheet of paper with your country code and your student code.*
9. *At the end of the exam place the books inside the envelope and leave everything on your desk.*

Astronomical and Physical Constants

Quantity	Value
Astronomical unit (AU)	149 597 870 691 m
Light year (ly)	9.4605×10^{15} m = 63,240 AU
Parsec (pc)	3.0860×10^{16} m = 206,265 AU
Sidereal year	365.2564 days
Tropical year	365.2422 days
Gregorian year	365.2425 days
Sidereal month	27.3217 days
Synodic month	29.5306 days
Mean sidereal day	$23^h56^m4^s.091$ of mean solar time
Mean solar day	$24^h3^m56^s.555$ of sidereal time
Mean distance, Earth to Moon	384 399 000 m
Earth mass ($M_\oplus$)	5.9736×10^{24} kg
Earth mean radius	6 371 000 m
Earth mean velocity in orbit	29 783 m/s
Moon mass ($M_☽$)	7.3490×10^{22} kg
Moon mean radius	1 738 000 m
Sun mass ($M_\odot$)	1.9891×10^{30} kg
Sun radius ($R_\odot$)	6.96×10^{8} m
Sun luminosity ($L_\odot$)	3.96×10^{26} J s^{-1}
Sun effective temperature ($T_{eff\odot}$)	5 800 ^{o}K
Sun apparent magnitude ($m_\odot$)	-26.8
Sun absolute magnitude ($M_\odot$)	4.82
Sun absolute bolometric magnitude ($M_{bol\odot}$)	4.72
Speed of light (c)	2.9979×10^{8} m/s
Gravitational constant (G)	6.6726×10^{-11} N m^2 kg^{-2}
Boltzmann constant (k)	1.3807×10^{-23} J K^{-1}
Stefan-Boltzmann constant (σ)	5.6705×10^{-8} J s^{-1} m^{-2} K^{-4}
Planck constant (h)	6.6261×10^{-34} J s
Electron charge (e)	1.602×10^{-19} C = 4.803×10^{-10} esu
Electron mass (m_e)	$5.48579903 \times 10^{-4}$ amu = 9.11×10^{-31} kg
Proton mass (m_p)	1.00727647 amu = 1.67268×10^{-27} kg

Neutron mass (m_n)	1.008664904 amu = 1.67499×10^{-27} kg
Deuterium nucleus mass (m_d)	2.013553214 amu = 3.34371×10^{-27} kg
Hydrogen mass	1.00794 amu = 1.67379×10^{-27} kg
Helium mass	4.002603 amu = 1.646723×10^{-27} kg

Conversion table

1 Å	0.1 nm = 10^{-10} m
1 barn	10^{-28} m^2
1 G	10^{-4} T
1 erg	10^{-7} J = 1 dyne cm
1 esu	3.3356×10^{-10} C
1 amu (atomic mass unit)	1.6606×10^{-27} kg
1 atm (atmosphere)	101,325 Pa = 1.01325 bar
1 dyne	10^{-5} N

THEORETICAL PART 1

(300 points for 15 Theoretical Part-1 questions, 20 points for each question)

Show your method of solution step by step in the answer sheets completely as your final answer. The scratch sheet is to be used for your personal calculation and will not be marked. Partial credits will be given for answers without showing method of solution.

1. Two persons, on the equator of the Earth separated by nearly 180° in longitude, observe the Moon's position with respect to the background star field at the same time. If the declination of the Moon is zero, sketch the situation and calculate the difference in apparent right ascension seen by those two persons.

2. On April 2, 2008 a telescope (10 cm diameter, *f*/10) at the Bosscha Observatory was used to observe the Sun and found an active region 0987 (based on the NOAA number) at 8° South and 40° West from the center of the solar disk. The region was recorded with a CCD SBIG ST-8 Camera (1600 × 1200 pixels, (9 μm × 9 μm)/ pixel) and its size was 5 × 4 pixels. According to the Astronomical Almanac, the solar diameter is 32'. How large is the corrected area of the active region in unit of millionth of solar hemisphere (msh)?

3. A full moon occurred on June 19, 2008 at $00^h\ 30^m$ West Indonesian Time (local civil time for western part of Indonesia with meridian of 105° E). Calculate the minimum and maximum possible values of duration of the Moon above the horizon for observers at Bosscha Observatory (longitude: 107° 35' 00″.0 E, latitude: 6° 49' 00″.0 S, Elevation: 1300.0 m). Time zone = UT $+7^h\ 00^m$.

4. Suppose a star has a mass of 20 $\mathfrak{M}_\odot$. If 20% of the star's mass is now in the form of helium, calculate the helium-burning lifetime of this star. Assume that the luminosity of the star is 100 $L_\odot$, in which 30% is contributed by helium burning. The carbon mass, ^{12}C, is 12.000000 amu. Helium burning to Carbon: $3\ ^4He \longrightarrow\ ^{12}C + \gamma$.

5. The average temperature of the Cosmic Microwave Background (CMB) is currently T = 2.73 K, and it yields the origin of CMB to be at redshift z_{CMB} =1100. The current densities of the Dark Energy, Dark Matter, and Normal Matter components of the Universe as a whole are ρ_{DE} = 7.1 × 10^{-30} g/cm^3, ρ_{DM} = 2.4 × 10^{-30} g/cm^3, and

$\rho_{NM} = 0.5 \times 10^{-30}$ g/cm^3, respectively. What is the ratio between the density of Dark Matter to the density of Dark Energy at the time CMB was emitted, if we assume that the dark energy is vacuum energy?

6. Radio wavelength observations of gas cloud swirling around a black hole in the center of our galaxy show that radiation from the hydrogen spin-flip transition (rest frequency = 1420.41 MHz) is detected at a frequency of 1421.23 MHz. If this gas cloud is located at a distance of 0.2 pc from the black hole and is orbiting in a circle, determine the speed of this cloud and whether it's moving toward or away from us and calculate the mass of the black hole.

7. A main sequence star at a distance 20 pc is barely visible through a certain space-based telescope which can record all wavelengths. The star will eventually move up along the giant branch, during which time its temperature drops by a factor of 3 and its radius increases 100 times. What is the new maximum distance at which the star can still be (barely) visible using the same telescope?

8. Gravitational forces of the Sun and the Moon lead to the raising and lowering of sea water surfaces. Let φ be the difference in longitude between points A and B, where both points are at the equator and A is on the sea surface. Derive the horizontal acceleration of sea water at position A due to Moon's gravitational force at the time when the Moon is above point B according to observers on the Earth (express it in φ, the radius $\boldsymbol{R}$ of Earth, and the Earth-Moon distance r).

9. The radiation incoming to the Earth from the Sun must penetrate the Earth's atmosphere before reaching the earth surface. The Earth also releases radiation to its environment and this radiation must penetrate the Earth's atmosphere before going out to the outer space. In general, the transmittance (t_1) of the Sun radiation during its penetration through the Earth's atmosphere is higher than that of the radiation from the Earth (t_2). Let $T_{\mathrm{eff}\,\odot}$ be the effective temperature of the Sun, $R_\odot$ the radius of the Sun, $r_\oplus$ the radius of the Earth, and x the distance between the Sun and the Earth. Derive the temperature of the Earth's surface as a function of the aforementioned parameters.

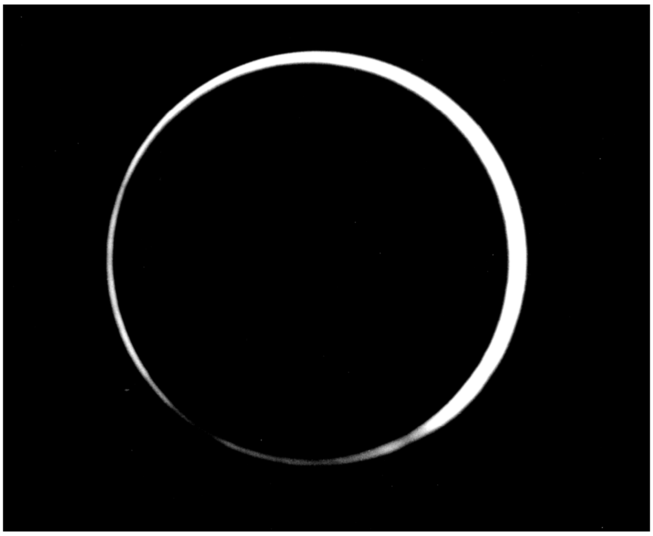
10. The coordinates of the components of Visual Binary Star μ Sco on August 22, 2008 are given in the table below

	α(RA)	δ(Dec)
μ Sco 1 (primary)	$20^h\ 17^m\ 38^s.90$	-12° 30' 30"
μ Sco 2 (secondary)	$20^h\ 18^m\ 03^s.30$	-12° 32' 41"

The stars are observed using Zeiss refractor telescope at the Bosscha Observatory with aperture and focal length are 600 mm and 10 780 mm, respectively. The telescope is equipped with 765 × 510 pixels CCD camera. The pixel size of the chip is 9 μm × 9 μm.

a. Can both components of the binary be inside the frame? ("YES" or "NO", show it in your computation!)
b. What is the position angle of the secondary star, with respect to the North?

11. Below is a picture on a 35 mm film of annular solar eclipse in Dumai, Riau, Indonesia on August 22, 1998, taken with a telescope having effective diameter 10 cm and f-ratio 15. The diameter of the Sun's disk in original picture on the film is 13.817 mm and the diameter of the Moon's disk is 13.235 mm. Determine the distances of the Sun and the Moon (expressed in km) from the Earth and the percentage of the solar disk covered by the Moon during the annular solar eclipse.



12. Consider a type Ia supernova, in a distant galaxy, which has a luminosity of $5.8 \times 10^9 L_\odot$ at maximum light. Suppose you observe this supernova using your telescope and find that its brightness is 1.6×10^{-7} times the brightness of Vega. The redshift of its host galaxy is known to be $z = 0.03$. Calculate the distance of this galaxy (in pc) using the data of the supernova and also the Hubble time.

13. In the journey of a space craft, scientists make a close encounter with an object and they would like to investigate the object more carefully using their on-board telescope. For simplicity, we assume this to be a two-dimensional problem and that the position of the space craft is stationary in (0,0). The shape of the object is a disk and the boundary has the equation

$$x^2 + y^2 - 10x - 8y + 40 = 0.$$

Find the exact values of maximum and minimum of tan φ where φ is the angle of the telescope with respect to the x direction during investigation from one edge to the other edge.

14. Consider a Potentially Hazardous Object (PHO) moving in a closed orbit under the influence of the Earth's gravitational force. Let u be the inverse of the distance of the object from the Earth and p be the magnitude of its linear momentum. As the object travels, the graph of u as a function of p passes through points A and B as shown in the following table. Find the mass and the total energy of the object, and express u as a function of p and sketch the shape of u curve from A to B.

	p ($\times 10^9$ kg m s^{-1})	u ($\times 10^{-8}$ m^{-1})
A	0.052	5.15
B	1.94	194.17

15. Galaxy NGC 2639 is morphologically identified as an Sa galaxy with measured maximum rotational velocity $v_{\max}$ of 324 km/s. After corrections for any extinction, its apparent magnitude in B is $m_B = 12.22$. It is customary to measure a radius R_{25} (in units of kpc) at which the galaxy's surface brightness falls to 25 $\text{mag}_B/\text{arcsec}^2$. Spiral galaxies tend to follow a typical relation:

$$\log R_{25} = -0.249 M_B - 4.00,$$

where M_B is the absolute magnitude in B. Apply the B-band Tully-Fisher relation for Sa spirals

$$M_B = -9.95 \log v_{max} + 3.15 \quad (v_{max} \text{ in km/s})$$

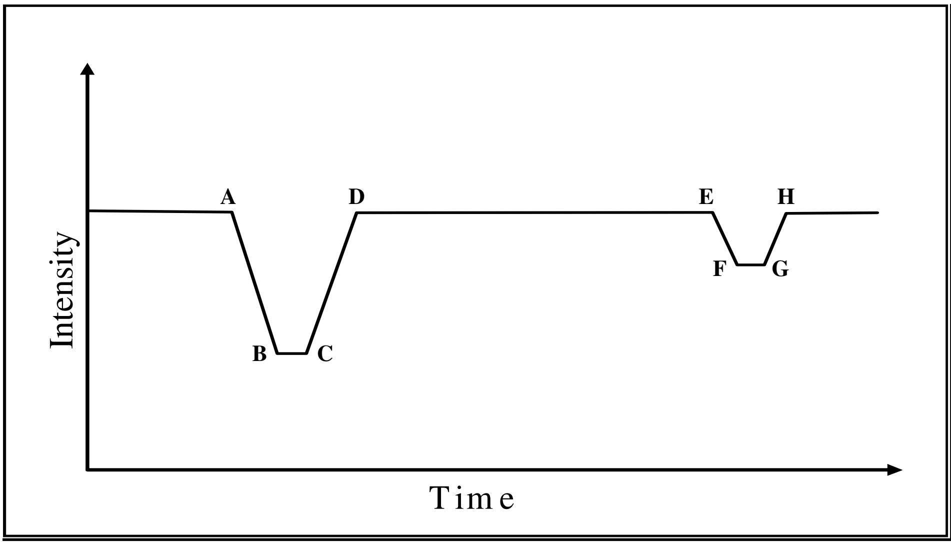
to calculate the mass of NGC 2639 out to R_{25}. If colour index of the sun is $(m_{B_\odot} - m_{V_\odot}) = 0.64$, write the mass (of NGC 2639) in units of solar mass $\mathfrak{M}_\odot$ and its luminosity B-band in unit of $L_\odot$.

THEORETICAL PART 2

(300 points for 3 Theoretical Part-2, 100 points for each question)

Show your method of solution step by step in the answer sheets completely as your final answer. The scratch sheet is to be used for your personal calculation and will not be marked. Partial credits will be given for answers without showing method of solution.

1. An eclipsing binary star system has a period of 30 days. The light curve in the figure below shows that the secondary star eclipses the primary star (from point A to point D) in 8 hours (measured from the time of first contact to final contact), whereas from point B to point C, the total eclipse period is 1 hour and 18 minutes. The spectral analysis yields the maximum radial velocity of the primary star to be 30 km/s and of the secondary star to be 40 km/s. If we assume that the orbits are circular and has an inclination of $i = 90^o$, determine the radii and the masses of both stars in unit of solar radius and solar mass.



2. A *UBV* photometric (*UBV* Johnson's) observation of a star gives U = 8.15, B = 8.50, and V = 8.14. Based on the spectral class, one gets the intrinsic color $(U - B)_o$ = -0.45. If the star is known to have radius of 2.3 $R_\odot$, absolute bolometric magnitude of -0.25, and bolometric correction (BC) of -0.15, determine:
   a. the intrinsic magnitudes *U, B*, and *V* of the star (take, for the typical interstellar matters, the ratio of total to selective extinction (color excess) R_V = 3.2),
   b. the effective temperature of the star,
   c. the distance to the star in pc.

Note: The relation between color excess of U - B and of $B - V$ is $E(U - B) = 0.72\ E(B - V)$.

Let A_v be the interstellar extinction and $R = 3.2$, then $A_v = 3.2\ E(B\text{-}V)$.

3. Measurement of the cosmic microwave background radiation (CMB) shows that its temperature is practically the same at every point in the sky to a very high degree of accuracy. Let us assume that light emitted at the moment of recombination ($T_r \approx$ 3000 K, $t_r \approx$ 300000 years) is only reaching us now ($T_o \approx$ 3 K, $t_o \approx 1.5 \times 10^{10}$ years). Scale factor S is defined as such $S_0 = S\left(t = t_o\right) = 1$ and $S_t = S\left(t < t_o\right) < 1$. Note that the radiation dominated period was between the time when the inflation stopped ($t = 10^{-32}$ seconds) and the time when the recombination took place, while the matter dominated period started at the recombination time. During the radiation dominated period S is proportional to $t^{1/2}$, while during the matter dominated period S is proportional to $t^{2/3}$.

a. Estimate the horizon distances when recombination took place. Assume that temperature T is proportional to $1/S$, where S is a scale factor of the size of the Universe.

b. Note: Horizon distance in degrees is defined as maximum separation between the two points in CMBR imprint such that the points could "see" each other at the time when the CMBR was emitted.

c. Consider two points in CMBR imprint which are currently observed at a separation angle $\alpha = 5°$. Could the two points communicate with each other using photon? (Answer with "YES" or "NO" and give the reason mathematically)

d. Estimate the size of our Universe at the end of inflation period.

The 2nd International Olympiad on Astronomy and Astrophysics

Bandung, Indonesia

Thursday, 21 August 2008

Practical Competition: Data Analysis

Please read this carefully:

1. *Every student receives problem sheets in English and/or in his/her native language, answer sheets, millimeter block papers, and scratch sheets.*
2. *The time available is five hours for the data analysis and observation competitions. There are three data analysis problems, and two observation problems.*
3. *Use only the materials provided.*
4. *Fill in the boxes at the top of each sheet of paper with your country code and your student code.*
5. *Use only the front side of* ***answer sheets****. Write only inside the boxed area.*
6. *Begin answering each question on a separate sheet.*
7. *Numerical results should be written with as many digits as are appropriate.*
8. *Write on* ***the answer sheets and the millimeter block papers*** *whatever you consider is required for the solution of each question. Please express your answer primarily in term of equations, numbers, figures, and plots. If necessary provide your answers with concise text.* ***.*** *Full credit will be given to correct answer with detailed steps for each question. Underline your final result.*
9. *At the end of the exam place the answer sheets and the millimeter block papers inside the envelope and leave everything on your desk.*

Astronomical and Physical Constants

Quantity	Value
Astronomical unit (AU)	149,597,870.691 km
Light year (ly)	9.4605×10^{17} cm = 63,240 AU
Parsec (pc)	3.0860×10^{18} cm = 206,265 AU
Sidereal year	365.2564 days
Tropical year	365.2422 days
Gregorian year	365.2425 days
Sidereal month	27.3217 days
Synodic month	29.5306 days
Mean sidereal day	$23^h56^m4^s.091$ of mean solar time
Mean solar day	$24^h3^m56^s.555$ of sidereal time
Mean distance, Earth to Moon	384,399 km
Earth mass ($\mathfrak{M}_\oplus$)	5.9736×10^{27} g
Earth's mean radius	6,371.0 km
Earth's mean velocity in orbit	29.783 km/s
Moon's mass ($\mathfrak{M}_☽$)	7.3490×10^{25} g
Moon's mean radius	1,738 km
Sun mass ($\mathfrak{M}_\odot$)	1.9891×10^{33} g
Mean Earth radius	6.3710×10^{6} cm
Sun radius ($R_\odot$)	6.96×10^{10} cm
Sun luminosity ($L_\odot$)	3.96×10^{33} erg s^{-1}
Sun effective temperature ($T_{eff\odot}$)	5 800 ^{o}K
Sun apparent magnitude ($m_\odot$)	-26.8
Sun bolometric magnitude ($m_{bol\odot}$)	-26.79
Sun absolute magnitude ($M_\odot$)	4.82
Sun absolute bolometric magnitude ($M_{bol\odot}$)	4.72
Speed of light (c)	2.9979×10^{10} cm/s
Gravitational constant (G)	6.6726×10^{-8} dyne cm^2 g^{-2}
Boltzmann constant (k)	1.3807×10^{-16} erg. K^{-1}
Stefan-Boltzmann constant (σ)	5.6705×10^{-5} erg cm^{-2} K^{-4} s^{-1}
Planck constant (h)	6.6261×10^{-27} erg s
Electron charge (e)	1.602×10^{-19} C = 4.803×10^{-10} esu
Electron mass (m_e)	$5.48579903 \times 10^{-4}$ amu

Proton mass (m_p)	1.007276470 amu
Neutron mass (m_n)	1.008664904 amu
Deuterium nucleus mass (m_d)	2.013553214 amu
Hydrogen mass	1.00794 amu
Helium mass	4.002603 amu
Carbon mass	12.01070 amu

Conversion table	
1 Å	0.1 nm
1 barn	10^{-28} m^2
1 G	10^{-4} T
1 erg	10^{-7} J = 1 dyne cm
1 esu	3.3356×10^{-10} C
1 amu (atomic mass unit)	1.6606×10^{-24} g
1 atm (atmosphere)	101,325 Pa = 1.01325 bar
1 dyne	10^{-5} N

300 points for 3 problems, 100 points for each problem

I. Virgo Cluster

The Virgo cluster of galaxies is the nearest large cluster which extends over nearly 10 degrees across the sky and contains a number of bright galaxies. It is very interesting to find the distance to Virgo and to deduce certain cosmological information from it. The table below provides the distance estimates using various distance indicators (listed in the left column). The right column lists the mean distance $d_i \pm$ the standard deviation s_i.

i	**Distance Indicator**	**Virgo Distance (Mpc**
1	Cepheids	14.9 ± 1.2
2	Novae	21.1 ± 3.9
3	Planetary Nebulae	15.2 ± 1.1
4	Globular Cluster	18.8 ± 3.8
5	Surface Brightness Fluctuation	15.9 ± 0.9
6	Tully-Fisher relation	15.8 ± 1.5
7	Faber-Jackson relation	16.8 ± 2.4
8	Type Ia Supernovae	19.4 ± 5.0

1. By applying a weighted mean, compute the average distance (which can be taken as an estimate to the distance to Virgo)

$$d_{avg} = \frac{\sum_i \frac{d_i}{s_i^2}}{\sum_i \frac{1}{s_i^2}}$$

   where the sum runs over the eight distance indicator used.

2. What is the uncertainty (rms) (in unit of Mpc) in that estimate?
3. Spectra of the galaxies in Virgo indicate an average recession velocity of 1136 km/sec for the cluster. Can you estimate the Hubble constant H_0 and its uncertainty (rms)?
4. What is the Hubble Time (age of the universe) using the value of Hubble constant you found and the uncertainty (rms)?

II. Determination of stellar masses in a visual binary system

The star α-Centauri (Rigel Kentaurus) is a triple star which consists of two main-sequence stars α-Centauri A and α-Centauri B representing visual binary system, and the third star, called Proxima Centauri, which is smaller and fainter than the other two stars. The angular distance between α-Centauri A and α-Centauri B is 17.59”. The binary system has an orbital period of 79.24 years. The visual magnitudes of α-Centauri A and α-Centauri B are -0.01 and 1.34 respectively. Their color indices are 0.65 and 0.85 respectively. Use the data below to answer the following questions.

Data for main-sequence stars

$(B-V)_0$	$T_{\rm eff}$	BC
-0.25	24500	2.30
-0.23	21000	2.15
-0.20	17700	1.80
-0.15	14000	1.20
-0.10	11800	0.61
-0.05	10500	0.33
0.00	9480	0.15
0.10	8530	0.04
0.20	7910	0
0.30	7450	0
0.40	6800	0
0.50	6310	0.03
0.60	5910	0.07
0.70	5540	0.12
0.80	5330	0.19
0.90	5090	0.28
1.00	4840	0.40
1.20	4350	0.75

BC=Bolometric Correction, $(B-V)_0$=Intrinsic Color

Questions:

1. Plot the curve BC versus $(B-V)_0$.

2. Determine the apparent bolometric magnitudes of α-Centauri A and α-Centauri B using the corresponding curve.
3. Calculate the mass of each star.

Notes:

1. **Bolometric correction** (BC) is a correction that must be made to the apparent magnitude of an object in order to convert an object's visible magnitude to its bolometric magnitude:

$$BC = m_v - m_{bol} \text{ or } BC = M_v - M_{bol}$$

2. **Luminosity mass relation** : $M_{bol} = -10.2 \log\left(\frac{M}{M_\odot}\right) + 4.9$

III. The Age of Meteorite

The basic equation of radioactive decay can be expressed as:

$$N(t) = N_0 \exp(-\lambda t)$$

where $N(t)$ and N_0 are the number of remaining atoms of the radioactive isotope (or parent isotope) at time t and its initial number at $t = 0$, respectively, while λ is the decay constant. The decay of the parent produces daughter nuclides $D(t)$, or radiogenics, which is defined as

$$D(t) = N_0 - N(t) .$$

Based on those ideas, a group of astronomers investigates a number of meteorite samples to determine their ages. They have two kinds of samples: allende chondrite (A) and basaltic achondrite (B). From the samples, they measure the abundance of ^{87}Rb and ^{87}Sr, where it is assumed that ^{87}Sr is entirely produced by the decay of ^{87}Rb. The value of λ is 1.42×10^{-11} per year for this isotopic decay. In addition, non-radiogenic element ^{86}Sr is also measured. Results of measurement are given in the table below, expressed in ppm (part per million).

Sample No	Meteorite type	^{86}Sr (ppm)	^{87}Rb (ppm)	^{87}Sr (ppm)
1	A	29.6	0.3	20.7
2	B	58.7	68.5	44.7
3	B	74.2	14.4	52.9
4	A	40.2	7.0	28.6
5	A	19.7	0.4	13.8
6	B	37.9	31.6	28.4
7	A	33.4	4.0	23.6
8	B	29.8	105.0	26.4
9	A	9.8	0.8	6.9
10	B	18.5	44.0	15.4

<u>Questions:</u>

1. Express the time t in term of $\dfrac{D(t)}{N(t)}$
2. Determine the half-life $t_{1/2}$, i.e., the time needed to obtain a half number of parents after decay.
3. Knowledge on the ratio between two isotopes is more valuable than just the absolute abundance of each isotope. It is quite likely that there was some initial strontium present. By taking $\left(\dfrac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right)$ as independent variable and $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)$ as dependent variable, estimate the simple linear regression model to represent the data.
4. Plot $\left(\dfrac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right)$ versus $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)$ and also the regression line (isochrone) for each type of the meteorites. (Please use minimum 7 decimal digits for intermediate calculations)
5. Subsequently, help this astronomer to determine the age of each type of the meteorites and its error. Which type is older?
6. Determine the initial value of $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)_0$ for each type of the meteorites and its error.

Glossary:

A simple linear regression line $y=a+bx$ can be fitted to a set of data (X_i,Y_i), $i=1,\ldots,n$, in which

$$b = \frac{SS_{xy}}{SS_{xx}}$$

$$a = \bar{y} - b\bar{x}$$

where

SS_{xx} : sum of square for $X = \sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2$

SS_{yy} : sum of square for $Y = \sum_{i=1}^{n} Y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} Y_i\right)^2$

SS_{xy} : sum of square for both X and $Y = \sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i$

Standard deviation of each parameter, a and b can be calculated by

$$S_a = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}} \times \sum_{i=1}^{n} X_i^2}$$

$$S_b = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}}}$$

2nd International Olympiad on Astronomy and Astrophysics

The 2nd International Olympiad on Astronomy and Astrophysics

Bandung, Indonesia

Thursday, August 21, 2008

Practical Competition: Observations

PART I: Naked Eye

Instructions:

1. *All participants will receive a naked-eye observation problem set, a writing board, a pen, a ruler and a flash light at the examination room.*
2. *Part I consists of three steps: distribution of problem sheets in examination room, naked eye observation on the observing ground (7 minutes), and returning to the examination room to answer questions (8 minutes). Bell ring will indicate the beginning and the end of each step.*
3. *All participants will be guided by assistants to go to the observing ground until return to the examination room. Assistants will collect the answer sheets from your table, after the 8 minute time is up.*
4. *Never forget to fill in the boxes at the top of each answer sheet with your country and your student codes. Otherwise, it will be ignored.*

Country Code	Student Code

PROBLEM

Figure 1 shows a part of the southern sky chart for August 21, 2008 at 07.00 p.m. local time. Unfortunately, a number of bright stars in Capricorn and Scorpio constellations are missing. Now, you have to find those missing bright stars in both constellations by looking at the sky directly. To help you remember, the common names of many bright stars are listed in Table 1. Draw small circles on the locations of the missing bright stars in the Capricorn and Scorpio constellations (Point: 60) and identify them by **putting the numbers on the sky chart**, as many as possible based on the Table 1 (Point: 60). Afterwards, draw on the sky chart, the border of Scorpio constellation (Point: 15) and Capricorn constellation (Point: 15).

Table 1: The number and common name of bright stars

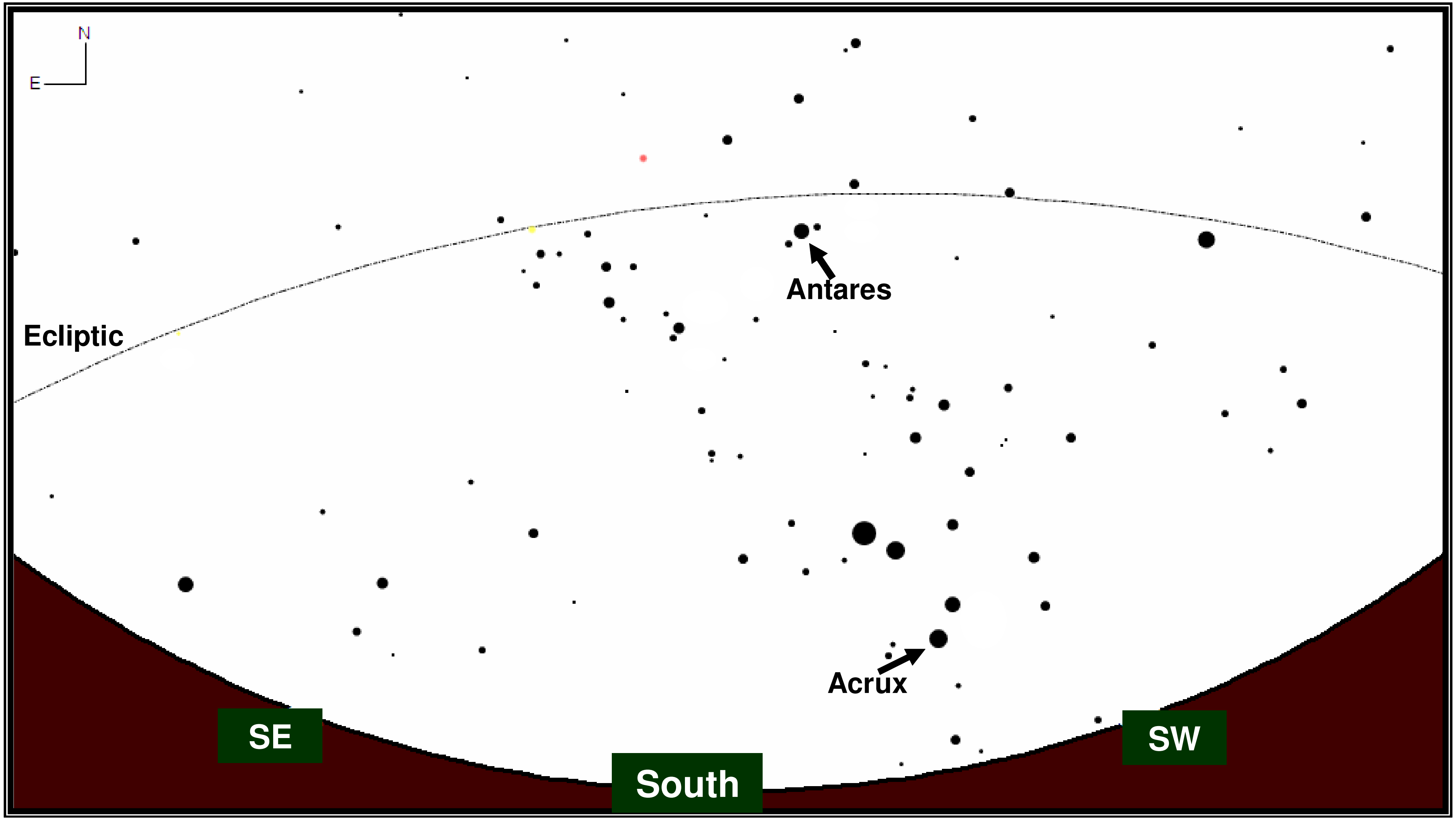
Number	Common Names	Number	Common Names
1	Rukbat (α Sgr)	18	Albali (ε Aqr)
2	Graffias (β Sco)	19	Altair (α Aql)
3	Nunki (σ Sgr)	20	Shaula (λ Sco)
4	Deneb (α Cyg)	21	Vrischika (π Sco)
5	Zaniah (η Vir)	22	Arich (γ Vir)
6	Tarazed (γ Aql)	23	Deneb Algedi (δ Cap)
7	Dabih (β Cap)	24	Heze (ζ Vir)
8	Girtab (κ Sco)	25	Nusakan (β CrB)
9	Spica (α Vir)	26	Wei (ε Sco)
10	Sabik (η Oph)	27	Syrma (ι Vir)
11	Dschubba (δ Sco)	28	Nashira (γ Cap)
12	Kaus Australis (ε Sgr)	29	Lesath (υ Sco)
13	Algiedi (α Cap)	30	Zavijava (β Vir)
14	Sadr (γ Cyg)	31	Arcturus (α Boo)
15	Vindemiatrix (ε Vir)	32	Megrez (δ UMa)
16	Antares (α Sco)	33	Chara (β CVn)
17	Yen (ζ Cap)	34	Sargas (θ Sco)

PART I: Naked Eye

Sky chart at 19.00 on August 21, 2008

Country and Student Code: ....

Figure 1



The 2nd International Olympiad on Astronomy and Astrophysics

Bandung, Indonesia

Thursday, August 21, 2008

Practical Competition: Observations

PART II: Using Telescope and CCD

Instructions:

1. *Part II consists of four steps: distributions of problem sheets in examination room, using telescope and CCD observations on the observing ground (20 minutes), print images and return to the examination room to answer questions (10 minutes). Bell ring will indicate the beginning and the end of each step. Works completed after the allocated time will not be considered.*
2. *All participants will be guided by assistants to go to the observing ground until return to the examination room. Assistants will collect the answer sheets from your table.*
3. *Never forget to fill in the boxes at the top of each answer sheet with your country and your student codes. Otherwise, it will be ignored.*

Country Code	Student Code

PROBLEM

Participants have to identify as many stars as possible in the field of a celestial photograph using the telescope and CCD provided by the committee. Participants have to choose one out of the five recommended regions in the sky listed below. Then, point the telescope to the direction of the selected sky region. Take three photographs with different exposure times and record the images of the sky by the CCD camera. Save the observational data. Transfer the data to the printing facilities to print out the result. Ask the technical assistant for a help. Choose the best prints-out and use the image to identify the stars in the field of observations. The following procedures are,

1. Choose only one out of the following five recommended regions to the directions of (marked by the following bright clusters):
   i. M7 ($\alpha=17^h53.^m3$, $\delta= -34^o46.'2$)
   ii. M8 ($\alpha=18^h04.^m2$, $\delta= -24^o22.'0$)
   iii. M20 ($\alpha=18^h02.^m4$, $\delta= -22^o58.'7$)
   iv. M21 ($\alpha=18^h04.^m2$, $\delta= -22^o29.'5$)
   v. M23 ($\alpha=17^h57.^m0$, $\delta= -18^o59.'4$)

   You may not change your choice.
2. Type in your country code, your student code, and your choice of region into the answer page provided in the computer.
3. Point the telescope to the chosen cluster by using telescope controller. If necessary you may move the telescope slightly to get the best position in the frame of the CCD by checking the display of CCDops software.
4. Display the region in The Sky map software provided in the computer, to confirm that the telescope is pointed to the selected object in the sky. You may change field of view of the sky chart.
5. You may invert the background images into white color, as in the chart mode. Copy and paste the sky chart from The Sky into the answer page. Use "Ctrl c" and "Ctrl v" buttons from the keyboard, respectively.

6. Type the equatorial coordinates of the centre of that object in the answer page as indicated in The Sky.
7. Take three photographs of the chosen object by using the attached CCD camera and CCDops software, with various exposure times.

   Choose exposure time in the range between 1 and 120 seconds. Image is automatically subtracted with dark frame with the same exposure time.
8. You must invert the background images into white color. Copy and paste images from CCDops into the answer page. Use “Ctrl c” and “Ctrl v” buttons from the keyboard, respectively.
9. Save your answer page into hard disk in a Microsoft Word file format.
10. Print your answer page which consists of the photographs and the corresponding sky chart.
11. Go to the identification room and bring with you the prints-out of the sky chart and the photographs.

    Ask some help from technical assistant, if necessary.
12. Choose the best out of the three printed images and identify as many objects as possible on it.
13. Use the assigned computer and The Sky software to identify objects. Type your identification to your answer page.
14. Type on the answer page the names (or catalogue number), RA, Dec, and magnitude of each identified star and put the sequential number on the photograph.

    Make sure that you list the stars on the answer page following the same order and number as on the photograph.
15. Estimate the limiting magnitude of the photograph you choose, empirically.

IOAA 2009 – Tehran, Iran

The third IOAA was held from 17$^{\text{th}}$ to 27$^{\text{th}}$ October 2009. The observational exam was held in the desert, in the Caravanserai of Deh Namak. 20 countries participated including first time participants Kazakhstan and Serbia.

3rd International Olympiad on Astronomy and Astrophysics

Theoretical Competition

Please read these instructions carefully:

1. Each student will receive problem sheets in English and/or in his/her native language.

2. The available time for answering theoretical problems is 5 hours. You will have 15 short problems (Theoretical Part 1, Problem 1 to 15), and 2 long problems (Theoretical Part 2, Problem 16 and 17).

3. Use only the pen that has been provided on your desk.

4. Do **Not** use the back side of your writing sheets. Write only inside the boxed area.

5. Yellow scratch papers are not considered in marking.

6. Begin answering each problem in separate sheet.

7. Fill in the boxes at the top of each sheet of your paper with your "country name", your "student code", "problem number", and total number of pages which is used to answer to that problem.

8. **Write the final answer for each problem in the box, labeled "Answer Sheet".**

9. Starting and the end of the exam will be announced by ringing a bell.

10. The final answer in each question part must be accompanied by units, which should be in SI or appropriate units. 20% of the marks available for that part will be deducted for a correct answer without units.

11. The required numerical accuracy for the final answer depends on the number of significant figures given in the data values in the problem. 20% of the marks available for the final answer in each question part will be deducted for answers without required accuracy as given in the problem. Use the constant values exactly as given in the table of constants.

12. At the end of the exam <u>put all papers, including scratch papers, inside the envelope</u> and leave everything on your desk.

Table of Constants

(All constants are in SI)

Parameter	Symbol	Value
Gravitational constant	G	
Plank constant	h	6.63×10^{-34} J s
Speed of light	c	
Solar Mass		
Solar radius		
Solar luminosity		
Apparent solar magnitude (V)		-26.8
Solar constant		
Mass of the Earth		
Radius of the Earth		
Mean density of the Earth		$5 \times 10^{3}\ kg\ m^{-3}$
Gravitational acceleration at sea level	g	
Tropical year		365.24 days
Sidereal year		365.26 days
Sidereal day		86164 s
Inclination of the equator with respect to the ecliptic	ε	
Parsec	pc	
Light year		
Astronomical Unit	AU	
Solar distance from the center of the Galaxy		
Hubble constant	H	
Mass of electron		
Mass of proton		
Central wavelength of V-band	λ	
Refraction of star light at horizon		
	π	3.1416

Useful mathematical formula:

3rd International Olympiad on Astronomy and Astrophysics

Theoretical Competition

Short Problems

Short Problems: (10 points each)

Problem 1: Calculate the mean mass density for a super massive black hole with total mass of 1×10^8 $M_\odot$ inside the Schwarzschild radius.

Problem 2: Estimate the number of photons per second that arrive on our eye at $\lambda = 550\,nm$ (V-band) from a G2 main sequence star with apparent magnitude of $m = 6$ (the threshold of naked eye visibility). Assume the eye pupil diameter is $6\,mm$ and all the radition from this star is in $\lambda = 550\,nm$.

Problem 3: Estimate the radius of a planet that a man can escape its gravitation by jumping vertically. Assume density of the planet and the Earth are the same.

Problem 4: In a typical Persian architecture, on top of south side windows there is a structure called "Tabeshband" (shader), which controls sunlight in summer and winter. In summer when the Sun is high, Tabeshband prevents sunlight to enter rooms and keeps inside cooler. In the modern architecture it is verified that the Tabeshband saves about 20% of energy cost. Figure (1) shows a vertical section of this design at latitude of $36°.0$ N with window and Tabeshband.

Using the parameters given in the figure, calculate the maximum width of the Tabeshband, "x", and maximum height of the window , "h" in such a way that:

i) No direct sunlight can enter to the room in the summer solstice at noon.
ii) The direct sunlight reaches the end of the room (indicated by the point **A** in the figure) in the winter solstice at noon.

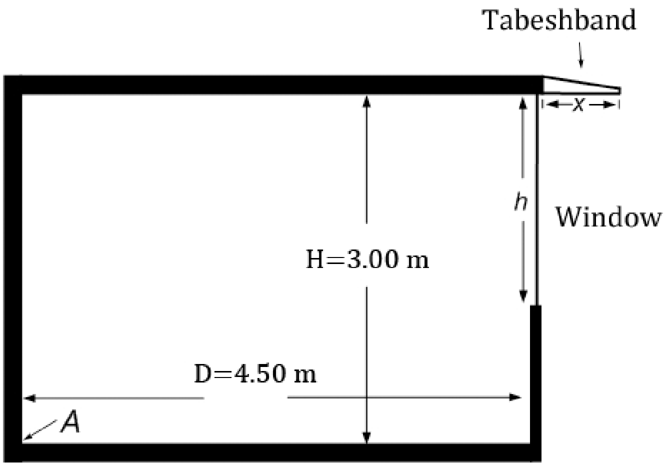


Figure (1)

Problem 5: The Damavand Mountain is located at the North part of Iran, in south coast of Caspian Sea. Consider an observer standing on the Damavand mountaintop (latitude = $35°\,57'$N; longitude = $52°\,6'$ E; altitude $5.6 \times 10^3\ m$ from the mean sea level) and looking at the sky over the Caspian Sea. What is the minimum declination for a star, to be seen marginally circumpolar for this observer. Geodetic radius of the Earth at this latitude is $6370.8\,km$. Surface level of the Caspian Sea is approximately equal to the mean sea level.

Problem 6: Derive a relation for the escape velocity of an object, launched from the center of a proto-star cloud. The cloud has uniform density with the mass of M and radius R. Ignore collisions between the particles of the cloud and the launched object. If the object were allowed to fall freely from the surface, it would reach the center with a velocity equal to $\sqrt{\frac{GM}{R}}$.

Problem 7: A student tries to measure field of view (FOV) of the eyepiece of his/her telescope, using rotation of the Earth. To do this job, the observer points the telescope towards Vega (alpha Lyr., RA: 18.5^h , Dec: +39°), turns off its "clock drive" and measures trace out time, t=5.3 minutes, that Vega crosses the full diameter of the FOV. What is the FOV of this telescope in arc-minutes?

Problem 8: Estimate the mass of a globular cluster with the radius of $r = 20\,pc$ and root mean square velocity of stars equal to $v_{rms} = 3\ kms^{-1}$.

Problem 9: The Galactic longitude of a star is $l = 15°$. Its radial velocity with respect to the Sun is $V_r = 100\ kms^{-1}$. Assume stars in the disk of the Galaxy are orbiting the center with a constant velocity of $V_0 = 250\ kms^{-1}$ in circular orbits in the same sense in the galactic plane. Calculate distance of the star from the center of the Galaxy.

Problem 10: A main sequence star with the radius and mass of $R = 4R_\odot, M = 6M_\odot$ has an average magnetic field of 1×10^{-4} T. Calculate the strength of the magnetic field of the star when it evolves to a neutron star with the radius of $20\ km$.

Problem 11: Assume the mass of neutrinos is $m_\nu = 10^{-5} m_e$. Calculate the number density of neutrinos (n_ν) needed to compensate the dark matter of the universe. Assume the universe is flat and 25 % of its mass is dark matter.

Hint: Take the classical total energy equal to zero

Problem 12: Calculate how much the radius of the Earth's orbit increases as a result of the Sun losing mass due to the thermo-nuclear reactions in its center in 100 years. Assume the Earth's orbit remains circular during this period.

Problem 13: Assume that you are living in the time of Copernicus and do not know anything about Kepler's laws. You might calculate Mars-Sun distance in the same way as he did. After accepting the revolutionary belief that all the planets are orbiting around the Sun, not around the Earth, you measure that the orbital period of Mars is 687 days, then you observe that 106 days after opposition of Mars, the planet appears in quadrature. Calculate Mars-Sun distance in astronomical unit (AU).

Problem 14: A satellite is orbiting around the Earth in a circular orbit in the plane of the equator. An observer in Tehran at the latitude of $\varphi = 35.6^\circ$ observes that the satellite has a zenith angle of $z = 46.0°$, when it transits the local meridian. Calculate the distance of the satellite from the center of the Earth (in the Earth radius unit).

Problem 15: An eclipsing close binary system consists of two giant stars with the same sizes. As a result of mutual gravitational force, stars are deformed from perfect sphere to the prolate spheroid with $a = 2b$, where a and b are semi-major and semi-minor axes (the major axes are always co-linear). The inclination of the orbital plane to the plane of sky is $90°$. Calculate the amplitude of light variation in magnitude (Δm) as a result of the orbital motion of two stars. Ignore temperature variation due to tidal deformation and limb darkening on the surface of the stars.

Hint: A prolate spheroid is a geometrical shape made by rotating of an ellipse around its major axis, like rugby ball or melon.

3rd International Olympiad on Astronomy and Astrophysics

Theoretical Competition

Long Problems

Problem 16: High Altitude Projectile (45 points)

A projectile which starts from the surface of the Earth at the sea level is launched with the initial speed of $v_\circ = \sqrt{(GM/R)}$ and with the projecting angle (with respect to the local horizon) of $\theta = \frac{\pi}{6}$. M and R are the mass and radius of the Earth respectively. Ignore the air resistance and rotation of the Earth.

a) Show that the orbit of the projectile is an ellipse with a semi-major axis of $a = R$.
b) Calculate the highest altitude of the projectile with respect to the Earth surface (in the unit of the Earth radius).
c) What is the range of the projectile (distance between launching point and falling point) in the units of the earth radii?
d) What is eccentricity (e) of this elliptical orbit?
e) Find the time of flight for the projectile.

Problem 17: Apparent number density of stars in the Galaxy (45 points)

Let us model the number density of stars in the disk of Milky Way Galaxy with a simple exponential function of $n = n_0 \exp(-\frac{r-R_0}{R_d})$, where r represents the distance from the center of the Galaxy, R_0 is the distance of the Sun from the center of the Galaxy, R_d is the typical size of disk and n_0 is the stellar density of disk at the position of the Sun. An astronomer observes the center of the Galaxy within a small field of view. We take a particular type of Red giant stars (red clump) as the standard candles for the observation with approximately constant absolute magnitude of $M = -0.2$,

a) Considering a limiting magnitude of $m = 18$ for a telescope, calculate the maximum distance that telescope can detect the red clump stars. For simplicity we ignore the presence of interstellar medium so there is no extinction.
b) Assume an extinction of $0.7\ mag/kpc$ for the interstellar medium. Repeat the calculation as done in the part (a) and obtain a rough number for the maximum distance these red giant stars can be observed.
c) Give an expression for the number of these red giant stars per magnitude within a solid angle of Ω that we can observe with apparent magnitude in the range of m and $m + \Delta m$, (i.e. $\frac{\Delta N}{\Delta m}$). Red giant stars contribute fraction f of overall stars. In this part assume no extinction in the interstellar medium as part (a). Assume the size of the disk is larger than maximum observable distance.

3rd International Olympiad of Astronomy and Astrophysics

Data Analysis Competition

Problem 1: CCD Image Processing (60 points)

As an exercise of image processing, this problem involves use of a simple calculator and tabular data (table 1.1) which contains the pixel values of an image during the given exposure time. This image, which is a part of a larger CCD image, was taken by a small CCD camera, installed on an amateur telescope and using a V band filter. Figure 1.1 shows this 50×50 pixels image that contains 5 stars.

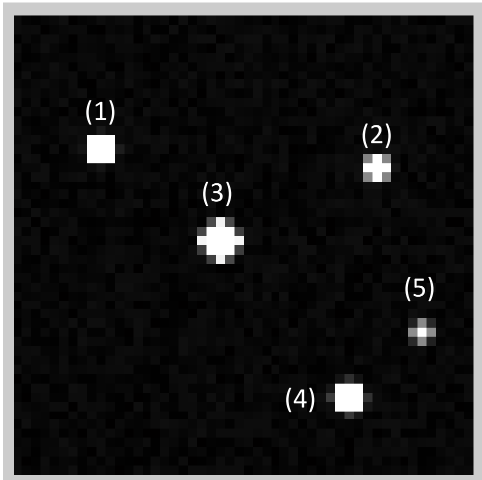


Figure 1.1

In table 1.1 the first row and column indicates the pixels' x and y coordinates. Table 1.2 gives the telescope and the image specifications.

Table 1.2

Telescope focal length	$1.20\ m$
CCD pixel size	$25 \times 25\ \mu m$
Exposure time	$450\ s$
Telescope zenith angle	25°
Average extinction coefficient in V band	0.3 mag/airmass

The observer identified stars 1, 3 and 4 by comparing this image with standard star catalogues. Table 1.3 shows stars true magnitudes (m_t) as given in the catalogue.

Table 1.3

Star	m_t
1	9.03
3	6.22
4	8.02

a. Using the available data, determine the instrumental magnitudes of the stars in the image. Assume the dark current is negligible and the image is flat fielded. For simplicity you can use a square aperture.
Hint: The instrumental magnitude is calculated using the difference between the measured flux from the star in the aperture and the flux from an equivalent area of dark sky.

b. The instrumental magnitude of a star in a CCD image is related to true magnitude as

$$m_I = m_t + KX - Zmag$$

where K is extinction, X is airmass, m_I and m_t are respectively instrumental and true magnitude of star and $Zmag$ is zero point constant. Calculate the zero point constant ($Zmag$) for identified stars. Calculate average zero point constant ($Zmag$).

Hint: Zero point constant is the constant reducing extinction-free magnitudes to the true magnitude.

c. Calculate true magnitudes of stars 2 and 5.

d. Calculate CCD pixel scale for the CCD camera in units of arcsec.

e. Calculate average brightness of dark sky in magnitude per square arcsec (m_{sky}).

f. Use a suitable plot to estimate astronomical seeing in arcsec.

Problem 2: Venus

An observer in Deh-Namak (you will be taking the observational part of the exam in this region tonight) has observed Venus for seven months, started from September 2008 and continued until March 2009. During the observation, a research CCD camera and an image processing software were used to take high resolution images and to extract high precision data. Table 2.1 shows the collected data during the observation.

Table 2.1 description:

Column 1	Date of observation.
Column 2	Earth-Sun distance in astronomical unit (AU) for observation date and time. This value is taken from high precision tables.
Column 3	Phase of Venus, Percent of Venus disk illuminated by the Sun as observed from the Earth.
Column 4	Elongation of Venus, the angular distance between center of the Sun and center of Venus in degrees as observed from the Earth.

a) Using given data in table 2.1, calculate the Sun-Venus-Earth angle($\angle SVE$) . This is angular separation between the Sun and the Earth as seen from Venus. Write $\angle SVE$ angle in column 2 of Table 2.2 in your answer sheet for the all observing dates.
Hint: Remember that the line between light and shadow, in the phases, is an ellipse arc.

b) Calculate Sun – Venus distance in AU and write it down in column 3 of table 2.2 for all observation dates.

c) Plot Sun – Venus distance versus observing date.

d) Find perihelion ($r_{v,min}$) and aphelion ($r_{v,max}$) distances of Venus from the Sun.

e) Calculate semi-major axis (a) of the Venus orbit.

f) Calculate eccentricity (e) of Venus orbit.

Table 2.1

Column 1	Column 2	Column 3	Column 4
Date	Earth - Sun Distance (AU)	Phase (%)	Elongation (SEV) (°)
20/9/2008	1.0043	88.4	27.56
10/10/2008	0.9986	84.0	32.29
20/10/2008	0.9957	81.6	34.53
30/10/2008	0.9931	79.0	36.69
9/11/2008	0.9905	76.3	38.71
19/11/2008	0.9883	73.4	40.62
29/11/2008	0.9864	70.2	42.38
19/12/2008	0.9839	63.1	45.29
29/12/2008	0.9834	59.0	46.32
18/1/2009	0.9838	49.5	47.09
7/2/2009	0.9863	37.2	44.79
17/2/2009	0.9881	29.6	41.59
27/2/2009	0.9904	20.9	36.16
19/3/2009	0.9956	3.8	16.08

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7	6	6	4	6	5	2	10	4
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7	1	0	9	2	5	8	5	2
2	8	7	5	1	2	8	2	8
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3	7	1	4	5	2	5	8	1
10	6	10	4	9	7	3	1	9
8	2	8	7	8	2	0	9	4
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8	5	10	8	1	3	2	6	4
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4	3	0	6	3	7	4	8	10
6	7	5	5	9	6	2	8	9
1	9	6	4	8	3	5	5	1
4	7	7	2	8	6	7	9	9
4	5	8	9	5	6	7	8	6
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5	7	4	8	1	7	7	2	4
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4	0	2	9	1	5	7	8	5
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7	9	7	2	5	4	8	0	2
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6	2	8	4	6	7	5	3	1

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5	1	4	2	4	9	8	10	3
2	3	4	5	1	9	9	3	9
2	6	3	4	8	2	9	3	3
5	2	4	4	2	5	3	4	4

2	2	2	5	1
1	3	4	0	6
9	2	3	9	8
3	8	5	7	9
9	1	4	8	8
2	2	2	2	3
5	8	1	5	7
2	2	9	4	1
5	0	5	2	7
3	2	3	1	3
4	4	4	6	10
4	7	3	6	4
7	4	5	7	6
3	3	4	8	7
1	5	6	5	3
4	0	3	4	1
3	1	8	3	9
4	4	0	9	3
2	8	7	7	8
8	6	9	3	5
1	6	10	9	2
3	7	8	7	4
3	5	7	9	1
9	6	5	6	1
5	10	3	3	6
1	1	1	5	0
8	8	6	9	1
7	7	9	0	2
1	1	4	6	6
3	8	3	8	5
5	3	2	2	4
7	7	1	3	4
9	4	9	2	3
24	2	2	5	10
96	7	7	8	8
25	8	7	0	8
1	6	2	10	2
6	6	7	3	0
8	2	8	6	2
4	5	4	3	4
8	10	2	1	1
2	5	2	0	8
3	6	2	2	7
9	2	7	3	1
6	3	8	1	1
8	9	8	7	0
8	1	2	8	5
0	2	3	5	1
6	2	0	9	8
7	7	0	1	6

3rd International Olympiad of Astronomy and Astrophysics

Observational Competition

Personal Information

Name:	**Country:**
Student Code:	

Please read these instructions carefully:

1. All participants will receive a problem set, a writing board, a pen, a ruler and a headlight by the organizers.

2. This competition consists of two parts:

i) Two questions on "Naked Eye observation". You Have 12 minutes to answer these two questions.

ii) One question on "Using a telescope". Each part of this question has a specific time, which is <u>mentioned in your question sheet</u>.

3. All participants will be guided by assistants to the observing site until returning to the waiting hall. Assistants will collect the answer and problem sheets.

4. Do not forget to fill out the boxes at the top of each answer sheet with your <u>country name</u> and your <u>student code</u>.

5. You have 2 minutes to familiarize yourself with Observing ground and darkness of your environment, just before starting the exam time in observing ground.

6. Examiner's alarm will indicate the beginning and the end of each part of your exam.

7. Each problem has a specific guideline which helps you during the exam.

3rd International Olympiad of Astronomy and Astrophysics

Observational Competition

Naked Eye Observations

Time: 12 Minutes

You Have 12 minutes to answer the questions of the Naked Eye Observations (Question 1 and Question 2)

Question 1

Part 1: Figure 1 (frame size $\cong 100^{\circ} \times 70^{\circ}$) shows a part of the sky, for 22 October 2009 at 21:00 local time. Four bright stars in Perseus and Andromeda constellations are missing in this chart. Find these missing stars by looking at the sky. Then, draw a cross on the location of each missing bright star in these two constellations on the chart (i.e. figure 1). Use numbers in table 1-1 to indicate these crosses**.**

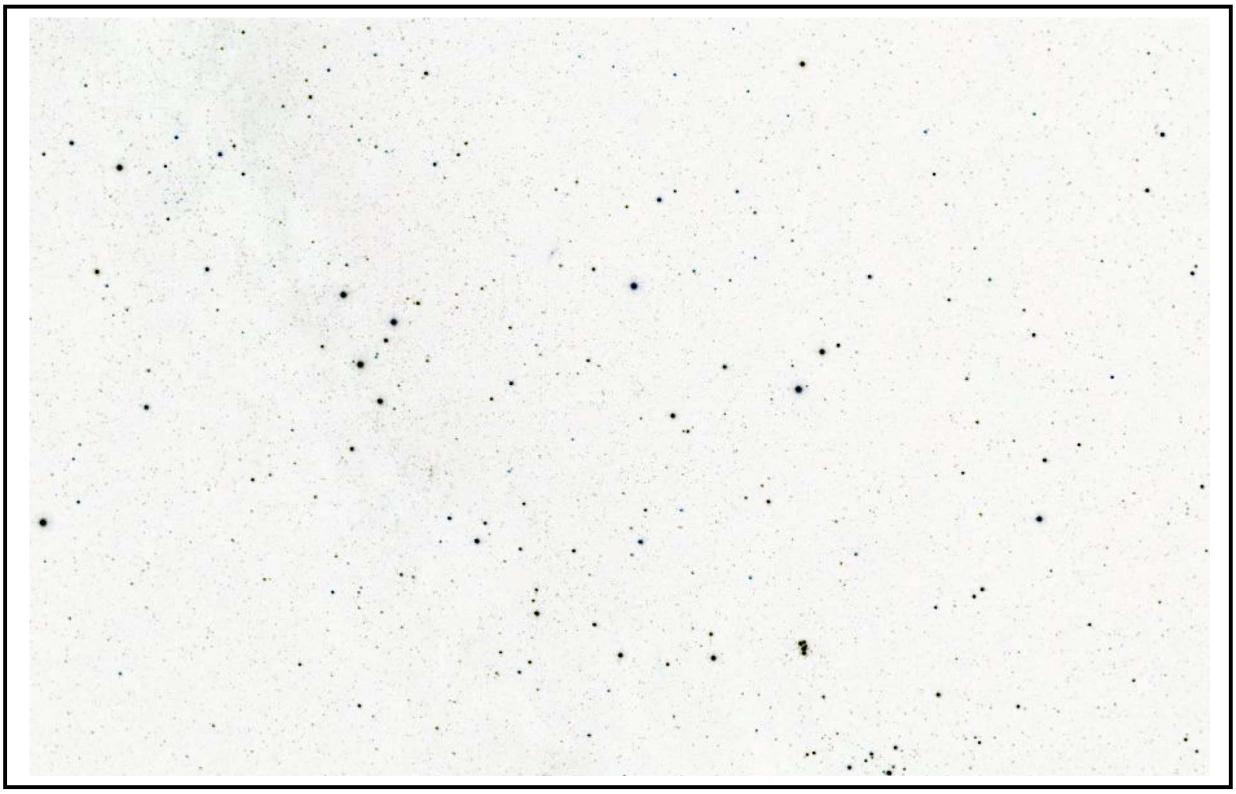
(40 Points)

Table 1-1

Number	Common Name	Bayer Names
1	Mirfak	Alpha Persei
2	Alpheratz	Alpha Andromeda
3	-	Epsilon Persei
4	Menkib	Xi Persei
5	-	Gamma Persei
6	Algol	Beta Persei
7	Almach	Gamma Andromeda
8	-	Delta Andromeda
9	-	51 Andromeda
10	Mirach	Beta Andromeda
11	Atik	Zeta Persei

Name:	Country:
Student Code:	

Question 1 - Figure 1



Question 2

Part 1: Figure 2 shows a part of the sky which contains **Cephei constellation**, for 22 October 2009 at 22:00 local time. Five bright stars in Cephei constellation are identified by numbers (1, 2, … , 5) and common names. Estimate the angular distances (in units of degrees) between two pairs of stars shown in table 2-1 and complete this table with your answers. **(40 Points)**

Tables 2-1

Angular Distance	
Pairs of stars	**Angular Distance (degrees)**
1 (Errai) and 2 (Alfirk)	
1 (Errai) and 3 (Alderamin)	

Part 2: Use table 2-2 and figure 2, then Estimate the "apparent visual magnitude" of stars 2 (Alfirak) and 3 (Alderamin) and complete Table 2-3. **(40 Points)**

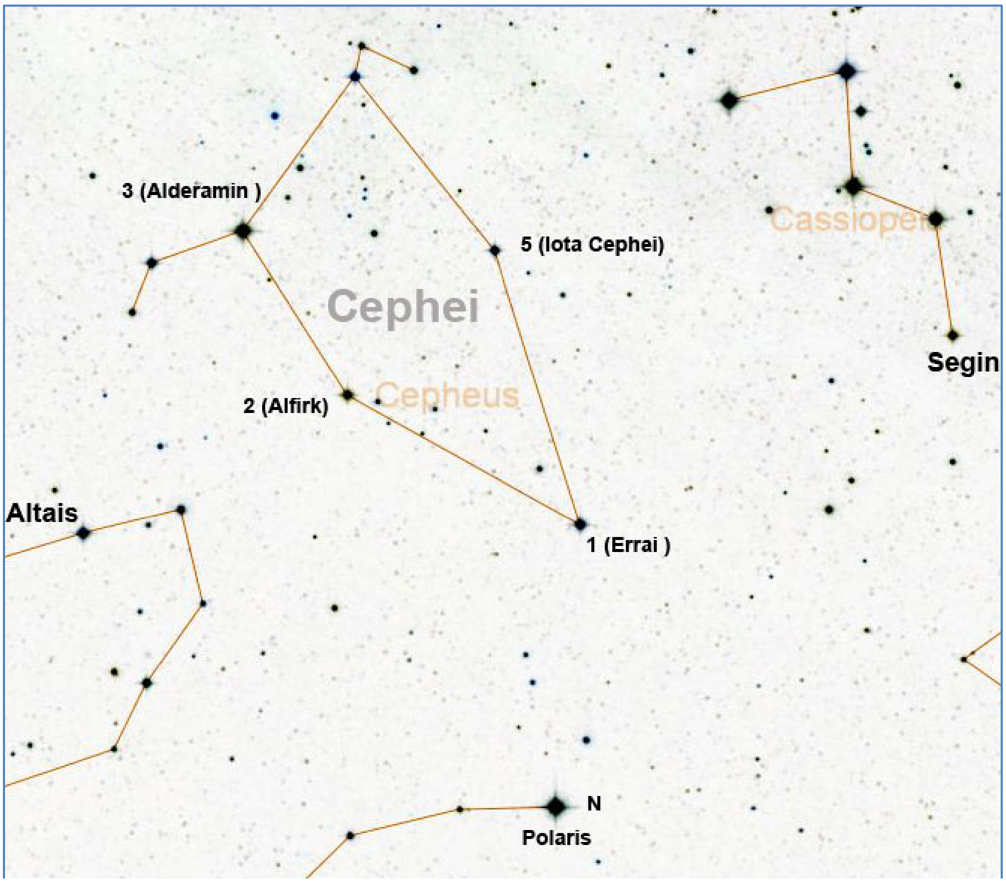
Table 2-2

Star Name	Apparent Visual Magnitude
Polaris	1.95
Altais	3.05
Segin	3.34
All of these stars, are marked in the figure 2	

Table 2-3

Magnitude Estimation		
Star Number	**Star Name**	**Apparent Visual Magnitude**
2	Alfirk	
3	Alderamin	

Question 2 - Figure 2



3rd International Olympiad of Astronomy and Astrophysics

Observational Competition

Telescopic Observations

Time: 9 Minutes

Note: You have only 9 Minutes to answer all parts of this Question.

Name:	Country:
Student Code:	Examiner Code:

Question 3

Before starting this part, please note: The telescope is pointed by the examiner towards Caph (α Cas). Please note the readings on the grade circles before moving the telescope (to be used in 3.2).

3.1:

Choose one of the 4 recommended stars listed below; write down the name of the selected star in table 3-1 and point the telescope to that star. Then, notify the examiner to check it. **(40 Points)**

1- Deneb (Alpha Cygni)
2- Alfirk (Beta Cephei)
3- Algol (Beta Persei)
4- Capella (Alpha Aurigae)

(You have 6 Minutes to answer 3.1)

Table 3-1

Name of selected star

Name:	Country:
Student Code:	

3.2: The Telescope was parked to Caph in Cassiopeia constellation (RA: 0h:9.7m ; Dec: 59°:12’). Using the clock beside the telescope write down the local time (with the format of HH:MM:SS) in the appropriate field in Table B. Then, by using the graded circle on the telescope mount, estimate the “declination” and the “hour angle” of the target measured from South, which you chose in part one of this question. Then, complete Table B. **(40 Points)**

(You have 7 Minutes to answer 3.2)

Table 3-2

Name and Coordinates of the Selected Star			Local Time :
Name of Selected Star	**Hour Angle (hh:mm)**	**Declination (°:’)**	

IOAA 2010 – Beijing, China

The fourth IOAA was held from 12$^{\text{th}}$ to 21$^{\text{st}}$ September 2010. One-hundred fourteen competitors from 23 countries participated in the event. It was a first time participation for Czech Republic, Philippines and Russia.

The 4th International Olympiad on Astronomy and Astrophysics
Beijing, China
Wednesday, 15 September 2010

Theoretical Competition

Please read these instructions carefully:

1. Each student will receive problem sheets in English and/or in his/her native language.
2. The time available for answering theoretical problems is 5 hours. You will have 15 short problems (Theoretical Part 1, Problem 1 to 15), and 2 long problems (Theoretical Part 2, Problem 16 and 17).
3. Use only the pen that has been provided on your desk.
4. Begin answering each problem on a new page of the notebook. Write down the number of the problem at the beginning.
5. Write down your “country name” and your “student code” on the cover of the notebook.
6. The final answer in each question or part of it must be accompanied by units and the correct number of significant digits (use SI or appropriate units). At most 20% of the marks assigned for that part will be deducted for a correct answer without units and/or with incorrect significant digits.
7. At the end of the exam put all papers and the notebook inside the envelope and leave everything on your desk.
8. Please write down logically step by step with intermediate equations/calculations to get the final solution.

Short Problem

Note: 10 points for each problem

1. In a binary system, the apparent magnitude of the primary star is 1.0 and that of the secondary star is 2.0. Find the maximum combined magnitude of this system.

2. If the escape velocity from a solar mass object's surface exceeds the speed of light, what would be its radius?

3. The observed redshift of a QSO is $z = 0.20$, estimate its distance. The Hubble constant is 72 km s^{-1} Mpc^{-1}.

4. A binary system is 10 pc away, the largest angular separation between the components is $7.0''$, the smallest is $1.0''$. Assume that the orbital period is 100 years, and that the orbital plane is perpendicular to the line of sight. If the semi-major axis of the orbit of one component corresponds to $3.0''$, that is $a_1 = 3.0''$, estimate the mass of each component of the binary system, in terms of solar mass.

5. If 0.8% of the initial total solar mass could be transformed into energy during the whole life of the Sun, estimate the maximum possible life time for the Sun. Assume that the solar luminosity remains constant.

6. A spacecraft landed on the surface of a spherical asteroid with negligible rotation, whose diameter is 2.2 km, and its average density is 2.2 g/cm^3. Can the astronaut complete a circle along the equator of the asteroid on foot within 2.2 hours? Write your answer "YES" or "NO" on the answer sheet and explain why with formulae and numbers.

7. We are interested in finding habitable exoplanets. One way to achieve this is through the dimming of the star, when the exoplanet transits across the stellar disk and blocks a fraction of the light. Estimate the maximum luminosity ratio change for an Earth-like planet orbiting a star similar to the Sun.

8. The Galactic Center is believed to contain a super-massive black hole with a mass $M = 4 \times 10^6 M_\odot$. The astronomy community is trying to resolve its event horizon, which is a challenging task. For a non-rotating black hole, this is the Schwarzschild radius, $R_S = 3(M/M_\odot)$ km. Assume that we have an Earth-sized telescope (using Very Long Baseline Interferometry). What wavelengths should we adopt in order to resolve the event horizon of the black hole? The Sun is located at 8.5 kpc from the Galactic Center.

9. A star has a measured I-band magnitude of 22.0. How many photons per second are detected from this star by the Gemini Telescope (8 m diameter)? Assume that the overall quantum efficiency is 40% and the filter passband is flat.

Filter	λ_0 (nm)	$\Delta\lambda$ (nm)	F_{VEGA} (W m^{-2} nm^{-1})
I	8.00×10^2	24.0	8.30×10^{-12}

10. Assuming that the G-type main-sequence stars (such as the Sun) in the disc of the Milky Way obey a vertical exponential density profile with a scale height of 300 pc, by what factor does the density of these stars change at 0.5 and 1.5kpc from the mid-plane relative to the density in the mid-plane?

11. Mars arrived at its great opposition at UT 17^h56^m Aug.28, 2003. The next great opposition of Mars will be in 2018, estimate the date of that opposition. The semi-major axis of the orbit of Mars is 1.524 AU.

12. The difference in brightness between two main sequence stars in an open cluster is 2 magnitudes. Their effective temperatures are 6000 K and 5000 K respectively. Estimate the ratio of their radii.

13. Estimate the effective temperature of the photosphere of the Sun using the naked eye colour of the Sun.

14. An observer observed a transit of Venus near the North Pole of the Earth. The transit path of Venus is shown in the picture below. A, B, C, D are all on the path of transit and marking the center of the Venus disk. At A and B, the center of Venus is superposed on the limb of the Sun disk; C corresponds to the first contact while D to the fourth contact, $\angle AOB = 90°$, MN is parallel to AB. The first contact occurred at 9:00 UT. Calculate the time of the fourth contact.

$T_{\text{Venus}} = 224.70$ days, $T_{\text{Earth}} = 365.25$ days, $a_{\text{Venus}} = 0.723$ AU, $r_{\text{Venus}} = 0.949 r_{\oplus}$

15. On average, the visual diameter of the Moon is slightly less than that of the Sun, so the frequency of annular solar eclipses is slightly higher than total solar eclipses. For an observer on the Earth, the longest total solar eclipse duration is about 7.5 minutes, and the longest annular eclipse duration is about 12.5 minutes. Here, the longest duration is the time interval from the second contact to the third contact. Suppose we count the occurrences of both types of solar eclipses for a very long time, estimate the ratio of the occurrences of annular solar eclipses and total solar eclipses. Assume the orbit of the Earth to be circular and the eccentricity of the Moon's orbit is 0.0549. Count all hybrid eclipses as annular eclipses.

Long Problem

Note: 30 points for each problem

16. A spacecraft is launched from the Earth and it is quickly accelerated to its maximum velocity in the direction of the heliocentric orbit of the Earth, such that its orbit is a parabola with the Sun at its focus point, and grazes the Earth orbit. Take the orbit of the Earth and Mars as circles on the same plane, with radius of $r_E = 1$ AU and $r_M = 1.5$ AU, respectively. Make the following approximation: during most of the flight only the gravity from the Sun needs to be considered.

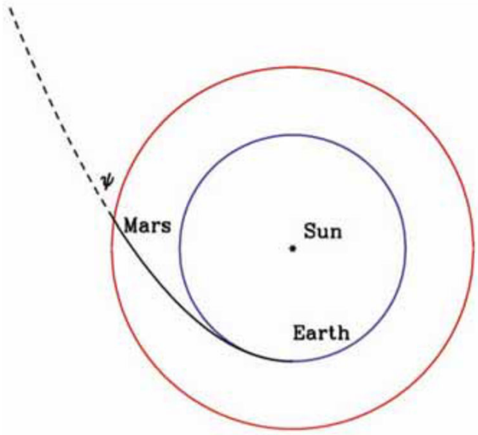


Figure 1:
The trajectory of the spacecraft (not in scale). The inner circle is the orbit of the Earth, the outer circle is the orbit of Mars.
Questions:

(a) What is the angle ψ between the path of the spacecraft and the orbit of the Mars (see Fig. 1) as it crosses the orbit of the Mars, without considering the gravity effect of the Mars?

(b) Suppose the Mars happens to be very close to the crossing point at the time of the crossing, from the point of view of an observer on Mars, what is the approaching velocity and direction of approach (with respect to the Sun) of the spacecraft before it is significantly affected by the gravity of the Mars?

17. The planet Taris is the home of the Korribian civilization. The Korribian species is a highly intelligent alien life form. They speak Korribianese language. The Korribianese-English dictionary is shown in Table 1; read it carefully! Korriban astronomers have been studying the heavens for thousands of years. Their knowledge can be summarized as follows:

⋆ Taris orbits its host star Sola in a circular orbit, at a distance of 1 Tarislength.

⋆ Taris orbits Sola in 1 Tarisyear.

⋆ The inclination of Taris's equator to its orbital plane is 3°.

⋆ There are exactly 10 Tarisdays in 1 Tarisyear.

⋆ Taris has two moons, named Endor and Extor. Both have circular orbits.

⋆ The sidereal orbital period of Endor (around Taris) is exactly 0.2 Tarisdays.

⋆ The sidereal orbital period of Extor (around Taris) is exactly 1.6 Tarisdays.

⋆ The distance between Taris and Endor is 1 Endorlength.

⋆ Corulus, another planet, also orbits Sola in a circular orbit. Corulus has one moon.

⋆ The distance between Sola and Corulus is 9 Tarislengths.

⋆ The tarisyear begins when Solaptic longitude of the Sola is zero.

Korribianese	**English Translation**
Corulus	A planet orbiting Sola
Endor	(i) Goddess of the night; (ii) a moon of Taris
Endorlength	The distance between Taris and Endor
Extor	(i) God of peace; (ii) a moon of Taris
Sola	(i) God of life; (ii) the star which Taris and Corulus orbit
Solaptic	Apparent path of Sola and Corulus as viewed from Taris
Taris	A planet orbiting the star Sola, home of the Korribians
Tarisday	The time between successive midnights on the planet Taris
Tarislength	The distance between Sola and Taris
Tarisyear	Time taken by Taris to make one revolution around Sola

Table 1: Korribianese-English dictionary

Questions:

(a) Draw the Sola-system, and indicate all planets and moons.

(b) How often does Taris rotate around its axis during one Tarisyear?

(c) What is the distance between Taris and Extor, in Endorlengths?

(d) What is the orbital period of Corulus, in Tarisyears?

(e) What is the distance between Taris and Corulus when Corulus is in opposition?

(f) If at the beginning of a particular tarisyear, Corulus and taris were in opposition, what would be Solaptic longitude (as observed from Taris) of Corulus n tarisdays from the start of that year?

(g) What would be the area of the triangle formed by Sola, Taris and Corulus exactly one tarisday after the opposition?

The 4^{th} International Olympiad on Astronomy and Astrophysics

Beijing, China

Thursday, 16 September 2010

Practical Competition: Data Analysis

Please read these instructions carefully:

1. You should use the ruler and calculator provided by LOC.
2. The time available for answering data analysis problems is 4 hours. You will have 2 problems.
3. Use only the pen that has been provided on your desk.
4. Begin answering each problem on a new page of the notebook. Write down the number of the problem at the beginning.
5. Write down your "country name" and your "student code" on the cover of the notebook.
6. At the end of the exam put all paper and the notebook inside the envelope and leave everything on your desk.
7. Write down logically step by step with intermediate equations/calculations to get the final solution.

Problem I CCD image (35 points)

Information:

Picture 1 presents a negative image of sky taken by a CCD camera attached to a telescope whose parameters are presented in the accompanying table (which is part of the FITS datafile header).
Picture 2 consists of two images: one is an enlarged view of part of Picture 1 and the second is an enlarged image of the same part of the sky taken some time earlier.
Picture 3 presents a sky map which includes the region shown in the CCD images.

The stars in the images are far away and should ideally be seen as point sources. However, diffraction on the telescope aperture and the effects of atmospheric turbulence (known as 'seeing') blur the light from the stars.
The brighter the star, the more of the spread-out light is visible above the level of the background sky.

Questions:

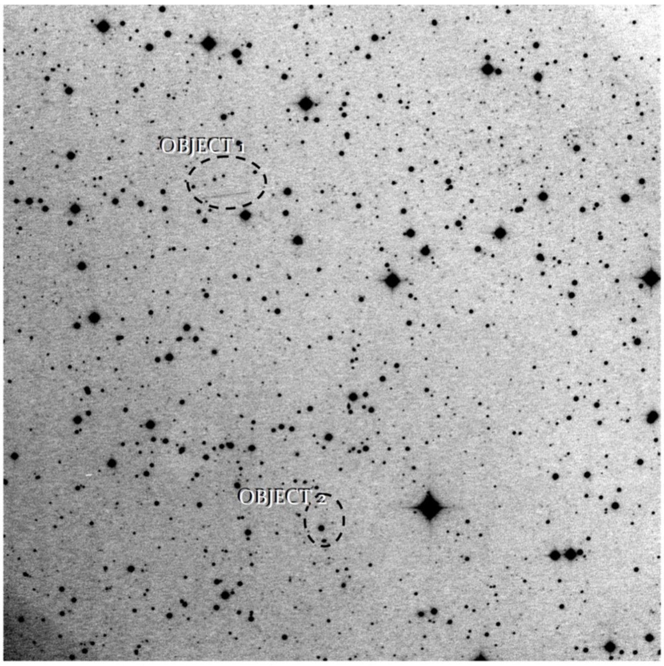
1. Identify any 5 bright stars (mark them by Roman numerals) from the image and mark them on both the image and map.
2. Mark the field of view of the camera on the map.
3. Use this information to obtain the physical dimensions of the CCD chip in mm.
4. Estimate the size of the blurring effect in arcseconds by examining the image of the star in Picture 2. (Note that due to changes in contrast necessary for printing, the diameter of the image appears to be about 3.5 times the full width at half maximum (FWHM) of the profile of the star.)
5. Compare the result with theoretical size of the diffraction disc of the telescope.
6. Seeing of 1 arcsecond is often considered to indicate good conditions. Calculate the size of the star image in pixels if the atmospheric seeing was 1 arcsecond and compare it with the result from question 4.
7. Two objects observed moving relative to the background stars have been marked on Picture 1. The motion of one ("Object 1") was fast enough that it left a clear trail on the image. The motion of the other ("Object 2") is more easily seen on the enlarged image (Picture 2A) and another image taken some time later (Picture 2B).
Using the results of the first section, determine the angular velocity on the sky of both objects.
Choose which of the statements in the list below are correct, assuming that the objects are moving on circular orbits. (Points will be given for each correct answer marked and deducted for each incorrect answer marked.) The probable causes of the different angular velocities are:
(a) different masses of the objects,
(b) different distances of the objects from Earth,
(c) different orbital velocities of the objects,
(d) different projections of the objects' velocities,
(e) Object 1 orbits the Earth while Object 2 orbits the Sun.

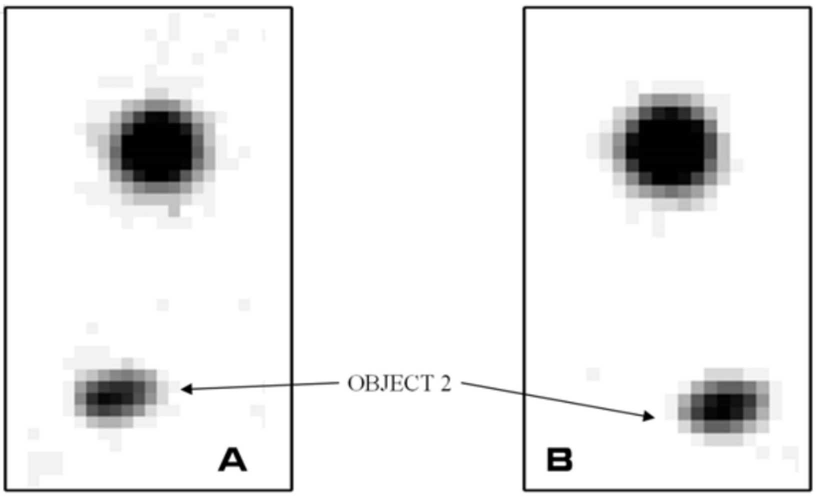
Data:

For Picture 1, the data are,

BITPIX =	16	/ Number of bits per pixel
NAXIS =	2	/ Number of axes
NAXIS1 =	1024	/Width of image (in pixels)
NAXIS2 =	1024	/Height of image (in pixels)
DATE-OBS=	'2010-09-07 05:00:40.4'	/Middle of exposure
TIMESYS =	'UT'	/Time Scale
EXPTIME =	300.00	/Exposure time (seconds)
OBJCTRA =	'22 29 20.031'	/RA of center of the image
OBJCTDEC =	'+07 20 00.793'	/DEC of center of the image
FOCALLEN=	'3.180 m'	/Focal length of the telescope
TELESCOP=	'0.61 m'	/Telescope aperture

Picture 1 for Problem I



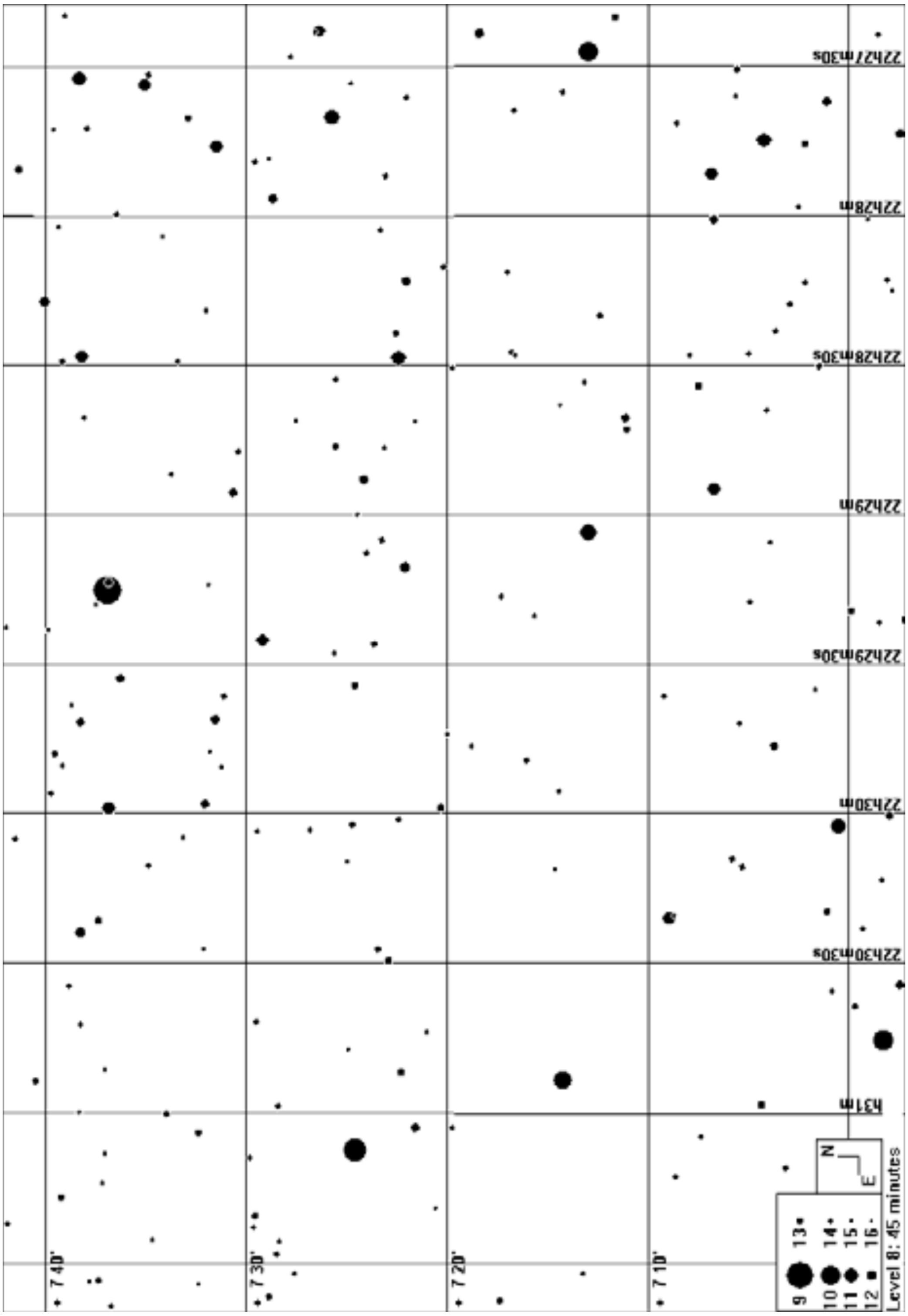


Picture 2 for Problem I:

A: The same area observed some time earlier. For this image the data are :
DATE-OBS= '2010-09-07 04:42:33.3' / Middle of exposure

B: Enlargement of Picture 1 around Object 2,

Picture 3 for Problem I:



Problem II: Light curves of stars (35 points)

A pulsating variable star KZ Hydrae was observed with a telescope equipped with a CCD camera. Figure 1 shows a CCD image of KZ Hya marked together with the comparison star and the check star. Table 1 lists the observation time in Heliocentric Julian dates, the magnitude differences of KZ Hya and the check star relative to the comparison star in V and R band.

The questions are:

(1) Draw the light curves of KZ Hya relative to the comparison star in V and R band, respectively.

(2) What are the average magnitude differences of KZ Hya relative to the comparison star in V and R, respectively?

(3) What are the photometry precisions in V and R, respectively?

(4) Estimate the pulsation periods of KZ Hya in V and R.

(5) Give the estimation of the pulsation amplitudes of KZ Hya in V and R

(6) What is the phase delay between the V and R bands, in term of the pulsation period?

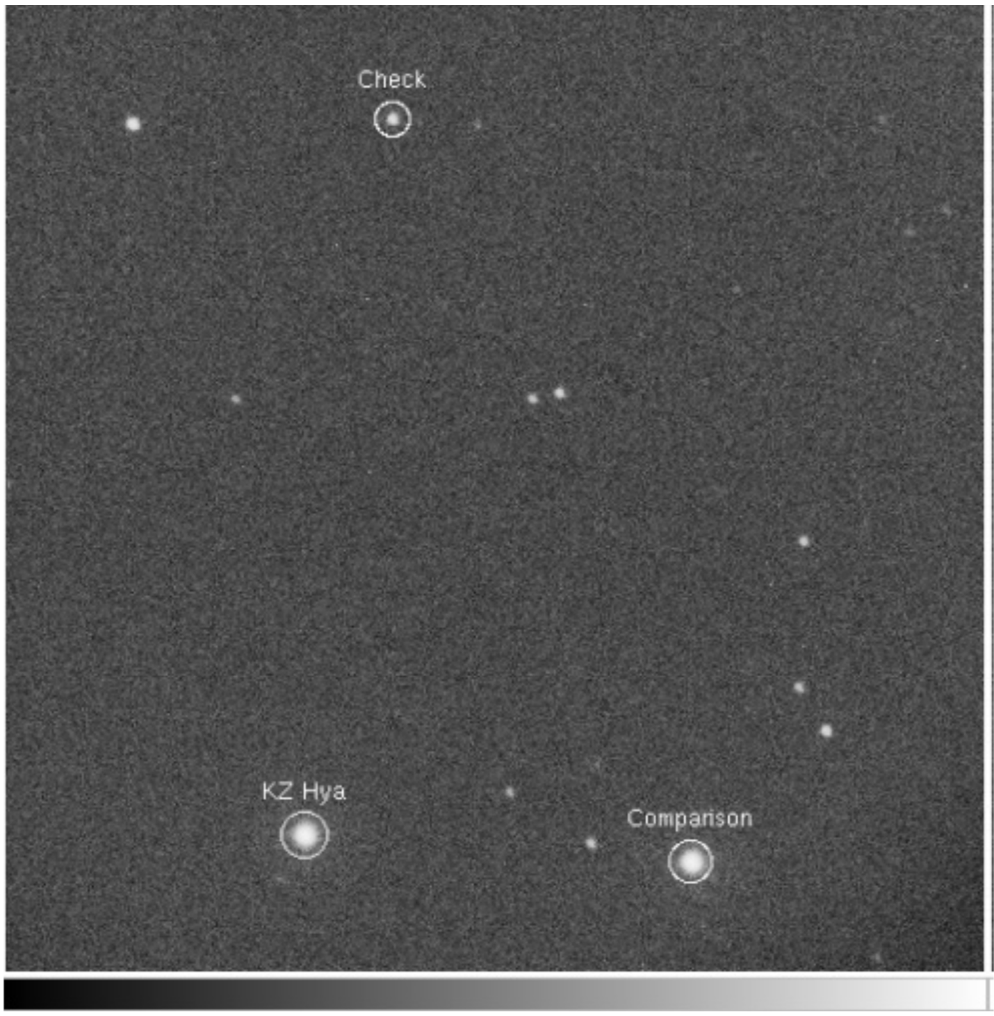


Fig. 1 for Problem II: A CCD image of KZ Hya.

Table 1 for Problem II: Data for the light curves of KZ Hya in V and R. ΔV and ΔR are KZ Hya relative to the comparison in V and R. ΔV_{chk} and ΔR_{chk} are the check star relative to the comparison in V and R.

HJD-2453800 (t)	ΔV (mag)	ΔV_{chk}	HJD-2453800 (t)	ΔR (mag)	ΔR_{chk}
3.162	0.068	4.434	3.1679	0.260	2.789
3.1643	0.029	4.445	3.1702	0.185	2.802
3.1667	–0.011	4.287	3.1725	–0.010	2.789
3.1691	–0.100	4.437	3.1749	–0.147	2.809
3.1714	–0.310	4.468	3.1772	–0.152	2.809
3.1737	–0.641	4.501	3.1796	-0.110	2.789
3.1761	–0.736	4.457	3.1820	–0.044	2.803
3.1784	–0.698	4.378	3.1866	0.075	2.805
3.1808	–0.588	4.462	3.1890	0.122	2.793
3.1831	–0.499	4.326	3.1914	0.151	2.793
3.1855	–0.390	4.431	3.1938	0.177	2.782
3.1878	–0.297	4.522	3.1962	0.211	2.795
3.1902	–0.230	4.258	3.1986	0.235	2.796
3.1926	–0.177	4.389	3.2011	0.253	2.788
3.195	–0.129	4.449	3.2035	0.277	2.796
3.1974	–0.072	4.394	3.2059	0.288	2.783
3.1998	–0.036	4.362	3.2083	0.296	2.796
3.2023	–0.001	4.394	3.2108	0.302	2.791
3.2047	0.016	4.363	3.2132	0.292	2.806
3.2071	0.024	4.439	3.2157	0.285	2.779
3.2096	0.036	4.078	3.2181	0.298	2.779
3.2120	0.020	4.377	3.2206	0.312	2.787
3.2145	0.001	4.360	3.2231	0.313	2.804
3.2169	0.001	4.325	3.2255	0.281	2.796
3.2194	0.005	4.355	3.2280	0.239	2.795
3.2219	0.041	4.474	3.2306	0.115	2.792
3.2243	0.009	4.369	3.2330	–0.111	2.788
3.2267	–0.043	4.330	3.2354	–0.165	2.793
3.2293	–0.183	4.321	3.2378	–0.152	2.781
3.2318	–0.508	4.370	3.2403	–0.088	2.787
3.2342	–0.757	4.423	3.2428	–0.014	2.780
3.2366	–0.762	4.373	3.2452	0.044	2.766
3.2390	–0.691	4.427	3.2476	0.100	2.806
3.2415	–0.591	4.483	3.2500	0.119	2.791
3.2440	–0.445	4.452	3.2524	0.140	2.797
3.2463	–0.295	4.262	3.2548	0.190	2.825

I. Telescope Tests

1. Find M15, M27 or one specified star.

2. Estimate the magnitude of a specified star.

3. Evaluate the angle distance of two stars.

II. Tests in the Planetarium

1. The showing is the night sky in Beijing on 21 o'clock tonight. You have two minutes to observe it.

The examiner will point 5 constellations using the laser pen one by one. Each constellation will be pointed about 1 minute. Write down the name of the five constellations. 25 points in total and 5 points per constellation.

2. Write down any five constellations that lie on current celestial equator. 10 minutes, 25 points. More than five constellations, no additional points.

3. The showing is the night sky in Beijing on a specified night. Determine the month that the night belongs to. What's the age of the moon for this night? Be accurate to one unit. 10 minutes, 20 points.

Assembling Telescope (indoor)

The Problem

Every team is given 10 minutes to assemble a telescope with an equatorial mount, so that it is ready for tonight's observation.

Once the competition starts, the assembling procedure will be monitored and judged by a jury, for any mistake in the process. And the assembling process will be timed. When the assembling is finished, the students of the group should raise their hands to indicate the assembling is completed. The jury should record the time taken for the assembling, after which the students should not be allowed to touch the telescope again. After the jury has checked the assembled telescope for the assembling quality, the participating group should take apart the telescope assembly and restore the various parts to the condition as they were before the assembling process.

The coordinates of Beijing is $(116°48', 40°32')$

Procedure:

The competition is divided into 4 rounds, with each round having 6 teams participating. The team with highest overall score wins.

Marking scheme

1. Time taken for the assembly: 50%
2. Team participation and collaborating skills: 20%
3. Major mistakes: 30%:
   (a) The balance of the telescope, in both axes.
   (b) Is the parts corrected put together: finder scope, fine adjustment knobs in both axes, and eyepieces, etc.
   (c) Are all the screws and knobs securely fastened?
   (d) Is the polar axis roughly adjusted? (The participants will be given the rough condition of the North.)

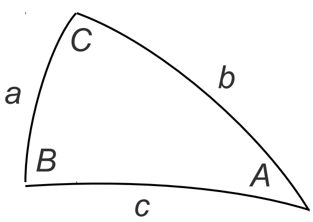
IOAA 2011 – Krakow-Katowice-Chorzow, Poland

The fifth IOAA was held from 25^{th} August to 4^{th} September 2011. This was the first IOAA to be held in Europe. The participants came from 26 countries, including the first time participation by Bulgaria, Colombia, Croatia, Hungary and Portugal.

Astronomical and physical constants

Astronomical unit (AU)	1.4960×10^{11} m
Light year (ly)	9.4605×10^{15} m = 63 240 AU
Parsec (pc)	3.0860×10^{16} m = 206 265 AU
1 Sidereal year	365.2564 solar days
1 Tropical year	365.2422 solar days
1 Calendar year	365.2425 solar days
1 Sidereal day	$23^{h}\ 56^{m}\ 04^{s}.091$
1 Solar day	$24^{h}\ 03^{m}\ 56^{s}.555$ units of sidereal time
Mass of Earth	5.9736×10^{24} kg
Mean radius of Earth	6.371×10^{6} m
Equatorial radius of Earth	6.378×10^{6} m
Mean velocity of Earth on its orbit	29.783 km s^{-1}
Mass of Moon	7.3490×10^{22} kg
Radius of Moon	1.737×10^{6} m
Mean Earth – Moon distance	3.844×10^{8} m
Mass of Sun	1.98892×10^{30} kg
Radius of Sun	6.96×10^{8} m
Effective temperature of the Sun	5 780 K
Luminosity of the Sun	3.96×10^{26} J s^{-1}
Solar constant	1366 W m^{-2}
Brightness of the Sun in V-band	−26.8 mag.
Absolute brightness of the Sun in V-band	4.75 mag.
Absolute bolometric brightness of Sun	4.72 mag.
Angular diameter of the Sun	$30'$
Speed of light in vacuum (c)	2.9979×10^{8} m s^{-1}
Gravitational constant (G)	6.6738×10^{-11} N m^{2} kg^{-2}
Boltzmann constant (k)	1.381×10^{-23} m kg s^{-2} K^{-1}
Stefan−Boltzmann constant (σ)	5.6704×10^{-8} kg s^{-3} K^{-4}
Planck constant (h)	6.6261×10^{-34} J s
Wien's constant (b)	2.8978×10^{-3} m K
Hubble constant (H_0)	70 km s^{-1} Mpc^{-1}
electron charge (e)	1.602×10^{-19} C
Current inclination of the ecliptic (ε)	$23^{o}\ 26.3'$
Coordinates of the northern ecliptic pole for epoch 2000.0 (α_E , δ_E)	$18^{h}\ 00^{m}\ 00^{s}$, $+ 66^{o}\ 33.6'$
Coordinates of the northern galactic pole for epoch 2000.0 (α_G , δ_G)	$12^{h}\ 51^{m}$, $+ 27^{o} 08'$

You can try to solve an equation $x = f(x)$ using iteration: $x_{n+1} = f(x_n)$.



Basic equations of spherical trigonometry

$$\sin a \sin B = \sin b \sin A \ ,$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \ ,$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \ .$$

Short theoretical questions

Each question max 10 points

1. Most single-appearance comets enter the inner Solar System directly from the Oort Cloud. Estimate how long it takes a comet to make this journey. Assume that in the Oort Cloud, 35 000 AU from the Sun, the comet was at aphelion.

2. Estimate the number of stars in a globular cluster of diameter 40 pc, if the escape velocity at the edge of the cluster is 6 km s^{-1} and most of the stars are similar to the Sun.

3. On 9 March 2011 the Voyager probe was 116. 406 AU from the Sun and moving at 17.062 km s^{-1}. Determine the type of orbit the probe is on: (a) elliptical, (b) parabolic, or (c) hyperbolic. What is the apparent magnitude of the Sun as seen from Voyager?

4. Assuming that Phobos moves around Mars on a perfectly circular orbit in the equatorial plane of the planet, give the length of time Phobos is above the horizon for a point on the Martian equator. Use the following data:
   Radius of Mars R_{Mars} = 3 393 km Rotational period of Mars T_{Mars} = 24.623 h. Mass of Mars $M_{\mathrm{Mars}} = 6.421 \times 10^{23}$kg Orbital radius of Phobos R_{P} = 9 380 km .

5. What would be the diameter of a radiotelescope working at a wavelngth of λ = 1 cm with the same resolution as an optical telescope of diameter D = 10 cm?

6. Tidal forces result in a torque on the Earth. Assuming that, during the last several hundred million years, both this torque and the length of the sidereal year were constant and had values of 6.0×10^{16} N m and 3.15×10^{7} s respectively, calculate how many days there were in a year 6.0 x 10^{8} years ago. Moment of inertia of a homogeneous filled sphere of radius R and mass m is $I = \frac{2}{5} m\, R^2$

7. A satellite orbits the Earth on a circular orbit. The initial momentum of the satellite is given by the vector **p**. At a certain time, an explosive charge is set off which gives the satellite an additional impulse $\Delta\mathbf{p}$, equal in magnitude to |**p**|. Let α be the angle between the vectors **p** and $\Delta\mathbf{p}$, and β between the radius vector of the satellite and the vector $\Delta\mathbf{p}$. By thinking about the direction of the additional impulse $\Delta\mathbf{p}$, consider if it is possible to change the orbit to each of the cases given below. If it is possible mark YES on the answer sheet and give values of α and β for which it is possible. If the orbit is not possible, mark NO.

   (a) a hyperbola with perigee at the location of the explosion.

(b) a parabola with perigee at the location of the explosion.

(c) an ellipse with perigee at the location of the explosion.

(d) a circle.

(e) an ellipse with apogee at the location of the explosion.

Note that for $\alpha = 180^{\circ}$ and $\beta = 90^{\circ}$ the new orbit will be a line along which the satellite will free fall vertically towards the centre of the Earth.

8. Assuming that dust grains are black bodies, determine the diameter of a spherical dust grain which can remain at 1 AU from the Sun in equilibrium between the radiation pressure and gravitational attraction of the Sun. Take the density of the dust grain to be $\varrho = 10^3$ kg m^{-3}.

9. Interstellar distances are large compared to the sizes of stars. Thus, stellar clusters and galaxies which do not contain diffuse matter essentially do not obscure objects behind them. Estimate what proportion of the sky is obscured by stars when we look in the direction of a galaxy of surface brightness $\mu = 18.0$ mag arcsec^{-2}. Assume that the galaxy consists of stars similar to the Sun.

10. Estimate the minimum energy a proton would need to penetrate the Earth's magnetosphere. Assume that the initial penetration is perpendicular to a belt of constant magnetic field 30 μT and thickness 1.0×10^4 km. Prepare the sketch of the particle trajectory. (Note that at such high energies the momentum can be replaced by the expression E/c. Ignore any radiative effects).

11. Based on the spectrum of a galaxy with redshift $z = 6.03$ it was determined that the age of the stars in the galaxy is from 560 to 600 million years. At what z did the epoch of star formation occur in this galaxy? Assume that the current age of the Universe is $t_0 = 13.7 \times 10^9$ years and that the rate of expansion of the Universe is given by a flat cosmological model with cosmological constant $\Lambda = 0$. (In such a model the scale factor $R \propto t^{2/3}$, where t is the time since the Big Bang.)

12. Due to the precession of the Earth's axis, the region of sky visible from a location with fixed geographical coordinates changes with time. Is it possible that, at some point in time, Sirius will not rise as seen from Krakow, while Canopus will rise and set? Assume that the Earth's axis traces out a cone of angle 47°. Krakow is at latitude 50.1° N; the current equatorial coordinates (right ascension and declination) of these stars are:
Sirius (α CMa) : $6^{h}\,45^{m}$, $-16^{\circ}\,43'$
Canopus (α Car) : $6^{h}\,24^{m}$, $-52^{\circ}\,42'$

13. The equation of the ecliptic in equatorial coordinates (α, δ) has the form:
$\delta = \arctan(\sin\alpha \tan\varepsilon)$,
where ε is the angle of the celestial equator to the ecliptic plane. Find an analogous relation $h = f(A)$ for the galactic equator in horizontal coordinates (A, h) for an observer at latitude $\varphi = 49^{\circ}\,34'$ at local sidereal time $\theta = 0^{h}\,51^{m}$.

14. Estimate the number of solar neutrinos which should pass through a 1 m^2 area of the Earth's surface perpendicular to the Sun every second. Use the fact that each fusion reaction in the Sun produces 26.8 MeV of energy and 2 neutrinos.

15. Given that the cosmic background radiation has the spectrum of a black body throughout the evolution of the Universe, determine how its temperature changes with redshift z. In particular, give the temperature of the background radiation at the epoch $z \approx 10$ (that of the farthest currently observed objects). The current temperature of the cosmic background radiation is 2.73 K .

Long theoretical questions

Instructions

1. You will receive in your envelope an English and native language version of the questions.
2. You have 5 hours to solve 15 short (tasks 1-15) and 3 long tasks.
3. You can use only the pen given on the desk.
4. The solutions of each task should be written on the answer sheets, starting each question on a new page. Only the answer sheets will be assessed.
5. You may use the blank sheets for additional working. These work sheets will not be assessed
6. At the top of each page you should put down your code and task number.
7. If solution exceeds one page, please number the pages for each task.
8. Draw a box around your final answer.
9. Numerical results should be given with appropriate number of significant digits with units.
10. You should use SI or units commonly used in astronomy. Points will be deducted if there is a lack of units or inappropriate number of significant digits.
11. At the end of test, all sheets of papers should be put into the envelope and left on the desk.
12. In your solution please write down each step and partial result.

Long theoretical questions (max 30 points each)

1. A transit of duration 180 minutes was observed for a planet which orbits the star HD209458 with a period of 84 hours. The Doppler shift of absorption lines arising in the planet's atmosphere was also measured, corresponding to a difference in radial velocity of 30 km/s (with respect to observer) between the beginning and the end of the transit. Assuming a circular orbit exactly edge-on to the observer, find the approximate radius and mass of the star and the radius of the orbit of the planet.

2. Within the field of a galaxy cluster at a redshift of $z = 0.500$, a galaxy which looks like a normal elliptical is observed, with an apparent magnitude in the B filter $m_B = 20.40$ mag.

   The luminosity distance corresponding to a redshift of $z = 0.500$ is $d_L = 2754$ Mpc.

   The spectral energy distribution (SED) of elliptical galaxies in the wavelength range 250 nm to 500 nm is adequately approximated by the formula:

$$L_\lambda(\lambda) \propto \lambda^4$$

   (i.e., the spectral density of the object's luminosity, known also as the monochromatic luminosity, is proportional to λ^4.)

   a) What is the absolute magnitude of this galaxy in the B filter ?

   b) Can it be a member of this cluster? (write YES or NO alongside your final calculation)

   Hints: Try to establish a relation that describe the dependence of the spectral density of flux on distance for small wavelength interval. Normal elliptical galaxies have maximum absolute magnitude equal to -22 mag.

3. The planetarium program 'Guide' gives the following data for two solar mass stars:

Star	1	2
Right Ascension	$14^h\,29^m\,44.95^s$	$14^h\,39^m\,39.39^s$
Declination	–62º 40′ 46.14″	–60 50′ 22.10″
Distance	1.2953 pc	1.3475 pc
Proper motion in R.A.	–3.776 arcsec / year	–3.600 arcsec / year
Proper motion in Dec.	0.95 arcsec / year	0.77 arcsec / year

   Based on these data, determine whether these stars form a gravitationally bound system. Assume the stars are on the main sequence. Write YES of bound or NO if not bound alongside your final calculation.

   Note: the proper motion in R.A. has been corrected for the declination of the stars.

Data Analysis questions

Instructions

1. In your envelope you will receive an English and native language versions of the questions.
2. You have 2.5 hours to solve 2 tasks.
3. You can get maximum 25 points for each task.
4. You can use only the pen and tools given on the desk.
5. The solutions of each task should be written on the answer sheets, starting each question on a new page. Only the answer sheets will be assessed.
6. You may use the blank sheets for additional working. These work sheets will not be assessed
7. At the top of each page you should put down your code and task number.
8. If solution exceeds one page, please number the pages for each task.
9. Draw a box around your final answer.
10. Numerical results should be given with appropriate number of significant digits with units.
11. You should use SI or units commonly used in astronomy. Points will be deducted if there is a lack of units or inappropriate number of significant digits.
12. At the end of the test, all sheets of paper should be put into the envelope and left on the desk.
13. In your solution please write down each step and partial results.
14. Graphs for tasks number 1 and 2 should be prepared on the plotting paper.

Data Analysis questions

1. Analysis of times of minima

Figure 1 shows the light curve of the eclipsing binary V1107 Cas, classified as a W Ursae Majoris type.

Table 1 contains a list of observed minima of the light variation. The columns contain: the number of the minimum, the date on which the minimum was observed, the heliocentric time of minimum expressed in Julian days and an error (in fractions of a day).

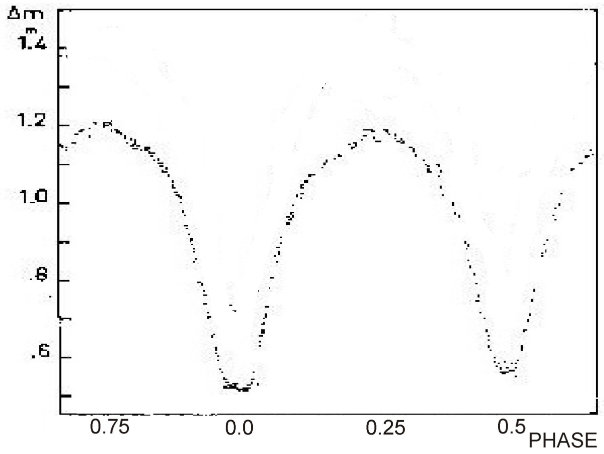


Fig. 1: Light curve of V1107 Cas.

Using these data:

(a) Determine an initial period of V1107 Cas, assuming that the period of the star is constant during the interval of observations. Assume that observations during one night are continuous. Duration of the transit is negligible.

(b) Make what is known as an (O−C) diagram (for "observed − calculated") of the times of minima, as follows: on the *x*-axis put the number of periods elapsed (the "epoch") since a chosen initial moment M_o; on the *y*-axis the difference between the observed moment of minimum M_{obs} and the moment of minimum calculated using the formula ("ephemeris"):

$$M_{calc} = M_o + P \times E$$

where *E,* the epoch, is exactly an integer or half-integer, and *P* is the period in days.

(c) Using this (O−C) diagram, improve the determination of the initial moment M_o and the period *P*, and estimate the errors in their values.

(d) Calculate the predicted times of minima of V1107 Cas given in heliocentric JD occurring between 19h, 1 September 2011 UT and 02h, 2 September 2011 UT.

No.	Date of minimum (UT)	Time of minimum (Heliocentric JD)	Error
1	22 December 2006	2 454 092.4111	0.0004
2	23 December 2006	2 454 092.5478	0.0002
3	23 September 2007	2 454 367.3284	0.0005
4	23 September 2007	2 454 367.4656	0.0005
5	15 October 2007	2 454 388.5175	0.0009
6	15 October 2007	2 454 388.6539	0.0011
7	26 August 2008	2 454 704.8561	0.0002
8	5 November 2008	2 454 776.4901	0.0007
9	3 January 2009	2 454 835.2734	0.0007
10	15 January 2009	2 454 847.3039	0.0004
11	15 January 2009	2 454 847.4412	0.0001
12	16 January 2009	2 454 847.5771	0.0004

Table 1: Observed times of minima of V1107 Cassiopeae

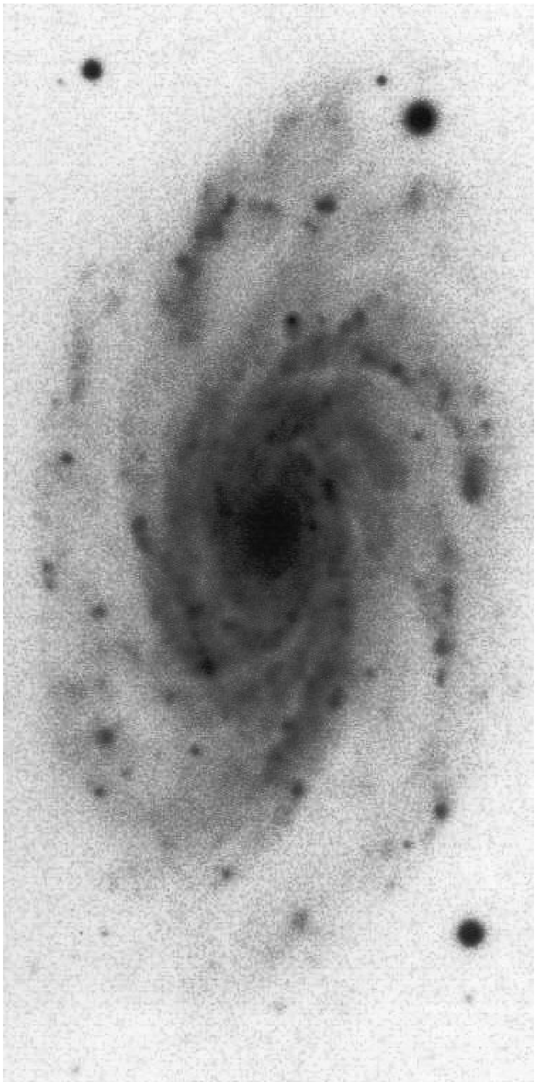
2. Weighing a galaxy

The attached images show a photograph of the spiral galaxy NGC 7083, which lies at a distance of 40 Mpc, and a fragment of its spectrum. The slit of the spectrograph was aligned with the major axis of the image of the galaxy. The *x*-axis of the spectrum represents wavelength, and the *y*-axis represents the angular distance of the emitting region from the core of the galaxy, where 1 pixel = 0.82 arcsec. Two bright emission lines are visible, with rest wavelengths of $\lambda_1 = 6564$ Å, $\lambda_2 = 6584$ Å.

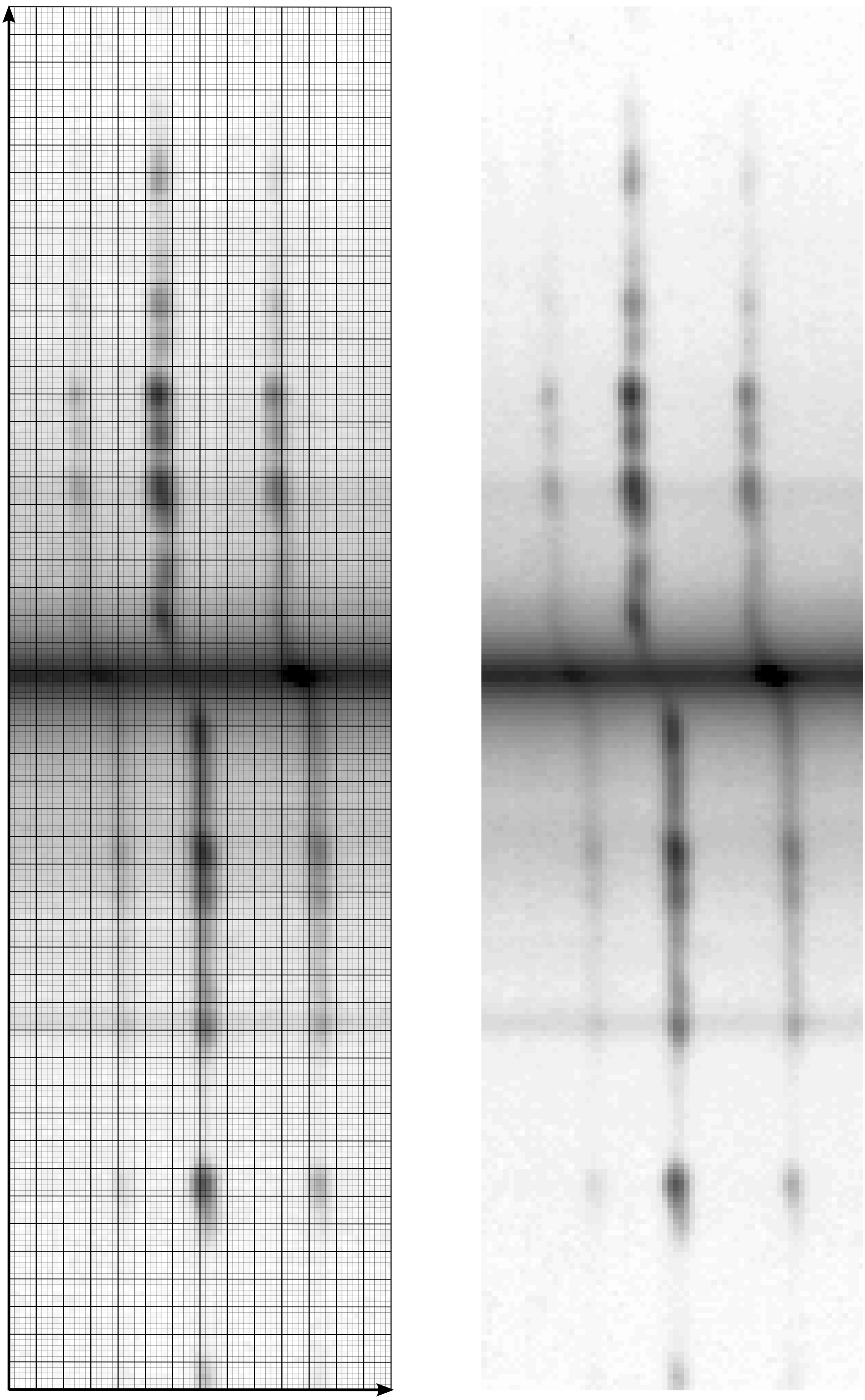
> Use the spectrum to plot the rotation curve of the galaxy and estimate the mass of the central bulge.

Assumption: central bulge is spherical.

The photograph of the galaxy has the correct proportions.



NGC 7083



Spectrum of NGC 7083. Grid marks pixels.

Observing competition – planetarium round

General instructions

1. There are 2 questions, each worth 25 points. You have **80** minutes to solve them, of which :

   (a) **20** minutes for reading the question and preparing for the observations,

   (b) **40** minutes to perform all the observations in the planetarium
   (20 minutes for each questions),

   (c) **20** minutes for calculations and finishing your work.

2. Additional time is allowed to move to and from the planetarium.
3. Along with the questions you will be given a map of the sky, for use with both questions. The map is for epoch J 2000.0, using a polar projection with a linear scale in declination, and covers stars down to about 5th magnitude. You will also be given paper for working and notes, writing implements, a pencil sharpener and an eraser.

   Please take everything from the desk in the first room with you to the planetarium dome, as you will be going to a different room afterwards to finish your work.
4. At your place in the dome you will find a torch and clipboard. Please leave these two items behind for the next contestant.
5. Only answers <u>given in the appropriate places on the question sheet and on the map</u> of the sky will be assessed. The additional worksheets will not be assessed.
6. Clearly mark every page with your code number.

About the questions

In Question 1 :

1. The sky is stationary, the observer is on the surface of the Earth.
2. Visible on the sky are: a comet, the Moon and a nova of about 2nd magnitude.
3. From the 11th minute, a grid representing horizontal coordinates will be projected on the sky, and will remain on until the end of the question.

In Question 2 :

1. Four consecutive days on the surface of Mars will be shown.
2. There is a Martian base visible on the horizon.
3. During the Martian daytime the sky will be slightly brightened.
4. The moons of Mars and the other planets will <u>not</u> be displayed.
5. The local meridian will be continuously visible on the sky.

<u>Note:</u> Azimuth is counted from 0° to 360° starting at S through W, N, E.

Observing competition – planetarium round

1. Earth

A) On the map of the sky, mark (with a cross) and label the nova (mark it “N”) and the Moon (mark it with a Moon symbol) and draw the shape and position of the comet.

B) In the table below, circle only those objects which are above the astronomical horizon. Note: you will lose 1 point for every incorrect answer.

M20 – Triffid Nebula	o Cet – Mira	δ CMa – Wezen
α Cyg – Deneb	M57 – Ring Nebula	β Per – Algol
δ Cep – Alrediph	α Boo – Arcturus	M44 – Praesepe (Beehive Cluster)

C) When the coordinate grid is visible, mark on the map the northern part of the local meridian (from the zenith to the horizon) and the ecliptic north pole (with a cross and marked “P”).

D) For the displayed sky, give the :

geographical latitude of the observer : $\varphi =$ . . . . ,

Local Sidereal Time : $\theta =$ . . . . ,

time of year, by circling the calendar month :

Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec.

E) Give the names of the objects, whose approximate horizontal coordinates are :

azimuth $A_1 = 45°$ and altitude $h_1 = 58°$: . . . . ,

azimuth $A_2 = 278°$ and altitude $h_2 = 20°$: . . . .

(If you can, use Bayer designations, IAU abbreviations and Messier numbers or English or Latin names.)

F) Give the horizontal coordinates (azimuth, altitude) of :

Sirius (α CMa) : $A_3 =$ . . . . ; $h_3 =$ . . . .

The Andromeda Galaxy (M 31) : $A_4 =$ . . . . ; $h_4 =$ . . . .

G) Give the equatorial coordinates of the star marked on the sky with a red arrow :

$\alpha =$ . . . . ; $\delta =$ . . . .

2. Mars

H) Give the areographic (Martian) latitude of the observer : $\varphi = \ldots\ldots\ldots$

I) Give the altitudes of upper (h_u) and lower (h_l) culmination of :

Pollux (β Gem) : $h_u = \ldots\ldots\ldots\ldots$; $h_l = \ldots\ldots\ldots\ldots$,

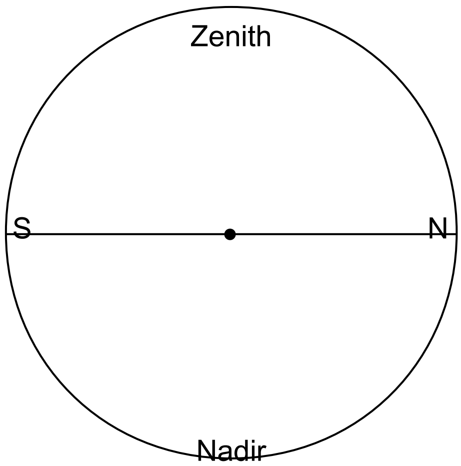
Deneb (α Cyg) $h_u = \ldots\ldots\ldots\ldots$; $h_l = \ldots\ldots\ldots\ldots$,

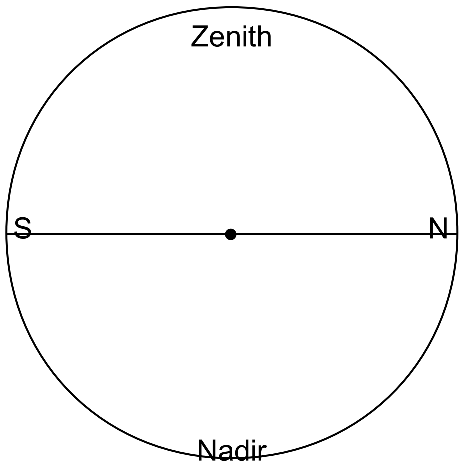
J) Give the areocentric (Martian) declination of :

Regulus (α Leo) $\delta = \ldots\ldots\ldots\ldots$.

Toliman (α Cen) $\delta = \ldots\ldots\ldots\ldots$.

K) Sketch diagrams to illustrate your working in questions (I) and (J) above :





L) on the map of the sky, mark (with a cross) and label ("M") the Martian celestial North Pole.

M) Give the azimuth of the observer as seen from the Martian base :

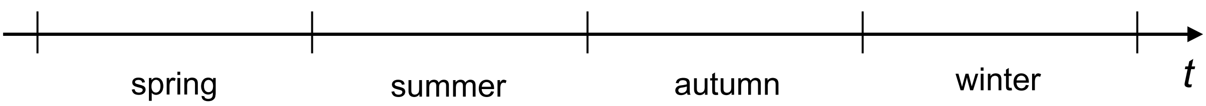
$A = \ldots\ldots\ldots\ldots$.

N) Estimate the location of the base on Mars, and circle the appropriate description :

a. near the Equator *b.* near the northern Tropic circle

c. near the northern Arctic circle *d.* near the North Pole

O) The time axis below shows the Martian year and the seasons in the northern hemisphere. Mark the date represented by the planetarium display on the axis.



Observing competition – night round

Instructions

1. There are 2 questions, each worth 25 points. You have **80** minutes to solve them, of which :
   (a) **25** minutes for reading the question and preparing for the observations,
   (b) **30** minutes to perform all the observations at the telescope (for both questions),
   (c) **25** minutes for calculations and finishing your work.

2. Additional time is allowed to move to and from the observing site.

3. Along with the questions you will receive a map of the sky, for use with both questions.

4. At the observing site you will find ready :
   (a) a refracting telescope with a right-angle mirror and an eyepiece with an illuminated reticle, which can be rotated about the optical axis,
   (b) a red torch, stopwatch, pencil, eraser and clipboard,
   (c) a chair.

   <u>Note:</u> the telscope is already aligned – do <u>not</u> change the position of the tripod!

   The brightness of the reticle can be adjusted by turning the on-off switch.

5. You are allowed to take only the questions, answer sheet and blank paper for additional work with you to the telescope.

6. Only the answer sheet will be assessed. The additional worksheets will not be assessed.

7. Clearly mark every page of the answer sheet with your code number.

8. If you have difficulty with the equipment (not related to the question) or disturb the alignment of the telescope, call an assistant.

Observing competition – night round

1. The Little Dolphin

An asterism known as the Little Dolphin lies near a line connecting the stars α Peg (Markab) and β Peg (Scheat). It is marked with a cicle on the large-scale map.

The map also shows the constellation of Delfinus, the Dolphin, with the brightest stars labelled with their Bayer designations (α, β, γ, δ and ε).

The coordinates of α and β Peg and the Little Dolphin (in right ascension order) are:

	Right Ascension α	Declination δ
Little Dolphin	$23^{h}\,02^{m}$	+23.0°
β Peg	$23^{h}\,04^{m}$	+28.1°
α Peg	$23^{h}\,05^{m}$	+15.2°

Based on your observations, make two drawings on the answer sheet :

On Drawing 1 :

Draw the view of the constellation **Delphinus** (Del) as seen through the finder scope. Include as many stars as you can see in the field of view.

With an arrow, mark the apparent direction of motion of the stars across the field of view of the finder scope caused by the rotation of the Earth.

Label the stars with the Bayer designations given on the map (α, β, γ, δ and ε).

Also label the brightest of these 5 stars "m_{max}".

Also label the faintest of these 5 stars "m_{min}".

On Drawing 2 :

Draw the view of the **Little Dolphin** as seen through the larger telescope. Include as many stars as you can see in the field of view.

With an arrow, mark the apparent direction of motion of the stars across the field of view of the telescope caused by the rotation of the Earth.

Label the stars of the Little Dolphin α′, β′, γ′, δ′ and ε′ such that they match the labels of the stars in the constellation Delphinus as given on the map.

Label the brightest of these 5 stars "m_{max}".

2. Determining declination

The two pictures on the next page show a small asterism, as seen directly on the sky and as a mirror image. Three stars are labelled: S1, S2 and S*x*. The position of the asterism is also marked with a rectangle on the larger-scale map of the sky.

Find this asterism and point your telescope to it.

Using the illuminated reticle as a fixed reference point, and the stopwatch, measure the time taken for the stars S1, S2 and *Sx* to move across the field. You may rotate the eyepiece so that the cross-hairs of the reticle are in the most convenient position for your measurement.

Use your measurements and the known declinations of stars S1 and S2 as given below to determine the declination of star *Sx.*

On the answer sheet, give your measurements and working, and estimate the random error in your result.

For each set of measurements you make, draw the view through the eyepiece on the answer sheet. (Use the blank circular field on the answer sheet.)

Mark the drawing with the compass directions N and E. Draw the reticle and the tracks of the stars to show the motion which you timed using the stopwatch.

Mark the ends of each timed track and show which time measurement refers to which track – for example, for measurement "T1" marking the ends "Start T1" and "End T1".

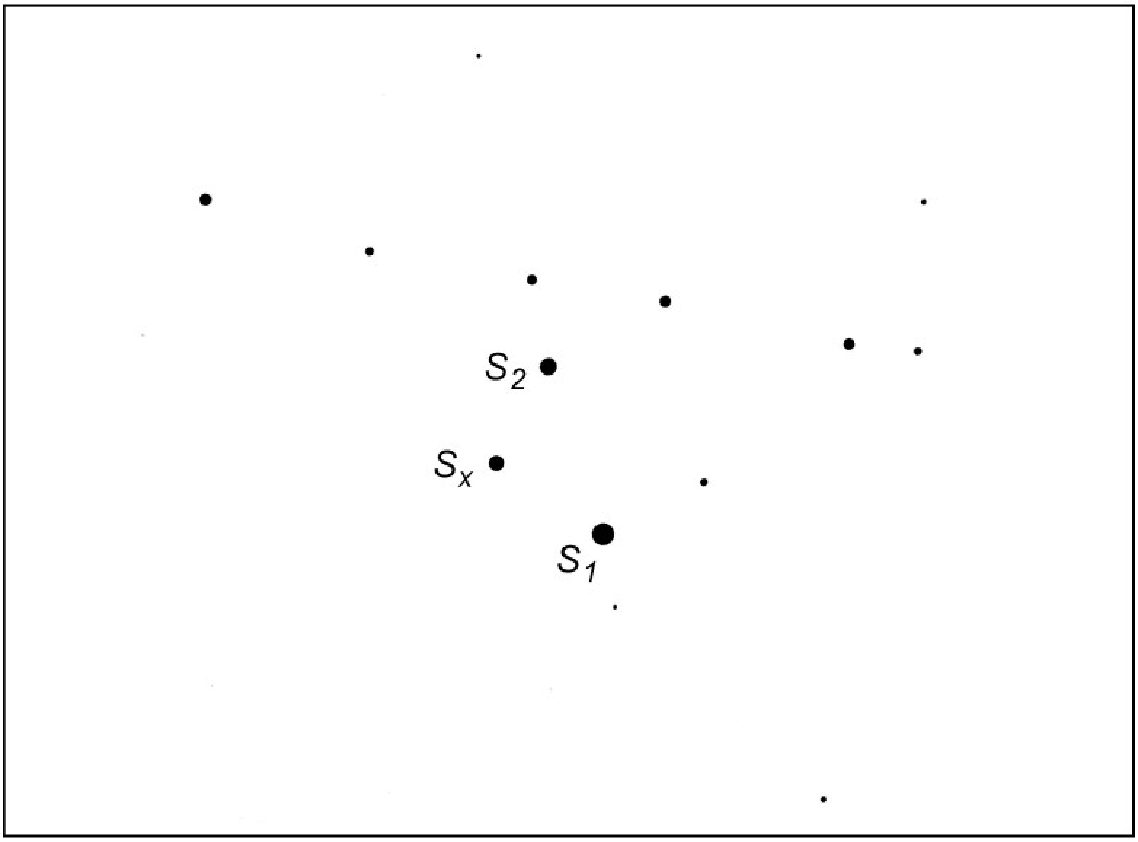
The angle of the reticle can be easily adjusted by rotating the eyepiece around its optical axis. If you change the angle of the reticle for a new measurement, draw a new diagram.

The declinations of the field stars S1 and S2 are :

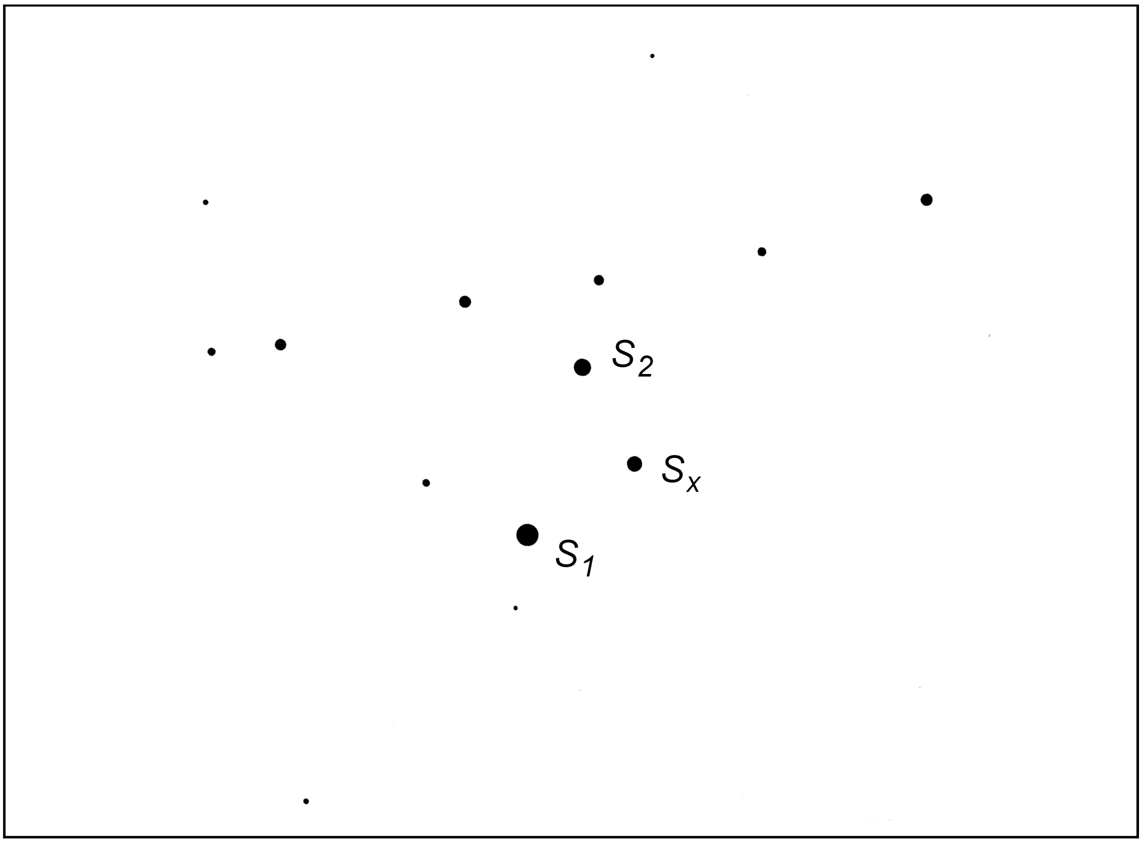
S_1 : $\delta = +19^\circ\, 48'\, 18''$ S_2 : $\delta = +20^\circ\, 06'\, 10''$

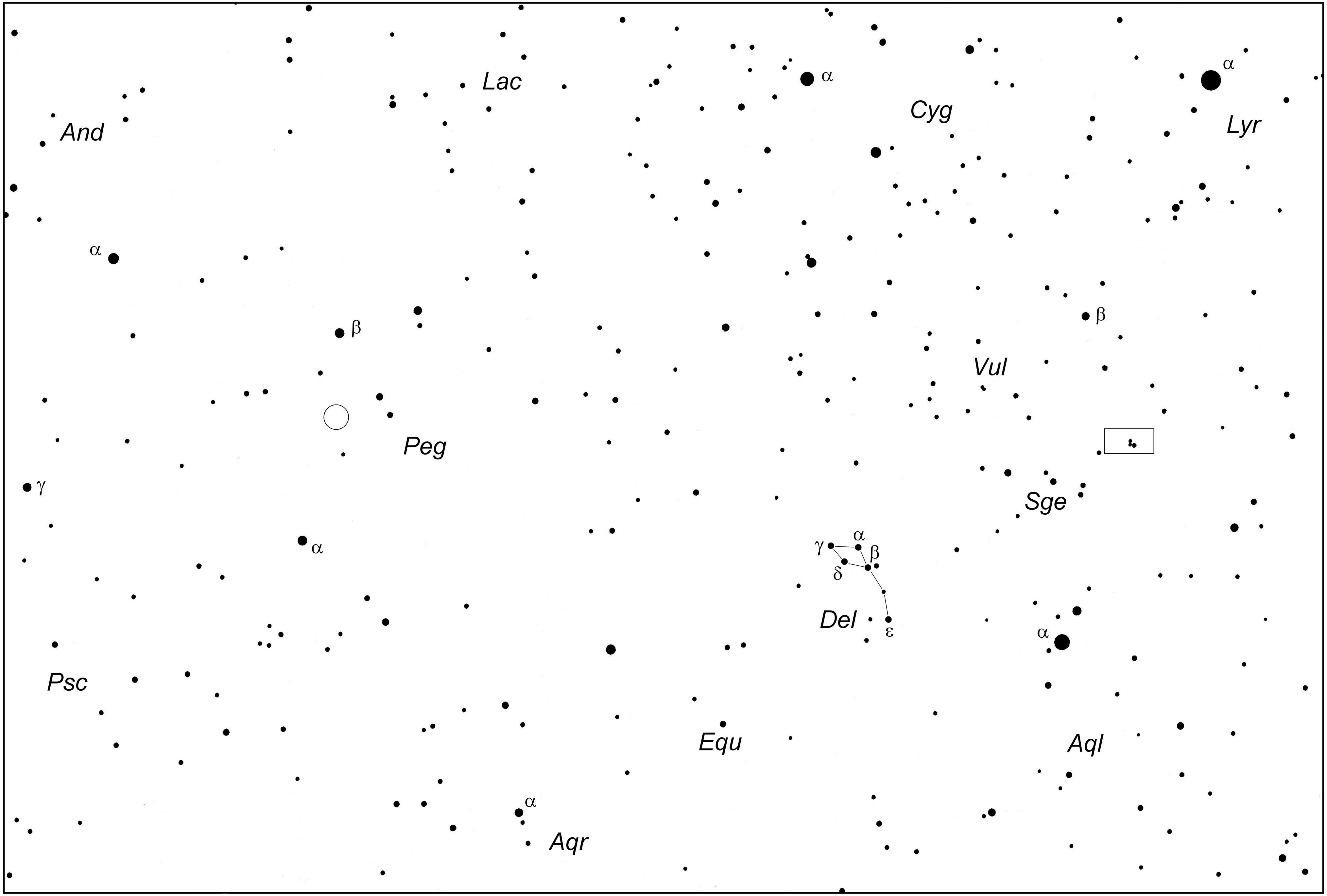
Assume that: $\delta(S_2) > \delta(Sx) > \delta(S_1)$.

Direct view:



Mirror image:





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IOAA 2012 – Rio de Janeiro & Vassouras, Brazil

The sixth IOAA was held from 4^{th} to 13^{th} August 2012. This was the first IOAA to be held in the western hemisphere. The participants came from 28 countries including reintroduction of Singapore and a solitary student representing United Arab Emirates.

« به نام یگانه ی هستی بخش »

سوالات ششمین المپیاد جهانی نجوم و اخترفیزیک

۲۰۱۲ برزیل

6th IOAA

Rio de Janeiro 2012

متن انگلیسی

6thioaa.blogsky.com

Theoretical Exam - Short Questions

1. At Brazil's **National Observatory**, located at the city of Rio de Janeiro (22° 54' S, 43° 12' W), there is a sundial above the door of the dome of the 32cm telescope, facing to the north. The dial lies on the plane East-Zenith-West and the rod is parallel to the Earth's axis. For which declinations of the sun and during what period of the year (months and seasons) the clock (i) does not work during, at least, some fraction of the day?, and (ii) does not work at all during the day?

2. Calculate the length of the sidereal day on Earth. What would be the length of the solar and sidereal days, in the current time measures (our solar hours, minutes and seconds), if the Earth would rotate in the opposite direction, but with the same rotation speed?

3. What is the time interval between two consecutive oppositions of Mars? Assume the orbit is circular.

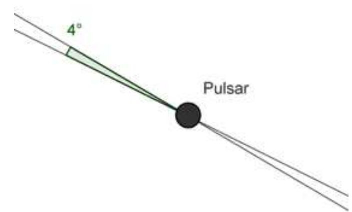
4. What would be full Moon's visual magnitude if its albedo were equal to 1?

5. Calculate the ratio between the average densities of the Earth and the Sun, using **ONLY** the dataset below:
   - the angular diameter of the Sun, as seen from Earth
   - the gravitational acceleration on Earth's surface
   - the length of the year
   - the fact that one degree in latitude at Earth's surface corresponds to 111 km

6. Most of the energy emitted by the Sun is generated in its core via the so-called proton-proton (p-p) nuclear chain reaction, which has three different branches. The most energetic branch transforms 2 He^3 into $He^4 + 2H^1$. Calculate the energy released (in MeV) and the fractional reduction of the mass of the particles involved in this reaction.

7. **Luminous Blue Variable** (LBV) stars greatly vary in visual brightness; however, the bolometric magnitude remains constant. Imagine a LBV star with a black body temperature of 5 000 K at its maximum visual brightness, and 30 000 K at its minimum visual brightness. Calculate the ratio of the star radius between both situations above.

8. A pulsar located 1000 pc far from Earth, 10 000 times more luminous than our Sun, emits radiation only from its two opposite poles, creating an homogeneous emission beam shaped as double cone with opening angle $\alpha = 4°$. Assuming the angle between the rotation axis and the emission axis is 30°, and assuming a random orientation of the pulsar beams in relation to an observer on Earth, what is the probability of detecting the pulses? In case we can see it, what is the apparent bolometric magnitude of the pulsar?



9. An old planetary nebula, with a white dwarf (WD) in its center, is located 50 pc away from Earth. Exactly in the same direction, but behind the nebula, lies another WD, identical to the first, but located at 150 pc from the Earth. Consider that the two WDs have absolute bolometric magnitude +14.2 and intrinsic color indexes B-V = 0.300 and U-V = 0.330. Extinction occurs in the interstellar medium and in the planetary nebula.
When we measure the color indices for the closer WD (the one who lies at the center of the nebula), we find the values B-V = 0.327 and U-B = 0.038. In this part of the Galaxy, the interstellar extinction rates are 1.50, 1.23 and 1.00 magnitudes per kiloparsec for the filters U, B and V, respectively. Calculate the color indices as they would be measured for the second star.

10. Assume that the universe currently is well described by a density parameter $\Omega_0 = 1$, there is no dark energy, and the current temperature of the universe is 2.73 K. Knowing that the temperature of the universe is inversely proportional to its radius (the scale factor), compute how long, starting from the present time, it will take to the Universe to cool down by 0.1 K

11. What is the angular amplitude of the oscillatory motion of the Sun, due to the existence of Jupiter, as measured by an observer located at Barnard's Star? What is the period of this oscillation?

12. What is the minimum diameter of a telescope, observing in the visible and near ultraviolet bands, located in one of the Lagrangian points L4 or L5 of the Sun-Earth system, in order to be able to detect the Earth's wobbling relative to the ecliptic plane caused by the gravitational action of the Moon?

13. An astronomer in the southern hemisphere contemplates the rise of the south ecliptic pole and wonders how fun it would be if the sky started spinning around the ecliptic pole, instead of the usual celestial pole. Sketch the displacement of this observer over the Earth's surface, to observe the stars revolving around south ecliptic pole in the same direction and with the same period that they usually revolve around the south celestial pole. Sketch the observer's trajectory for one entire day. Determine its velocity (direction and speed) when crossing the Equator for the first time.

14. An observer in Salonika ($\varphi = +40.65°$), Greece, quietly contemplates the starry sky when he realizes that a very bright object (α = 5h55min, $\delta = +7.41°$, $m = 0.45$), when reaching its upper culmination, mysteriously detaches from the celestial sphere and continues moving at the same tangential speed, remaining in this movement for all eternity, Assume that the Earth stands still and the celestial sphere rotates. Then, determine the final alt-azimuthal coordinates of the object. How long will it take for its apparent magnitude to change to 6.00?

15. Christ, the Redeemer is the most famous Brazilian monument. But there are many similar statues in other Brazilian cities and across the world. Imagine that an exact copy of the

monument was built on Borradaile Island, at latitude φ = -66.55°, the first place south of the Antarctic Circle reached by man.

Assume the island is exactly on the Antarctic Circle, and define a Cartesian coordinate system (Oxy) on the horizontal plane, with the origin O being at the base of the Christ, the Ox axis in the East-West direction and the Oy axis in the North-South direction. Determine the equation of the curve described by the tip of the Christ's head shadow on the horizontal plane, on a sunny solstice day and the minimum length of the shadow during that day (neglect the motion of the sun in declination during the day). Neglect the atmospheric effects.

Theoretical Exam - Long Questions

1. An astronomer on Earth observes a globular cluster, which has an angular diameter α and contains N stars, each one with the same absolute magnitude M_0, and is at a distance D from the Earth. A biologist is in the center of that cluster.
   1.1. What is the difference between the combined visual magnitudes of all stars observed by the astronomer and the biologist. **Consider that the spatial distribution of stars in the cluster is perfectly homogeneous and the biologist is measuring the combined magnitude of the entire cluster.**
   1.2. What is the diameter of the astronomer's telescope, considering he wants to visualize the cluster with the same brightness that the biologist sees?
   1.3. What would be the difference between the visual magnitudes observed by the two scientists, if the diameter of the field of view of the biologist is also α.

2. Astronomers studied a spiral galaxy with an inclination angle of 90º from the plane of the sky ("edge-on") and apparent magnitude 8.5. They measured the rotational velocity and radial distance from the Galactic center and plotted its rotation curve.
   2.1. Approximate the rotation curve in Figure 1 with a continuous function V(D) composed of two straight lines.
   2.2. Using the same observations, they estimated that the rotation period of the pressure wave in the galactic disk is half of the rotation period of the mass of the disk.

Estimate the time it takes for a spiral arm to take another turn around the galactic center (use the function constructed in 2.1).

2.3. Calculate the distance to the galaxy using the Tully-Fisher relation (see Table of Constants).

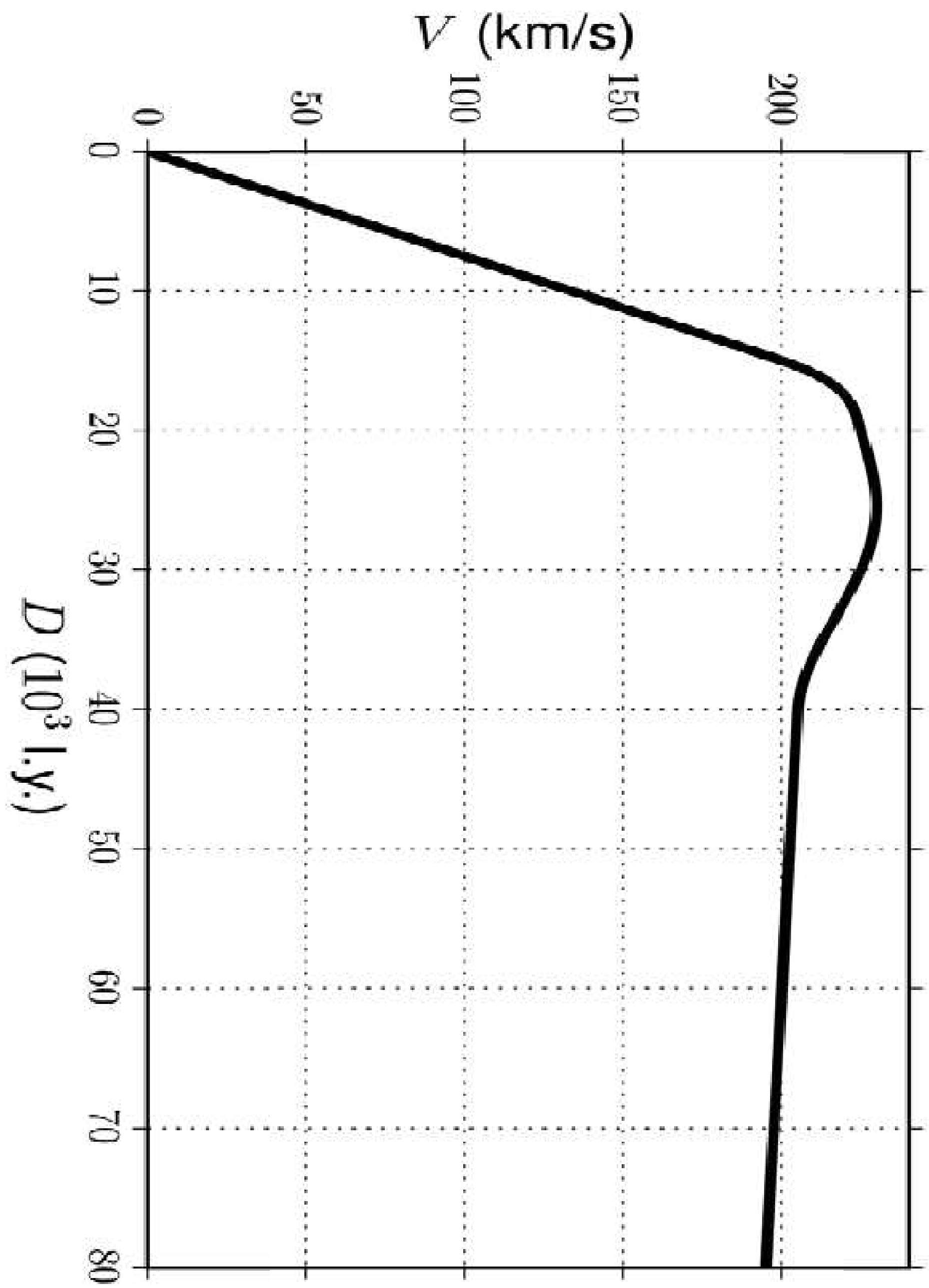
2.4. Calculate the maximum and minimum values of the observed wavelengths of the hydrogen lines corresponding to 656.28 nm in the spectrum of this galaxy. Hint: also take into account the cosmological expansion.

2.5. Using Figure 1, estimate the mass of the galaxy up to a radius of 3×10^3 light-years.

2.6. Estimate the number of stars of the galaxy, assuming that:

- the mean mass of the stars is equal to one solar mass and one third of the baryonic mass of the galaxy is in the form of stars , and;
- the fraction of baryonic to dark matter in the galaxy is the same as the fraction for the whole Universe (see Table of Constants).

FIGURE 1



Data analysis

Question 1

A few facts about photometry of asteroids:

- Asteroids are small, irregularly shaped objects of our Solar System that orbit the Sun in approximately elliptical orbits.
- Their brightness observed at a given instant from Earth depend on the surface area illuminated by the Sun and the part of the asteroid which is visible to the observer. Both vary as the asteroid moves.
- The way the sunlight is reflected by the surface of the asteroid depends on its texture and on the angle between the Sun, the asteroid and the observer (phase angle), which varies as the Earth and the asteroid move along their orbits. In particular, asteroids with surfaces covered by fine dust (called regoliths) exhibit a sharp increase in brightness at phase angles φ close to zero (i.e., when they are close to the opposition).
- Since the observed flux of any source decreases with the square of distance, the observed magnitude of an asteroid also depends on its distance from the Sun and from the observer at the time of the observation. Their apparent magnitude *m* outside the atmosphere is then

  $$M(t) = \mathrm{M_r}(t)+5\log(RD)$$

  where m_r is usually called reduced magnitude (meaning the magnitude the asteroid would have if its distances from the Sun and the Earth were reduced both to 1 *AU*) and depends only on the visible illuminated surface area and on phase angle effects. *R* and *D* are, respectivelly, the heliocentric and geocentric distances.

Consider now the following scenario. Light curves of a given asteroid were obtained at three different nights at different points of its orbit, and at each time a photometric standard was observed in the same frame of the asteroid. Table 1 shows the geometric configuration of the asteroid at each night (phase angle φ, in degrees, *R* and *D* in *AU*s), and the calibrated magnitude of the photometric standard star that was observed along with the asteroid. Consider the calibrated magnitude as the final apparent magnitude after all the effects are taken into account.

Tables 2, 3 and 4 contain, for each night, the time interval of each observation with respect to the first one (in hours) the air mass, the instrumental magnitude of the asteroid and the instrumental magnitude of the star.

Air mass is the dimensionless thickness of the atmosphere along the line of sight, and is equal to 1 when looking to zenith.

1. Plot the star magnitude *versus* air mass for each set
2. Calculate the extinction coefficient for each night (see Appendix A at the end of the text)
3. Were the observations affected by clouds in one night? Express your answer as: a) Night A, b) Night B, c) Night C, d) None of the nights.
4. Plot the calibrated magnitude *versus* time for each set of observations of the asteroid (see Appendix B).
5. Determine the rotation period for each night. Consider that the light curve for this asteroid has two minima and two maxima, and that the semi-period is the average of the intervals between the two maxima and the two minima.
6. Determine the amplitude (difference from maximum to minimum) of the light curve for each night
7. Plot the calibrated reduced magnitude M_r *versus* phase angle φ (use the mean value of each light curve)
8. Calculate the angular coefficient of the phase curve (the plot of the calibrated reduced magnitude versus the phase angle) considering only the points away from the opposition (See 3rd bullet of facts about photometry of asteroids above)
9. Is there any reason to assume a surface covered by fine dust (regolith)? Answer YES/NO.

Table 1

Night	*D*	*R*	φ	M_{star}
A	0.36	1.35	0.0	8.2
B	1.15	2.13	8.6	8.0
C	2.70	1.89	15.6	8.1

Table 2: Night A

Δt	Air mass	m_{ast}	m_{star}
0.00	1.28	7.44	8.67
0.44	1.18	7.38	8.62
0.89	1.11	7.34	8.59
1.33	1.06	7.28	8.58
1.77	1.02	7.32	8.58
2.21	1.00	7.33	8.56
2.66	1.00	7.33	8.56
3.10	1.01	7.30	8.56
3.54	1.03	7.27	8.58
3.99	1.07	7.27	8.58
4.43	1.13	7.31	8.61
4.87	1.21	7.37	8.63
5.31	1.32	7.42	8.67
5.76	1.48	7.49	8.73
6.20	1.71	7.59	8.81
6.64	2.06	7.69	8.92
7.09	2.62	7.87	9.14
7.53	3.67	8.21	9.49

Table 3: Night B

Δt	Air mass	m_{ast}	m_{star}
0.00	1.28	13.24	8.38
0.44	1.18	13.21	8.36
0.89	1.11	13.13	8.34
1.33	1.06	13.11	8.33
1.77	1.02	13.11	8.32
2.21	1.00	13.15	8.32
2.66	1.00	13.17	8.32
3.10	1.01	13.17	8.32
3.54	1.03	13.13	8.33
3.99	1.07	13.15	8.34
4.43	1.13	13.14	8.34
4.87	1.21	13.14	8.37
5.31	1.32	13.21	8.38
5.76	1.48	13.30	8.43
6.20	1.71	13.34	8.47
6.64	2.06	13.39	8.54
7.09	2.62	13.44	8.65
7.53	3.67	13.67	8.87

Table 4: Night C

Δt	Air mass	m_{ast}	m_{star}
0.00	1.28	11.64	8.58
0.44	1.18	11.53	8.54
0.89	1.11	11.56	8.60
1.33	1.06	11.49	8.52
1.77	1.02	11.58	8.48
2.21	1.00	11.79	8.63
2.66	1.00	11.67	8.53
3.10	1.01	11.53	8.46
3.54	1.03	11.47	8.48
3.99	1.07	11.63	8.67
4.43	1.13	11.51	8.51
4.87	1.21	11.65	8.55
5.31	1.32	11.77	8.61
5.76	1.48	11.88	8.75
6.20	1.71	11.86	8.78
6.64	2.06	12.03	9.03
7.09	2.62	12.14	9.19
7.53	3.67	12.63	9.65

Question 2 - Cepheids are very bright variable stars whose mean absolute magnitudes are functions of their pulsation periods. This allows astrophysicists to easily determine their intrinsic luminosities from the variation in their observed, apparent luminosities.

Below is a table with Cepheid data. P_0 is the pulsation period in days and $<M_V>$ is the mean absolute visual magnitude.

Cepheid	**P_0 (days)**	**$<M_V>$**
SU Cas	1.95	-1.99
V1726 Cyg	4.24	-3.04
SZ Tau	4.48	-3.09
CV Mon	5.38	-3.37
QZ Nor	5.46	-3.32
α UMi	5.75	-3.42
V367 Sct	6.30	-3.58
U Sgr	6.75	-3.64
DL Cas	8.00	-3.80
S Nor	9.75	-3.95
ζ Gem	10.14	-4.10
X Cyg	16.41	-4.69
WZ Sgr	21.83	-5.06
SW Vel	23.44	-5.09
SV Vul	44.98	-6.04

1) Plot all Cepheids in a scatter diagram. $\log_{10}(P_0)$ should be the abscissa and $<M_V>$ should be the ordinate.

2) Fit, using least squares, a straight line to the $<M_V>$ vs $\log_{10}(P_0)$ plot. This equation allows one to obtain the absolute magnitude from the pulsation period for any Cepheid.

3) Figures 1 and 2 show the light curves of two Cepheids. Use the available data to estimate the distances to each of these two Cepheids. Also estimate the uncertainty of the distance determination (exact formulae are not necessary).

4) Comparing the difference between the distances of the two stars with the typical size of a galaxy, would it be possible for these two stars to be in the same galaxy? Please mark "YES" ( ) or "NO" ( ).

Figure 1

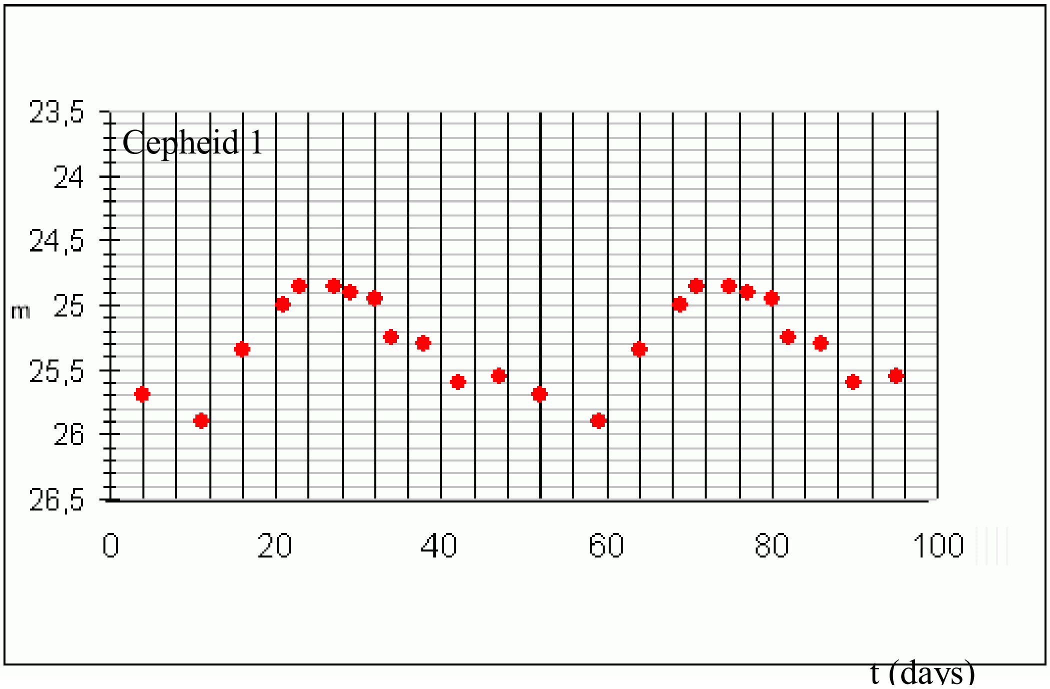
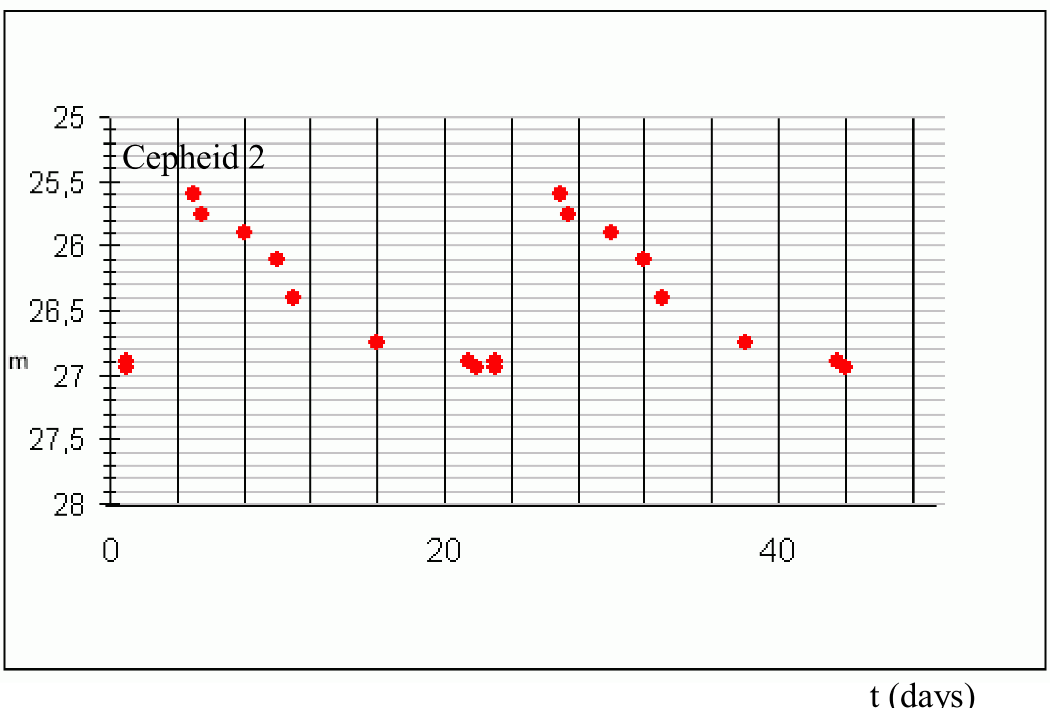

Cepheid 1
23,5
24
24,5
25
25,5
26
26,5
m
0
20
40
60
80
100
t (days)

Figure 2


Cepheid 2
25
25,5
26
26,5
27
27,5
28
m
0
20
40
t (days)

Appendix A: extinction coefficients

The magnitude outside the atmosphere is given by:

$$M = m - A.X + B$$

where A is the extinction coefficient and B is the zero point coefficient for the night, X is the airmass of the observation and m is the instrumental magnitude (that is, the magnitude obtained directly from the image)

Appendix B: calibrated differential magnitude

If the standard star is the same frame as the object, the calibrated magnitude of the object can be obtained by:

$$M_{ast} = m_{ast} - m_{star} + M_{star}$$

Appendix C: least squares estimation of angular coefficients

Given straight lines described by

$$y_i = \alpha + \beta x_i,\ i = 1..n$$

the least squares estimator of β is given by

$$\beta = \frac{\sum_i^n x_i y_i - \frac{1}{n}\sum_i^n x_i \sum_i^n y_i}{\sum_i^n x_i^2 - \frac{1}{n}\left(\sum_i^n x_i\right)^2}$$

Name: ____ Country: _____________

Observational Exam – 1st attempt

1 – Estimate the field of view of this telescope, using 10mm Plöss eyepiece and ***star chart-1***, showing nearby region of open cluster NGC 6231. Star chart 1 shows two angular distances. Use them as reference. Express your answer in arc minutes and tenths of it.

2 – Use ***star chart-2*** to estimate the magnitude of the missing star, shown as a cross, inside NGC 6231. Use the magnitude of other stars as reference.
Note: To avoid confusion between decimal dots and real stars, dots where supressed. So, magnitude 60 corresponds to magnitude 6.0. Give your answer using one decimal figure and 0.1 precision.

3 – Point your telescope to the binary star **ε - Trianguli Australis** using ***star chart-3*** as a guide. That pair components are magnitude 4.1 and 9.3 separated by 82". Choose the best option for the correct color of each star:

Brighter: White/blue ( ) Yellow ( ) Red ( )

Dimmer: White/blue ( ) Yellow ( ) Red ( )

4 – Identify objects pointed by the evaluator as Open Cluster **(OC)**, Globular Cluster **(GC)**, Emission Nebulae **(EN)** or Planetary Nebulae **(PN)**.

Object 1 ( ) Object 3 ( )

Object 2 ( ): Object 4 ( )

5 – Use your green laser pointer to spot the stars Antares, Vega, Altair and Peacock. Also point to the constellation Corona Australis.

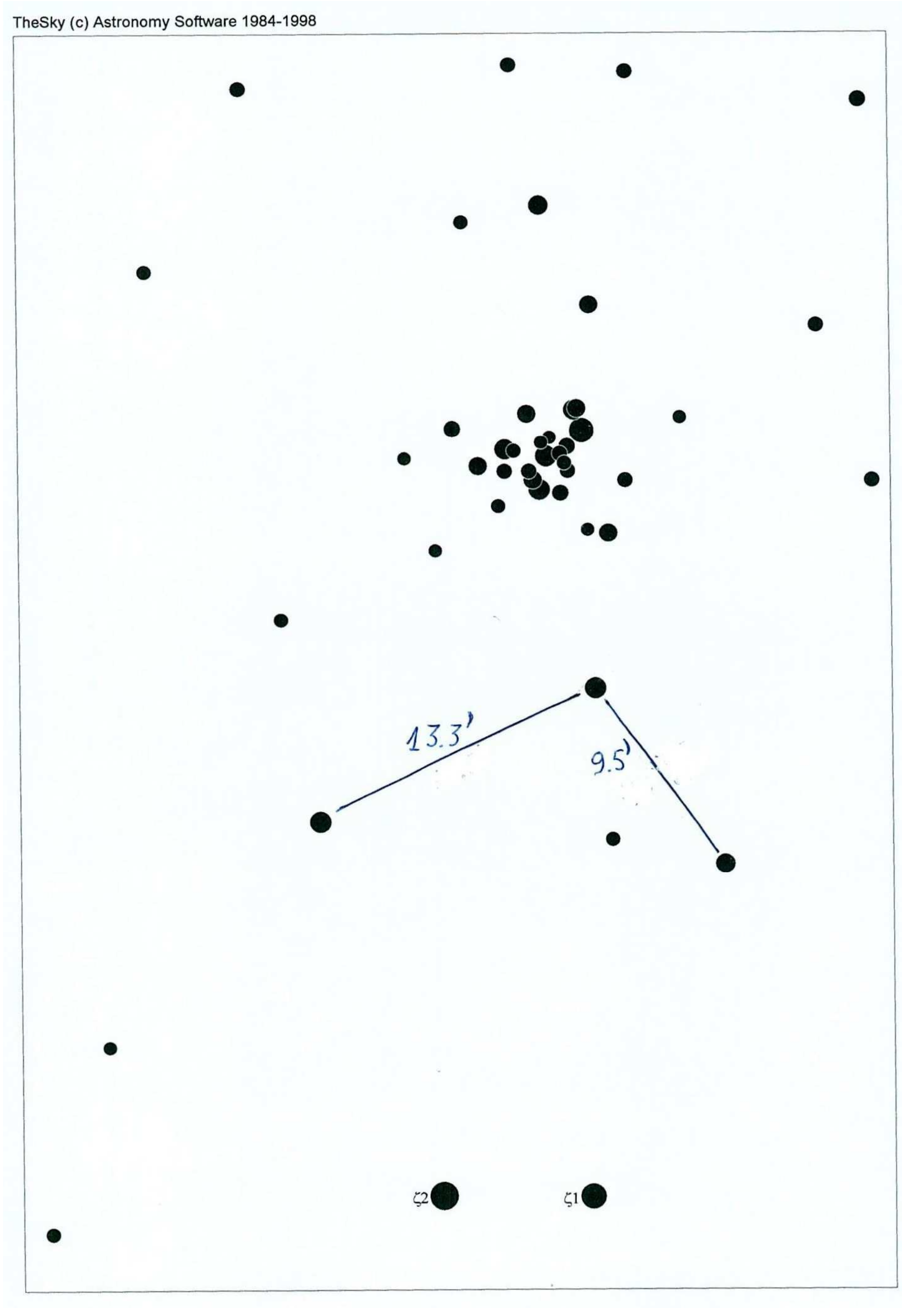
<u>Material needed for each student</u>:

Red flashlight, green laser pointer, chair, table, pencil, rubber and clipboard.

Name: ______________________________ Country: ____________

Observational Exam – 1st attempt

Chart 1 – NGC 6231 Field of view



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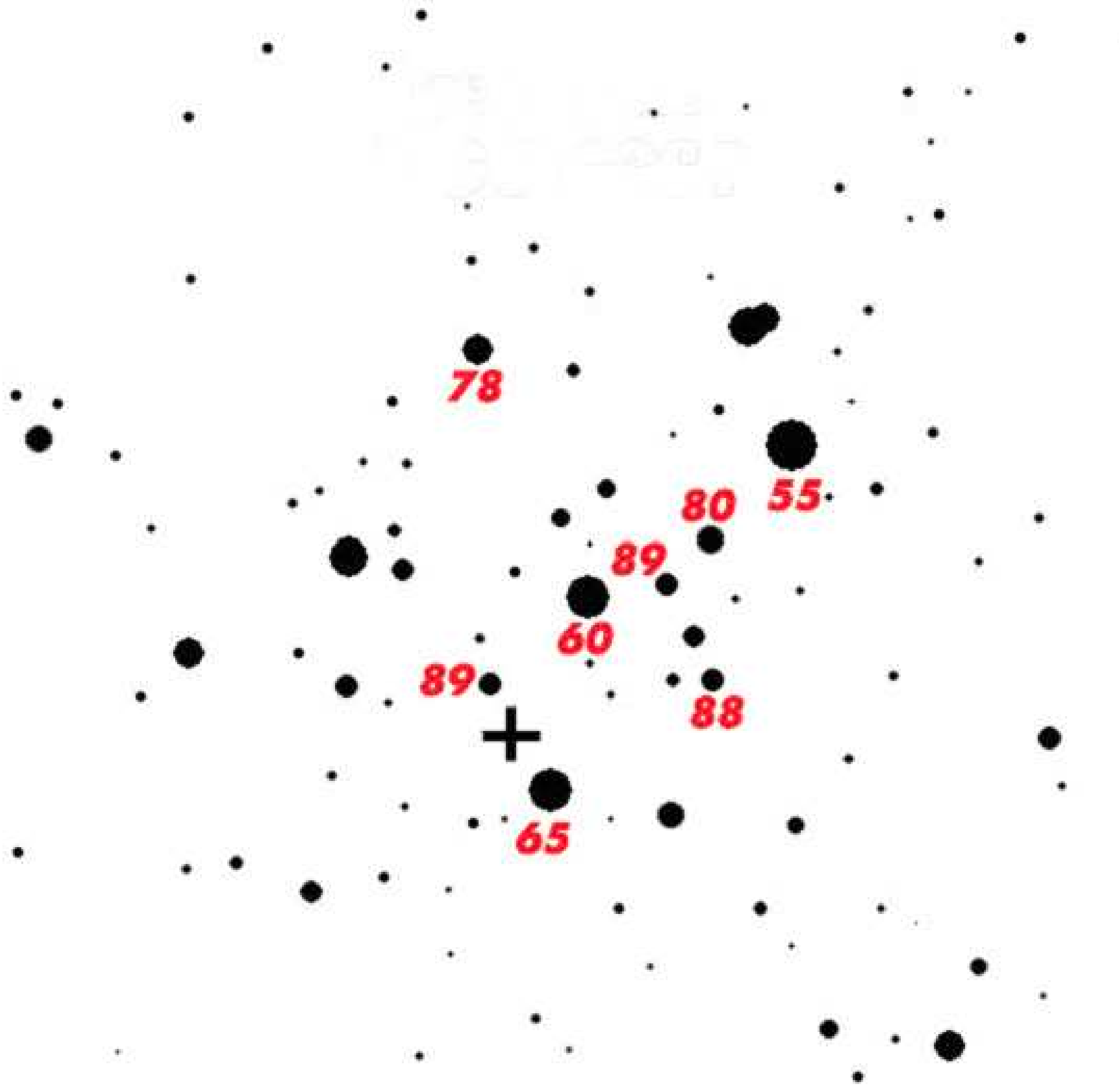
Observational Exam – 1st attempt

Chart 2 – NGC 6231



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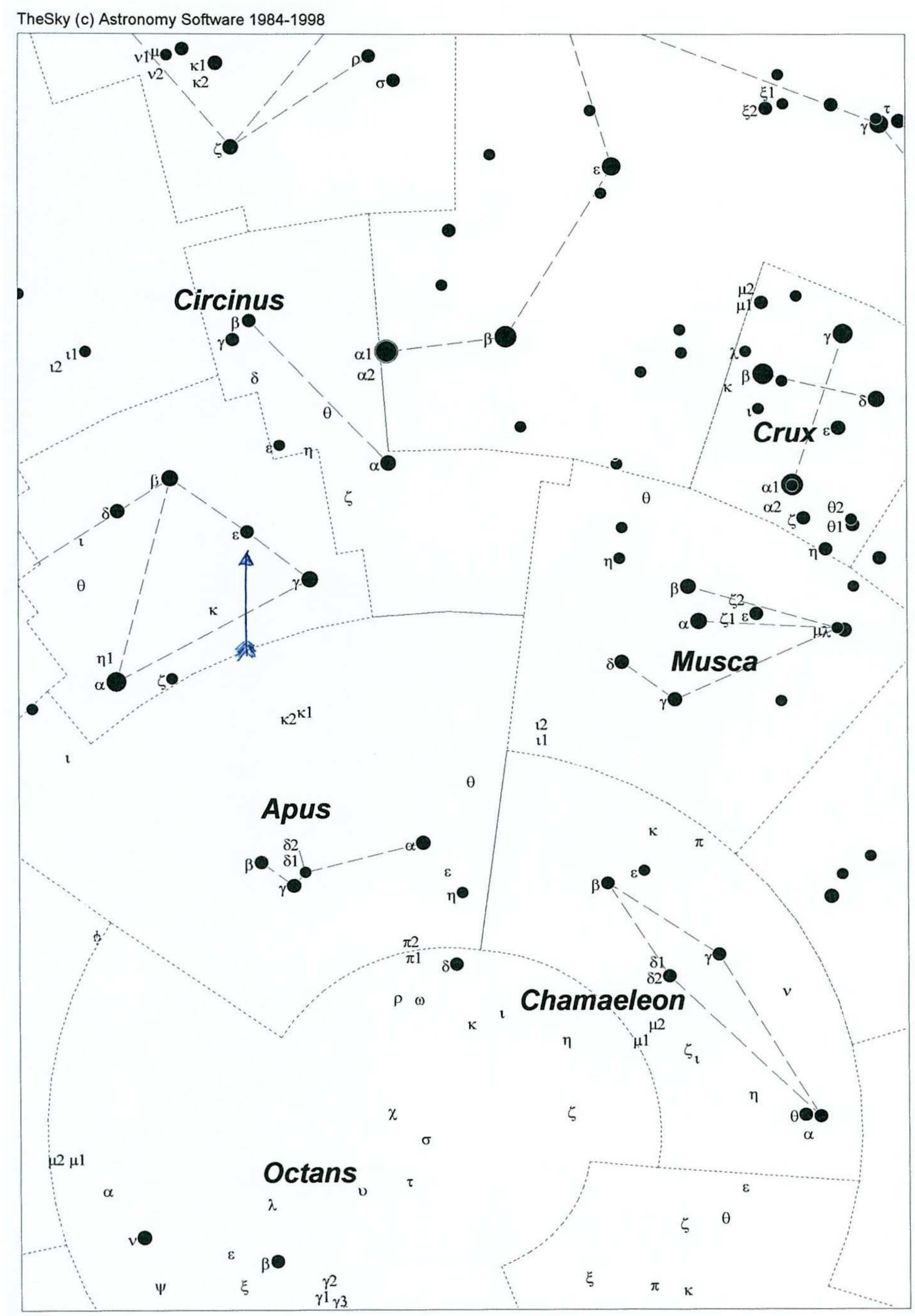
Observational Exam – 1st attempt

Chart 3 - □ Trianguli Australis location



Name: ____ Country: _____________

Observational Exam – 2nd attempt

1 – Use your green laser pointer to aim at 3 zodiacal constellations of your choice.

2 – Point to β and ν scorpii (**star chart-4**), two binary stars. Use 2x Barlow + 10mm eyepiece to determine the main difference on both stars, besides differences in distance between the components and magnitude.

3 – Point your telescope to the star SAO 209318 (**star chart-5**). Pay attention to a small nebulous patch close to that star. Use your 10mm or 10mm + 2x Barlow to estimate the distance between the star and the nebulous patch, in arc minutes. (coordinates to SAO 209318 are RA: 17h50m51s and Dec: -37°02'). Express your answer using 0.5' precision, knowing that field of view of the 10mm eyepiece on this telescope is 24 arcminutes or 0.4º.

4 – Point your telescope to the binary star Albireo (β-Cygni) using **star chart-6** as guide. That pair components are magnitude 3.2 and 4.7 separeted by 34.8" (2010). Choose the best option for the correct color of each star:

Brighter:	White ( )	blue ( )	Yellow ( )	Red ( )
Dimmer:	White ( )	blue ( )	Yellow ( )	Red ( )

5 – Identify objects pointed by the evaluator as Open Cluster (**OC**), Globular Cluster (**GC**), Emission Nebulae (**EN**) or Planetary Nebulae (**PN**).

Object 1	( )	Object 3	( )
Object 2	( ):	Object 4	( )

Material needed for each student:

Red flashlight, green laser pointer, chair, table, pencil, rubber and clipboard.

6th IOAA

6th International Olympiad on Astronomy and Astrophysics

04 to 14 August, 2012 - Rio de Janeiro - Brazil

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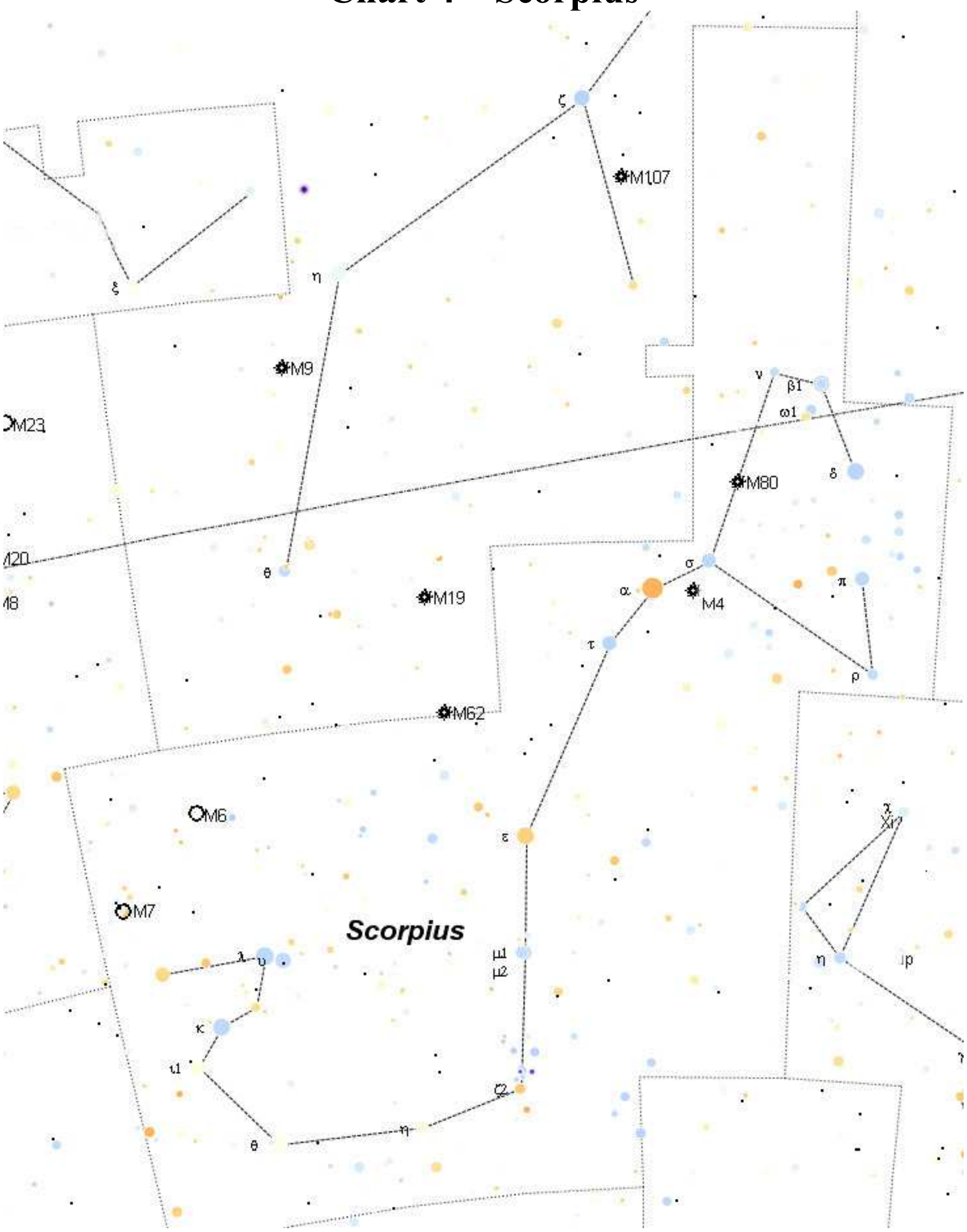
Observational Exam – 2nd attempt

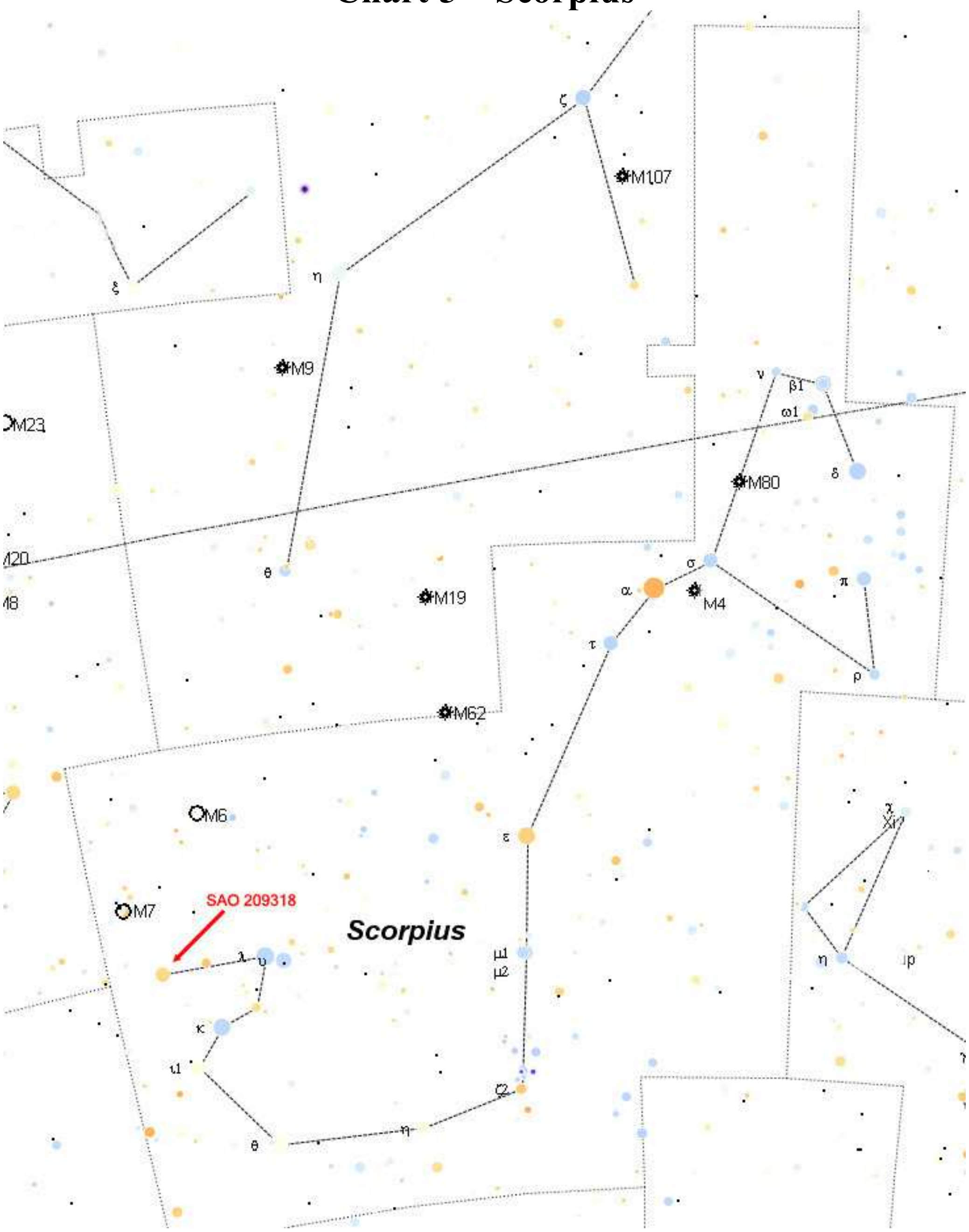
Chart 4 – Scorpius



Name: ____ Country: ____________

Observational Exam – 2nd attempt

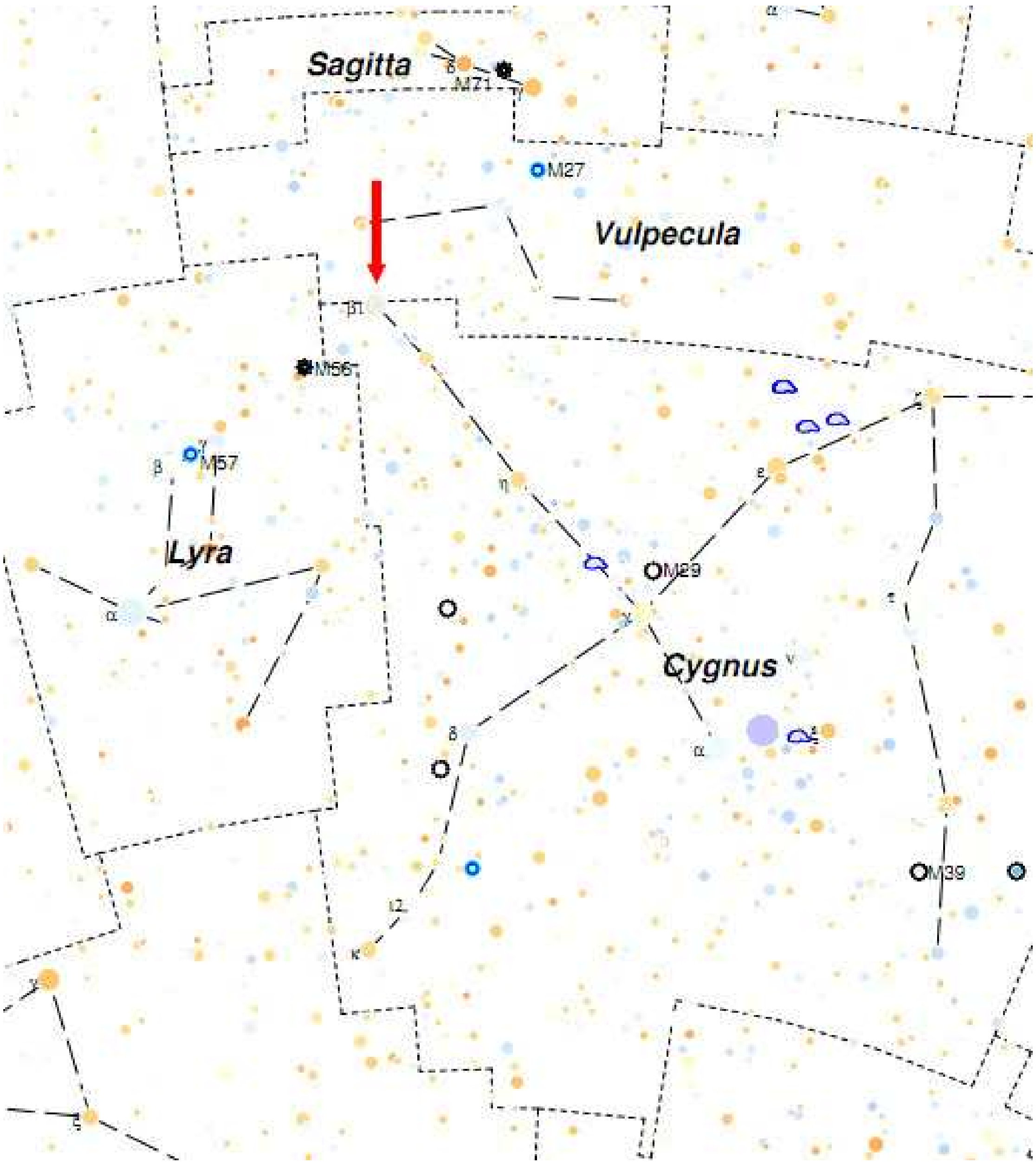
Chart 5 – Scorpius



Name: ____ Country: ____________

Observational Exam – 2nd attempt

Chart 6 – Cygnus



Team Problem

This is a team problem. The teams will have 4 days to conclude it and each team shall be able to allocate enough time to work, dividing the time between individual and group work time.

The solution must be delivered on August 11th at 9 AM. It must contain not more than **four sheets (eight pages), with all the essential assumptions, insights and results** from the team. In addition to the four main **sheets**, the teams can attach some appendices containing raw data (lists, tables) and auxiliary calculations/derivations/drawings.

Part I

Imagine a binary stellar system, consisting of two main sequence stars, with one star at least 30 times more massive than the other. Now imagine a planet located at one of the Lagrange points of the system, L4 or L5. Imagine the orbital plane of the system is aligned to our line of sight, i. e., it is perpendicular to the plane of the sky ("edge-on"), so you can detect it using its light curve.

1) Make a schematic drawing of this system.

2) Create a hypothetical light curve for this system and use it to understand the mechanics of the system. Do not take into account any instrumental effect.

3) Assuming the light curve from item (2) is accurate, calculate the values of the **relevant** physical parameters for each of the three bodies: mass, radius, orbital period, mean orbital radius, luminosity, effective temperature.

Part II

Now let's examine the planet from Part I, called Troll by its inhabitants, more closely. Troll has a rotation axis inclined by 30° with respect to the normal of the orbital plane, sidereal rotation period of five terrestrial days and mass of 0.5 terrestrial masses.

4) For an observer located at the Trollian equator, draw a sketch of the sky in the moment of the day when the incident light on the surface of the planet reaches its maximum intensity.

Team Problem

5) At which latitude during the Trollian year, is there at least one Trollian sidereal day that there is no night?

Part III

Suppose now that Troll has a moon that can't be detected from Earth and whose radius is ¼ of Troll's radius. The moon's orbital plane is inclined 15° with respect to the orbital plane of the planet. The orbital period of the moon is about 30 Trollian sidereal days.

6) For an observer at a Trollian latitude φ, what will be the longest time interval for which the moon will stay above the horizon? In which part of Trollian year will it happen?

7) Under the same conditions and in the same day of item (6), calculate the duration of the moon setting, that is, the time since the moon touches the Troll horizon until it disappears completely below it.

8) How many total (T) and how many partial (P) eclipses can be seen during a Trollian sidereal year?

Part IV

Assume that the binary system is at 277 pc from Earth, located at equatorial coordinates $\alpha = 3^h$ and $\delta = -15°$, as seen from Earth.

9) Is it possible for a Trollian observer to see the Sun being occulted by any of the other bodies of its system? Say **YES** if it possible and **NO** otherwise.

10) What is the "Trollian ecliptic latitude" of the Sun, to a Trollian observer?

11) In which conditions can the Trollian observer see a transit of the Jupiter in front of the Sun? Make a drawing to explain your answer.

12) Estimate the apparent magnitudes of the following stars, as seen from Troll sky: Sun, Vega, Sirius, Rigel, Aldebaran.

Team Problem

	α	δ	m	M
Vega	18h36m56s	+38°47'01"	0.03	0.58
Sirius	06h45m09s	-16°42'58"	-1.47	1.42
Rigel	05h14m32s	-08°12'06"	0.12	-7.84
Aldebaran	04h35m55s	+16°30'34"	0.75	-0.63

IOAA 2013 – Volos, Greece

The seventh IOAA was held from 27^{th} July to 4^{th} August 2013. Total 39 teams from 35 countries participated in the event. These included first time teams from Armenia, Canada, Cyprus, F.Y.R. of Macedonia, Malaysia, New Zealand, Slovenia and U.S.A.

Table of Constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity	9.8 m/s^2	
Obliquity of Ecliptic	$23°27'$	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo (a)	0.39	
Mass($M_☾$)	7.35×10^{22} kg	**Moon**
Radius ($R_☾$)	1.74×10^{6} m	
Mean distance from Earth	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	–12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	**Sun**
Luminosity ($L_\odot$)	3.83×10^{26} W	
Absolute Magnitude ($\mathcal{M}_\odot$)	4.72 mag	
Angular diameter	0.5 degrees	
Effective Surface Temperature	5800 K	
Jupiter's orbit semi-major axis	5.204 AU	**Jupiter**
Jupiter's orbital period	11.8618 yr	
Diameter of human pupil	6 mm	**Distances and sizes**
1 AU	1.50×10^{11} m	
1 pc	206,265 AU	
Distance from Sun to Barnard's Star	1.83 pc	
Mars orbit semi-major axis	1.52 AU	
Gravitational constant (G)	6.67×10^{-11} N m^2 kg^{-2}	**Physical constants**
Planck constant (h)	6.62×10^{-34} J s	
Boltzmann constant (k_B)	1.38×10^{-23} J K^{-2}	
Stefan-Boltzmann constant (σ)	5.67×10^{-8} W m^{-2} K^{-4}	
Hubble constant (H_0)	72 km s^{-1} Mpc^{-1}	
Speed of light (c)	299,792,458 m/s	
Proton mass	938.27 MeV c^{-2}	
Deuterium mass	1875.60 MeV c^{-2}	
Neutron mass	939.56 MeV c^{-2}	
Helium-3 mass	2808.30 MeV c^{-2}	
Helium-4 mass	3727.40 MeV c^{-2}	

The brightest stars visible from Greece

Star		α (2000) h m	δ (2000) ° ′	m(V) mag	M(V) mag	B–V	Spectral Type
Alpheratz	α And	00 08	+29 05	2.03	–0.9	–0.10	A0 p
Caph	β Cas	00 09	+59 09	2.26	+1.5	+0.34	F2 IV
Schedar	α Cas	00 40	+21 26	2.22	–1.0	+1.17	K0 II
Diphda	β Cet	00 44	–17 59	2.04	+0.2	+1.04	K0 III
Mirach	β And	01 10	+35 37	2.06	–0.4	+1.62	M0 III
Achernar	α Eri	01 38	–57 15	0.48	–1.6	–0.18	B5 IV
Almach	γ And	02 04	+42 20	2.13	–0.1	+1.20	K2 III
Hamal	α Ari	02 07	+23 28	2.00	+0.2	+1.15	K2 III
Mira	o Cet	02 19	–02 59	2.0	–1.0	+1.42	M6 e
Polaris	α UMi	02 32	+89 16	2.02	–4.6	+ 0.6	F8 Ib
Algol	β Per	03 08	+40 57	2.2	–0.3	–0.1	B8 V
Mirfak	α Per	03 24	+49 51	1.80	–4.3	+0.48	F5 Ib
Aldebaran	α Tau	04 36	+16 30	0.85	–0.3	+1.54	K5 III
Rigel	β Ori	05 15	–08 12	0.11	–7.0	–0.03	B8 Iα
Capella	α Aur	05 17	+46 00	0.08	+0.3	+0.80	G8 III
Bellalrix	γ Ori	05 25	+06 21	1.63	–3.3	–0.22	B2 III
EINath	β Tau	05 26	+28 36	1.65	–1.6	–0.13	B7 III
Mintaka	δ Ori	05 32	–00 18	2.19	–6.1	–0.21	O9.5II
Alnilam	ε Ori	05 36	–01 12	1.70	–6.2	–0.19	B0 Iα
Alnitak	ζ Ori	05 41	–01 57	1.79	–5.9	–0.21	O9.5 Ib
Saiph	κ Ori	05 48	–09 40	2.05	–6.8	–0.18	B0.5 Iα
Betelgeuse	α Ori	05 55	+07 24	0.50	–5.6	+1.86	M2 Iab
Menkalinan	β Aur	06 00	+44 57	1.90	+0.6	+0.03	A2 IV
Mirzam	β CMa	06 23	–17 57	1.98	–4.5	–0.24	Bl II
Canopus	α Car	06 24	–52 42	–0.73	–4.7	+0.16	F0 Ib
Alhena	γ Gem	06 38	+16 24	1.93	+0.0	+0.00	A0 IV
Sirius	α CMa	06 45	–16 43	–1.45	+1.4	+0.00	Al V
Adhara	ε CMa	06 59	–28 58	1.50	–5.0	–0.22	B2 II
Wezen	δ CMa	07 08	–26 24	1.84	–7.3	+0.67	F8 Ia
Castor	α Gem	07 35	+31 53	1.58	+0.8	+0.04	Al V
Procyon	α CMi	07 39	+05 14	0.35	+2.7	+0.41	F5 IV
Pollux	β Gem	07 45	+28 01	1.15	+1.0	+1.00	K0 III
Naos	ζ Pup	08 04	–40 00	2.25	–7.0	–0.27	O5.8
—	γ Vel	08 10	–47 20	1.83	–4.0	–0.26	WC 7
Avior	ε Car	08 23	–59 30	1.87	–2.1	+1.30	K0 II
—	δ Vel	08 45	–54 43	1.95	+0.1	+0.0	A0 V4
Suhail	λ Vel	09 08	–43 26	2.26	–4.5	+1.69	K5 Ib
Miaplacidus	β Car	09 13	–69 43	1.68	–0.4	+0.00	A0 III

The brightest stars visible from Greece (cont.)

Star			α (2000) h	m	δ (2000) ° ′	m (V) mag	M (V) mag	B–V	Spectral Type
Scutulum	ι	Car	09	17	–59 16	2.24	–4.5	+0.18	F0 Ib
Alphard	α	Hya	09	28	–08 40	1.99	–0.4	+1.43	K3 III
Regulus	α	Leo	10	08	+11 58	1.35	–0.6	–0.11	B7 V
Algeiba	γ	Leo	10	20	+19 51	2.1	–0.5	+1.12	K0 III
Dubhe	α	UMa	11	03	+61 45	1.79	–0.7	+1.06	K0 III
Denebola	β	Leo	11	49	+14 34	2.14	+1.6	+0.09	A3 V
Acrux	α	Cru	12	27	–63 06	0.9	–3.5	–0.26	Bl IV
Gacrux	γ	Cru	12	31	–57 07	1.64	–2.5	+1.60	M3 III
Muhlifain	γ	Cen	12	42	–48 58	2.16	–0.5	–0.02	A0 III
Mimosa	β	Cru	12	48	–59 41	1.26	–4.7	–0.24	B0 III
Alioth	ε	UMa	12	54	+55 57	1.78	–0.2	–0.02	A0 p
Mizar	ζ	UMa	13	24	+54 56	2.09	+0.0	+0.03	A2 V
Spica	α	Vir	13	25	–11 09	0.96	–3.4	–0.23	Bl V
—	ε	Cen	13	40	–53 28	2.30	–3.6	–0.23	Bl V
Alkaid	η	UMa	13	48	+49 19	1.86	–1.9	–0.19	B3 V
Hadar	β	Cen	14	04	–60 22	0.60	–5.0	–0.23	Bl II
Menkent	θ	Cen	14	07	–36 22	2.06	+1.0	+1.02	K0 III
Arcturus	α	Boo	14	16	+19 11	–0.06	–0.2	+1.23	K2 IIIp
Rigil Kent	α	Cen	14	40	–60 50	–0.1	+4.3	+0.7	G2 V
Kochab	β	UMi	14	50	+74 09	2.07	–0.5	+1.46	K4 III
Alphecca	α	CrB	15	35	+26 43	2.23	+0.5	–0.02	A0 V
Antares	α	Sco	16	29	–26 26	1.0	–4.7	+1.81	Ml Ib
Atria	α	TrA	16	49	–69 02	1.93	–0.3	+1.43	K4 III
—	ε	Sco	16	5O	–34 18	2.29	+0.7	+1.15	K2 III
Shaula	λ	Sco	17	34	–37 06	1.62	–3.4	–0.22	Bl IV
Ras–Alhague	α	Oph	17	35	+12 34	2.07	+0.8	+0.15	A5 III
—	θ	Sco	17	37	–43 00	1.87	–4.5	+0.40	FO I
Eltanin	γ	Dra	17	57	+51 29	2.22	–0.6	+1.52	K5 III
Kaus Australis	ε	Sgr	18	24	–34 23	1.83	–1.5	–0.02	B9 IV
Vega	α	Lyr	18	37	+38 47	0.04	+0.5	+0.00	AO V
Nunki	σ	Sgr	18	55	–26 18	2.08	–2.5	–0.20	B2 V
Altair	α	Aql	19	51	+08 52	0.77	+2.3	+0.22	A7 IV
Sadir	γ	Cyg	20	22	+40 15	2.23	–4.7	+0.67	F8 Ib
Peacock	α	Pav	20	26	–56 44	1.93	–2.9	–0.20	B3 IV
Deneb	α	Cyg	20	41	+45 17	1.25	–7.3	+0.09	A2 Ia
Al Na' ir	α	Gru	22	08	–46 58	1.74	+0.2	–0.14	B5 V
—	β	Gru	22	42	–46 53	2.20	–1.5	+ 1.6	M3 II
Fomalhaut	α	PsA	22	58	–29 37	1.16	+1.9	+0.09	A3 V

The brightest stars of the sky with m(V) ≤ 1.00

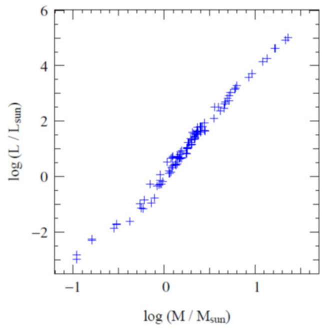
Star			α (2000)		δ (2000)	m(V)	M(V)	B–V	Spectral
			h	m	° ′	mag	mag		type
Sirius	α	CMa	06	45	–16 43	–1.45	+1.4	+0.00	Al V
Canopus	α	Car	06	24	–52 42	–0.73	–4.7	+0.16	F0 Ib
Rigil Kent	α	Cen	14	40	–60 50	–0.1	+4.3	+0.7	G2 V
Arcturus	α	Boo	14	16	+19 11	–0.0	–0.2	+1.23	K2 IIIp
Vega	α	Lyr	18	37	+38 47	0.04	+0.5	+0.00	AO V
Capella	α	Aur	05	17	+46 00	0.08	+0.3	+0.80	G8 III
Rigel	β	Ori	05	15	–08 12	0.11	–7.0	–0.03	B8 Iα
Procyon	α	CMi	07	39	+05 14	0.35	+2.7	+0.41	F5 IV
Achernar	α	Eri	01	38	–57 15	0.48	–1.6	–0.18	B5 IV
Betelgeuse	α	Ori	05	55	+07 24	0.50	–5.6	+1.86	M2 Iab
Hadar	β	Cen	14	04	–60 22	0.60	–5.0	–0.23	Bl II
Altair	α	Aql	19	51	+08 52	0.77	+2.3	+0.22	A7 IV
Aldebaran	α	Tau	04	36	+16 30	0.85	–0.3	+1.54	K5 III
Acrux	α	Cru	12	27	–63 06	0.90	–3.5	–0.26	Bl IV
Spica	α	Vir	13	25	–11 09	0.96	–3.4	–0.23	Bl V
Antares	α	Sco	16	29	–26 26	1.00	–4.7	+ 1.81	Ml Ib

Theoretical Exam - Short Questions

1. What would be the mean temperature on the Earth's surface if we ignore the greenhouse effect, assume that the Earth is a perfect black body and take into account its non-vanishing albedo? Assume that the Earth's orbit around the Sun is circular.

2. Let us assume that we observe a hot Jupiter planet orbiting around a star at an average distance $d = 5$ AU. It has been found that the distance of this system from us is $r = 250$ pc. What is the minimum diameter, D, that a telescope should have to be able to resolve the two objects (star and planet)? We assume that the observation is done in the optical part of the electromagnetic spectrum ($\lambda \sim 500$ nm), outside the Earth's atmosphere and that the telescope optics are perfect.

3. It is estimated that the Sun will have spent a total of about $t_1 = 10$ billion years on the main sequence before evolving away from it. Estimate the corresponding amount of time, t_2, if the Sun were 5 times more massive.



. 1

Figure 1: Mass to Luminosity relation

4. Figure 2 shows the relation between absolute magnitude and period for classical cepheids. Figure 3 shows the light curve (apparent magnitude versus time in days) of a classical cepheid in a local group galaxy . (a) Using these two figures estimate the distance of the cepheid from us. (b) Revise your estimate assuming that the interstellar extinction towards the cepheid is $A = 0.25$ mag.

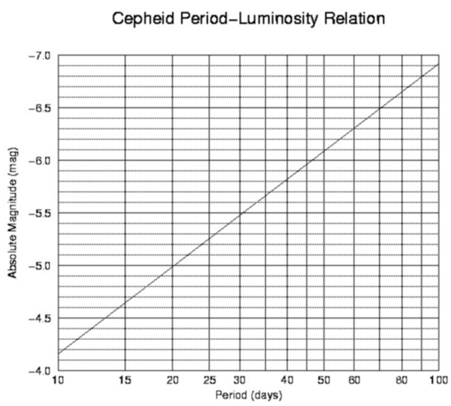


Figure 2

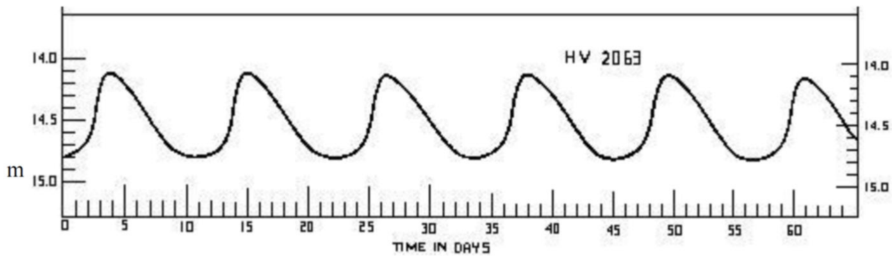


Figure 3

5. The optical spectrum of a galaxy, whose distance had been measured to be 41.67 Mpc, showed the Balmer Hα line (λ_0 = 656.3 nm) redshifted to λ = 662.9 nm. (a) Use this distance to calculate a value of the Hubble constant, H_0. (b) Using your results, estimate the Hubble time of the Universe.

6. A star has an effective temperature T_{eff} = 8700 K, absolute magnitude M = 1.6 and apparent magnitude m = 7.2. Find (a) the star's distance, r, (b) its luminosity, L, and (c) its radius, R. (Ignore extinction).

7. A star has visual apparent magnitude m_V = 12.2 mag, parallax $\pi = 0''.001$ and effective temperature T_{eff} = 4000 K. Its bolometric correction is B.C. = –0.6 mag. (a) Find its luminosity as a function of the solar luminosity. (b) What type of star is it? (i) a *red giant*? (ii) a *blue giant*? or (iii) a *red dwarf*? Please write (i), (ii) or (iii) in your answer sheet.

8. A binary system of stars consists of star (a) and star (b) with brightness ratio 2. The binary system is difficult to resolve and is observed from the Earth as <u>one star</u> of 5th magnitude. Find the apparent magnitude of each of the two stars (m_a, m_b).

9. Find the equatorial coordinates (*hour angle* and *declination*) of a star at Madrid, geographic latitude $\varphi = 40°$, when the star has zenith angle $z = 30°$ and azimuth $A = 50°$ (azimuth as measured from the South)

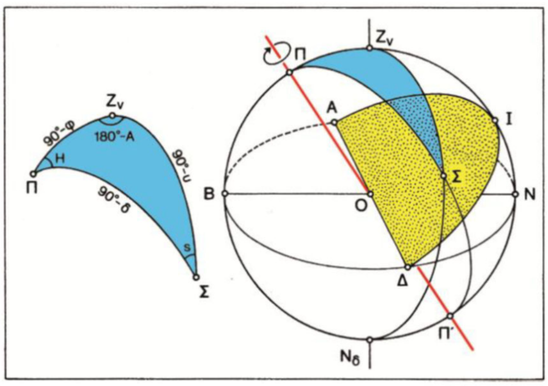


Figure 3. The position triangle

10. In the centre of our Galaxy, in the intense radio source Sgr A*, there is a black hole with large mass. A team of astronomers measured the angular distance of a star from Sgr A* and its orbital period around it. The maximum angular distance was $0.12''$ (arcsec) and the period was 15 years. Calculate the mass of the black hole in solar masses, assuming a circular orbit.

11. What is the maximum altitude, $a_{\rm M}$ (max), at which the Full Moon can be observed from Thessaloniki? The geographical latitude of Thessaloniki is $\varphi_\Theta = 40°37'$. Take into account as many factors as possible.

12. *Sirius A*, with visual magnitude $m_{\rm V} = -1.47$ (the brighter star on the sky) and with stellar radius $R_{\rm A} = 1.7R_\odot$, is the primary star of a binary system. The existence of its companion, *Sirius B*, was deduced from astrometry in 1844 by the well known mathematician and astronomer Friedrich Bessel, before it was directly observed. Assuming that both stars were of the same spectral type and that *Sirius B* is fainter by 10 mags ($\Delta m = 10$), calculate the radius of *Sirius B*.

13. Recently in London, because of a very thick layer of fog, the visual magnitude of the Sun, became equal to the (usual – as observed during cloudless nights) magnitude of the full Moon. Assuming that the reduction of the intensity of light due to the fog is given by an exponential equation, calculate the exponential coefficient, τ, which is usually called *optical depth*.

14. What is the hour angle, H, and the zenith angle, z, of *Vega* ($d = 38°47'$) in Thessaloniki ($\lambda_1 = 1^{\rm h}32^{\rm m}$, $\varphi_1 = 40°37'$), at the moment it culminates at the local meridian of Lisbon ($\lambda_2 = 0^{\rm h}36^{\rm m}$, $\varphi_2 = +39°43'$)?

15. The Doppler shift of three remote galaxies has been measured with the help of Spectral observations:

Galaxy	Redshift, z
3C 279	0.536
3C 245	1.029
4C41.17	3.8

(a) Calculate their apparent recession velocity (1) using the classical approach, (2) using the approximate formula $v = c\ \ln(1+z)$, that is often used by cosmologists and (3) using the special relativistic approach.

(b) For all three formulae, at what percentage of the speed of light do they appear to recede?

(c) Which of (1) classical, (2) special relativity (3) approximate cosmological.

Theoretical Exam - Long Questions

Question 1

In a homogeneous and isotropic universe, the matter (baryonic matter + dark matter) density parameter $\Omega_m = \dfrac{\rho_m}{\rho_c} = 32\%$, where ρ_m is the matter density and ρ_c is the critical density of the Universe.

(1) Calculate the average matter density in our local neighbourhood.

(2) Calculate the escape velocity of a galaxy 100 Mpc away from us. Assume that the recession velocity of galaxies in Hubble's law equals the corresponding escape velocity at that distance, for the critical density of the Universe that we observe.

(3) The particular galaxy is orbiting around the centre of our cluster of galaxies on a circular orbit. What is the angular velocity of this galaxy on the sky?

(4) Will we ever discriminate two such galaxies that are initially at the same line of sight, if they are both moving on circular orbits but at different radii (answer "Yes" or "No")?

[Assume that the Earth is located at the centre of our local cluster.]

Question 2

A spacecraft is orbiting the Near Earth Asteroid (2608) *Seneca* (staying continuously very close to the asteroid), transmitting pulsed data to the *Earth*. Due to the relative motion of the two bodies (the asteroid and the Earth) around the *Sun*, the time it takes for a pulse to arrive at the ground station varies approximately between 2 and 39 minutes. The orbits of the Earth and *Seneca* are coplanar. Assuming that the *Earth* moves around the *Sun* on a circular orbit (with radius $a_{\text{Earth}} = 1$ AU and period $T_{\text{Earth}} = 1$ yr) and that the orbit of *Seneca* does not intersect the orbit of the *Earth*, calculate:

(1) the semi-major axis, a_{Sen} the eccentricity, e_{Sen} of *Seneca's* orbit around the *Sun*

(2) the period of *Seneca's* orbit, TSen and the average period between two consecutive oppositions, T_{syn} of the *Earth-Seneca* couple

(3) an approximate value for the mass of the planet Jupiter, M_{Jup} (assuming this is the only planet of our Solar system with non-negligible mass compared to the Sun). Assume that the presence of Jupiter does not influence the orbit of Seneca.

Question 3

(1) Using the *virial theorem* for an isolated, spherical system, i.e, that $-2\langle K\rangle = \langle U\rangle$, where "$K$" is the average kinetic energy and "U" is the average potential energy of the system, determine an expression for the total mass of a cluster of galaxies if we know the radial velocity dispersion, σ, of the cluster's galaxy members and the cluster's radius, R. Assume that the cluster is isolated, spherical, has a homogeneous density and that it consists of galaxies of equal mass.

(2) Find the *virial mass*, i.e. the mass calculated from the *virial theorem*, of the Coma cluster, which lies at a distance of 90 Mpc from us, if you know that the radial velocity dispersion of its member galaxies is $\sigma_{v_r} = 1000$ km/s and that its angular diameter (on the sky) is about 4°.

(3) From observations, the total luminosity of the galaxies comprising the cluster is approximately $L = 5 \times 10^{12} L_\odot$. If the mass to luminosity ratio, M/L, of the cluster is ~ 1 (assume that all the mass of the cluster is visible mass), this should correspond to a total mass $M \sim 5 \times 10^{12} M_\odot$ for the mass of the cluster. Give the ratio of the luminous mass to the total mass of the cluster you derived in question (2).

Data analysis

Question 1.

In Figure 1, part of the constellation of Ursa Major is shown. It was taken with a digital camera with a large CCD chip (17mm × 22mm). Find the focal length, f, of the optical system and give the error of your results.

Figure 1. Part of the constellation of Ursa Major.

Question 2.

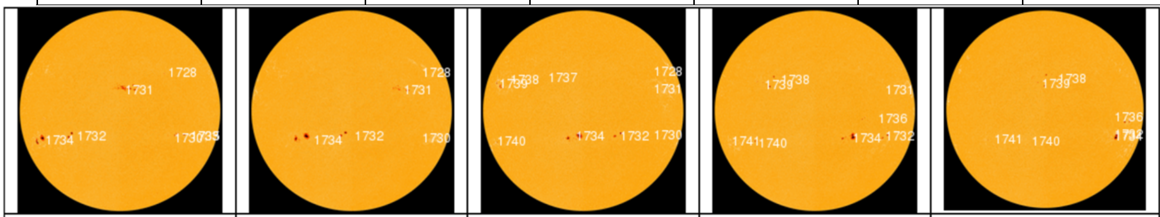
You are given 5 recent photographs of the solar photosphere shot at exactly the same time every two days (May 1 – May 9, 2013) in equatorial coordinates. You are also given two transparent Stonyhurst grids, which display heliocentric coordinates (heliocentric longitude, $\ell_{\odot}$, and heliocentric latitude, $b_{\odot}$). They cover the interval between April 28 to May 15. As the Earth does not orbit exactly around the Sun's equator, so, through the year, the solar equator seems to move up and down a little more than 7 degrees from the centre of the solar disc. This angle, B_0, varies sinusoidally through the year. Furthermore, the axis of rotation of the Sun, as seen from the Earth, does not coincide with the axis of rotation of the Earth. The angle on the plane of the sky between the two axes, P_0, also varies though the year. The numerical value of these angles (B_0 and P_0) are indicated on each of the 5 image of the Sun.

(1) Mark the axis of rotation of the Sun on each photograph.

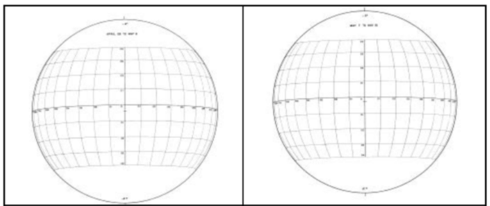
(2) Choose 3 prominent sunspots that can be followed in all (or most) photographs and mark them as *S1*, *S2* and *S3* on the photos. Using the appropriate Stonyhurst grids, find their coordinates ($\ell_{\odot}$, $b_{\odot}$) for every day (May 1 to May 9) and note them down in Table 1.

Table 1

Date	Sunspot S1		Sunspot S2		Sunspot S3	
	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$
May 1						
May 3						
May 5						
May 7						
May 9						



1 May 2013	3 May 2013	5 May 2013	7 May 2013	9 May 2013



April 28 to May 06	May 07 to May 15

Figure 3A. Five photographs of the solar photosphere (top) and the corresponding Stonyhurst grids

(3) Construct the diagrams $\Delta\ell_\odot/\Delta t$ for each sunspot.

(4) Calculate its synodic period (P) of rotation **in days** for each sunspot. Write down the result for each sunspot, P_{S1}, P_{S2}, P_{S3}.

(5) Calculate the average synodic period ($P_\odot$) of rotation of the Sun in days.

Question 3.

Figure 2 shows a photograph of the sky in the vicinity of the Hyades open cluster. The V-filter in the Johnson's photometric system was used. Figure 3 is a chart of the region with known V-magnitudes (m_V) of several stars (note that in order to avoid confusion with the stars, no decimal point is used, i.e. a magnitude $m_V = 8.1$ is noted as "81"). Hint: some of the stars may not be in the chart.

(a) Identify as many of the *stars shown with a number and an arrow* in Figure 3 and mark them on Figure 2.

(b) Comparing the V-magnitudes of the known stars in Figure 2, estimate the V-magnitudes of the *stars shown with a number and an arrow* in Figure 3.

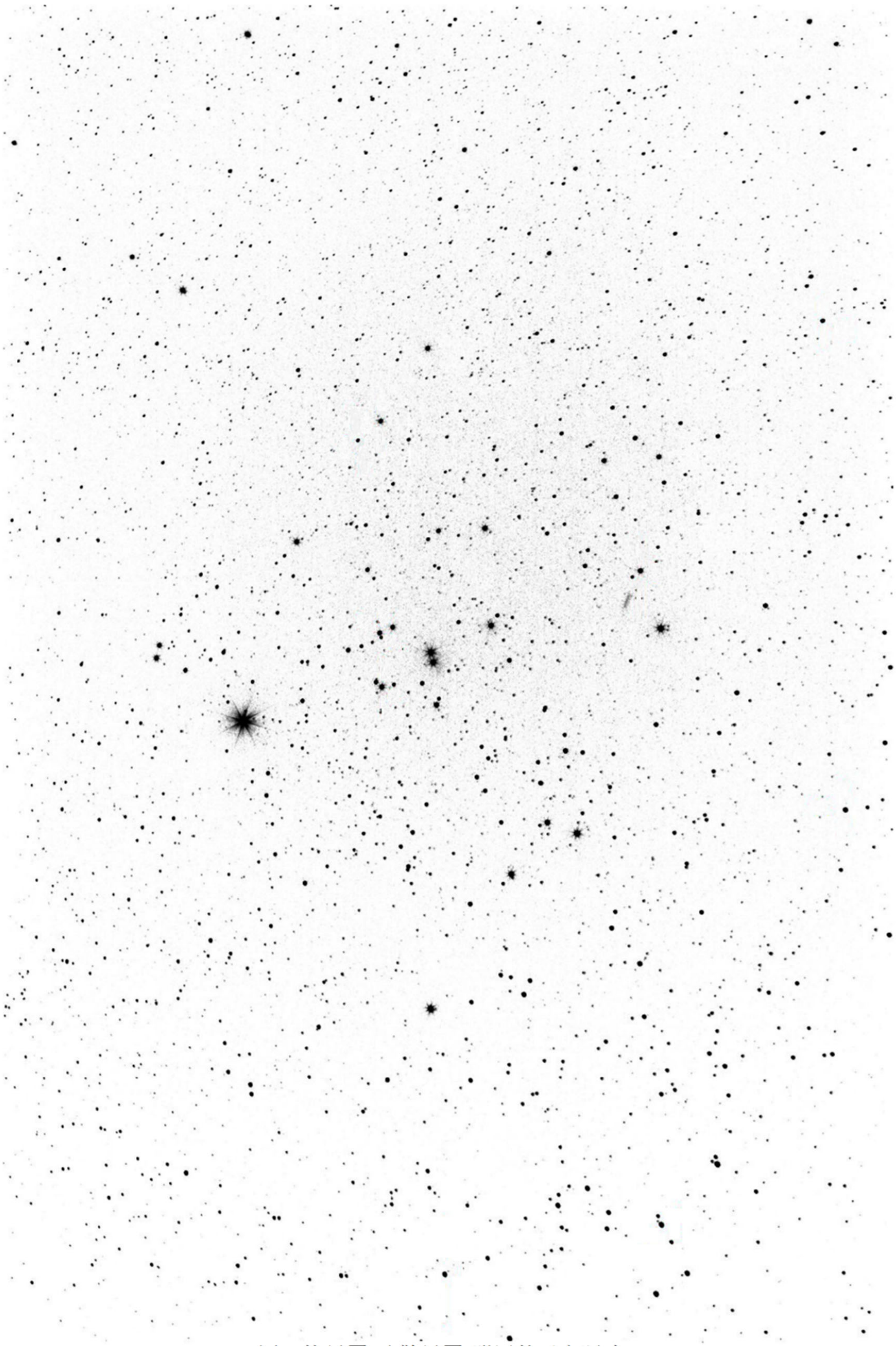


Figure 2. Photograph of the sky in the vicinity of the Hyades open cluster

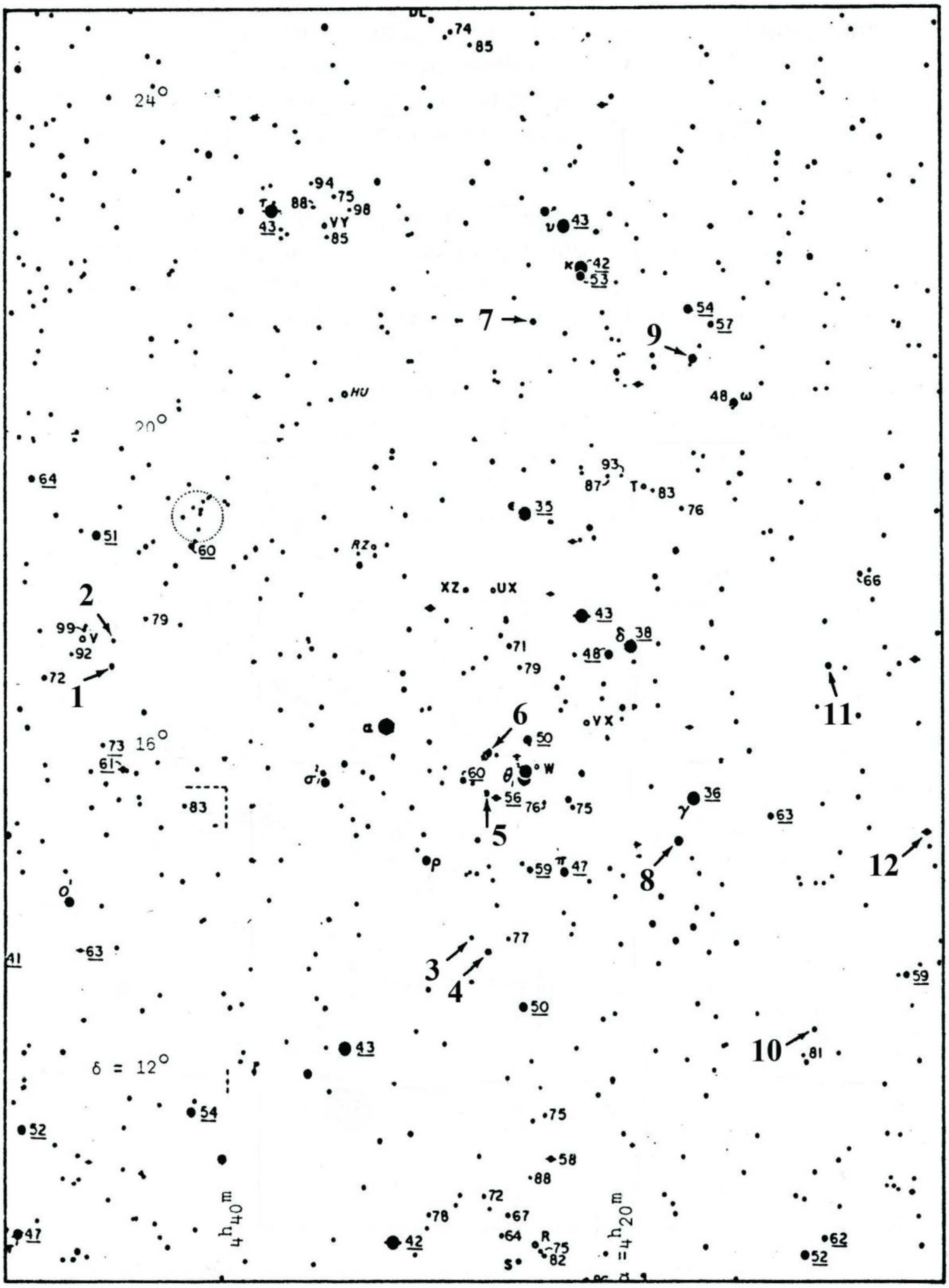


Figure 3. Sky chart of the region in the vicinity of the Hyades open cluster

Question 4.

Calculate the distance of the Hyades cluster using the *moving cluster* method (Figure 5).

1. In a *Text* file (*Hyades-stars.txt*) you are given a list of 35 stars from the field of the Hyades open cluster, observed by the Hipparcos space telescope.

The information listed in the columns of the text file for each of the 35 stars is: (a) The Hipparcos catalogue number (*HIP*). (b) Their *right ascension* (alpha – α) [h m s]. (c) Their *declination* (delta – d) [° ′ ″]. (d) Their *trigonometric parallax* (p – π) [$'' \times 10^3$]. (e) Their *proper motion in right ascension* multiplied by cos d (mu_axcosd – $\mu_\alpha \times \cos d$) [$'' \times 10^3$/yr]. (f) Their *proper motion in declination* (mu_d – μ_d) [$'' \times 10^3$/yr]. (g) Their radial velocity (v_r – v_r) [km/s].

```
  HIP       alpha        delta        p     mu_axcosd mu_d     v_r
=====================================================================
 13834  2 58  5.08  20 40  7.7  31.41  234.79 -31.64   28.10
 14838  3 11 37.67  19 43 36.1  19.44  154.61  -8.39   24.70
 18170  3 53  9.96  17 19 37.8  24.14  143.97 -29.93   35.00
 18735  4  0 48.69  18 11 38.6  21.99  129.49 -28.27   31.70
 19554  4 11 20.20   5 31 22.9  25.89  146.86   5.00   36.60
 20205  4 19 47.53  15 37 39.7  21.17  115.29 -23.86   39.28
 20261  4 20 36.24  15  5 43.8  21.20  108.79 -20.67   36.20
 20400  4 22  3.45  14  4 38.1  21.87  114.04 -21.40   37.80
 20455  4 22 56.03  17 32 33.3  21.29  107.75 -28.84   39.65
 20542  4 24  5.69  17 26 39.2  22.36  109.99 -33.47   39.20
 20635  4 25 22.10  22 17 38.3  21.27  105.49 -44.14   38.60
 20711  4 26 18.39  22 48 49.3  21.07  108.66 -45.83   35.60
 20713  4 26 20.67  15 37  6.0  20.86  114.66 -33.30   40.80
 20842  4 28  0.72  21 37 12.0  20.85   98.82 -40.59   37.50
 20885  4 28 34.43  15 57 44.0  20.66  104.76 -15.01   40.17
 20889  4 28 36.93  19 10 49.9  21.04  107.23 -36.77   39.37
 20894  4 28 39.67  15 52 15.4  21.89  108.66 -26.39   38.90
 20901  4 28 50.10  13  2 51.5  20.33  105.17 -15.08   39.90
 21029  4 30 33.57  16 11 38.7  22.54  104.98 -25.14   41.00
 21036  4 30 37.30  13 43 28.0  21.84  108.06 -19.71   38.80
 21039  4 30 38.83  15 41 31.0  22.55  104.17 -24.29   39.56
 21137  4 31 51.69  15 51  5.9  22.25  107.59 -32.38   36.00
 21152  4 32  4.74   5 24 36.1  23.13  114.15   6.17   39.80
 21459  4 36 29.07  23 20 27.5  22.60  109.97 -53.86   43.30
 21589  4 38  9.40  12 30 39.1  21.79  101.73 -14.90   44.70
 21683  4 39 16.45  15 55  4.9  20.51   82.40 -19.53   35.60
 22044  4 44 25.77  11  8 46.2  20.73   98.87 -13.47   39.60
 22157  4 46  1.70  11 42 20.2  12.24   67.48  -7.09   43.00
 22176  4 46 16.78  18 44  5.5  10.81   73.03 -69.79   44.11
 22203  4 46 30.33  15 28 19.6  19.42   91.37 -24.72   42.42
 22565  4 51 22.41  18 50 23.8  17.27   79.66 -32.76   36.80
 22850  4 54 58.32  19 29  7.6  14.67   63.32 -28.41   38.40
 23497  5  3  5.70  21 35 24.2  20.01   68.94 -40.85   38.00
 23983  5  9 19.60   9 49 46.6  18.54   63.54  -7.87   44.16
 24019  5  9 45.06  28  1 50.2  18.28   55.86 -60.57   44.90
```

Import the *txt* file in *MS Excel*

2. Convert the coordinates in degrees (with 4 decimal points).
3. Calculate the angular distance, φ, between each of the stars and the point of convergence, which is at ($\alpha_c = 6^h7^m$, $\delta_c = +6°56'$).
4. Calculate the proper motion of each star, μ [$''$/yr], using $\mu_\alpha \cos\delta$ and μ_δ given in the list.
5. Use the above data to calculate the distance, r_μ, for each star using the following equation:

$$r_\mu = \frac{v_r \tan\varphi}{4.74047\mu}$$

where r_μ is the distance of the star in parsecs, v_r is the radial velocity of the star in km/sec, φ is the *angular distance* between the star and the point of convergence that you have already estimated in step 3, while μ is the total proper motion estimated in step 4. Do all stars belong to the Hyades cluster? You can assume that any stars whose distance from the centre of the cluster ($r_\mu = 46.34$ pc) is larger than 10 pc, are not part of the cluster.
6. Independently, calculate the distance, r_π, of each star in the list using the trigonometric parallax angle, π.
7. Find the average distance of the Hyades cluster, $\overline{r_\mu}$ and $\overline{r_\pi}$, and its standard deviation, σ_μ and σ_π, for each method (*moving cluster* and *trigonometric parallax* methods).
8. Which method is more accurate: (*i*) the *moving cluster* method, (*ii*) the *trigonometric parallax* method? Please answer with (*i*) or (*ii*).

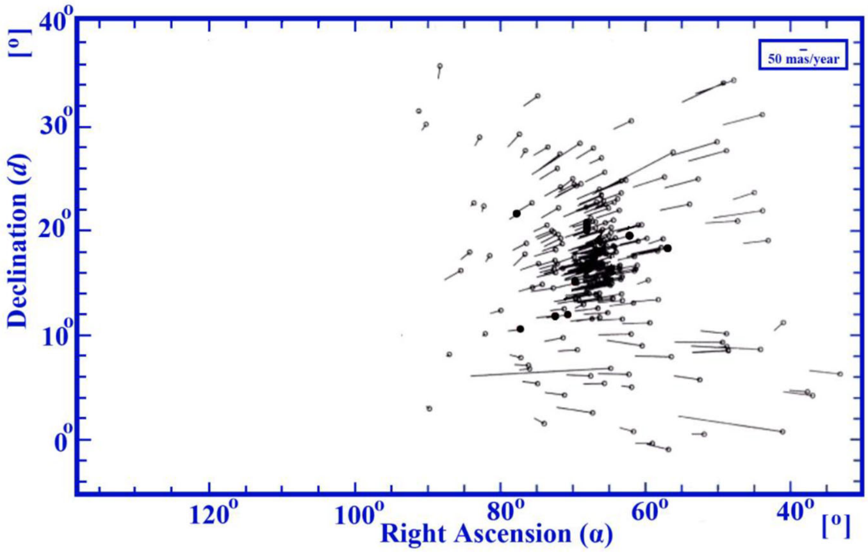


Figure 4. The distance of the Hyades open cluster using the *moving cluster* method

Observational test

Question:

At 14 o'clock local time in the morning of the spring equinox a rare transit of Mercury is going to take place. A team of astronomers reaches a mountain top, early in the morning, in order to align his telescope and then observe the transit. The site is new and they do not know the geographical coordinates.
Unfortunately the sky is covered with clouds. No stars are visible. The telescope cannot be aligned. The sky is overcast until 11 o'clock. The Sun becomes visible. An experienced astronomer manages to roughly align the telescope in less than 2 minutes! He only uses a water bubble.

You are given the telescope of the 7th IOAA and a water level. Assume that it is spring equinox and that the time is 12 o'clock. A fake Sun is shining. Could you align the telescope?

(Note: Obviously for this exercise, a telescope tube is not necessary, therefore, for the sake of convenience, the telescope will be equipped with a rough paper-tube and without counter weights.

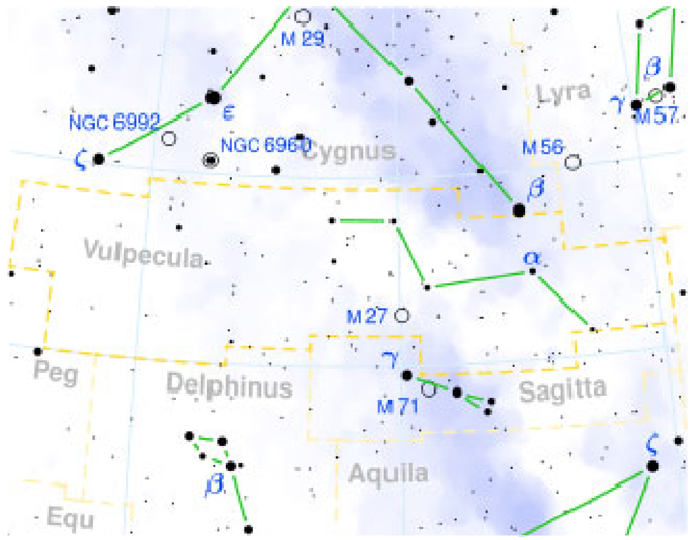
Observational test

Question 1.

Find the field of view of the telescope with the eyepiece provided by the attendant.

Question 2.

Locate the bright star γ *Sagitta* ($RA = 19^h58^m45.39^s$, $Dec = +19°29'31.5''$), which lies between the constellations of Lyra and Delphinus. Then aim and locate the famous Dumbbell Nebula, M27 ($RA = 19^h59^m36.34^s$, $+22°43'16.09''$) in the center of the field of view. The observing spot is rather dark and you cannot read the setting circles!



The field in the vicinity of the Dimbell Nebula

Question 3:

At 14 o'clock local time in the morning of the spring equinox a rare transit of Mercury is going to take place. A team of astronomers reaches a mountain top, early in the morning, in order to align his telescope and then observe the transit. The site is new and they do not know the geographical coordinates. Unfortunately the sky is covered with clouds. No stars are visible. The telescope cannot be aligned. The sky is overcast until 11 o'clock. The Sun becomes visible. An experienced astronomer manages to roughly align the telescope in less than 2 minutes! He only uses a water bubble.

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IOAA 2014 – Suceava & Gura Humorului, Romania

The eighth IOAA was held from 1^{st} to 11^{th} August 2014. Total 42 teams from 37 countries participated in the event. These included first time teams from Mexico, Moldova, Montenegro, Nepal and Pakistan.

The Olympiad's emblem consists of the coat of arms of Moldavia, the head of the Dacian wolf and the constellation of Cygnus.

The flag and coat of arms of Moldavia, one of the Danubian Principalities, together with Wallachia, which formed the basis of the Romanian state, were subject to numerous changes throughout their history. The coat of arms of Moldavia, acknowledged in 1392, is traditionally represented by the bull head, an animal which has star. symbolising the reign of wisdom.is placed between the wild bulls horns, with a heraldic rose on the left, as a symbol of faith. On the right, the Moon in its crescent phase stands for rebirth. The background is cinnabar (red in heraldry), and it symbolises bravery.

The wolfs head represents the Dacian Draco, which was the standard ensign of the Dacian troops, and can be seen in the hands of Decebalus' soldiers in several scenes on Trajan's Column in Rome, Italy. It has the form of a dragon whose open mouth contains several metal tongues. Its hollow head was mounted on a pole, with fabric tube affixed at the rear. In use, the Draco was held up into the wind, or above the head of a horseman, with the wind passing through it, giving the impression that it was alive, while making a shrill sound. The Dacians regarded it both as a protective and religious symbol. The Draco shows religious syncretism between the wolf, the dragon and the serpent. At the same time, it represents the Draco (the dragon) constellation.

The third element of the emblem consists of the Cygnus constellation. "Cygnus" is a scientific association affiliated with UNESCO, which supported the organisation of the International Olympiad on Astronomy and Astrophysics. Its name also has a mythological meaning. In Greek mythology, Orpheus was transformed into a swan after his death and ascended to heaven with his lyre.

Short problems

Problem 1. Lagrange Points

The *Lagrange* points are the five positions in an orbital configuration (assume circular orbits), where a small object is stationary relative to two big bodies, only gravitationally interacting with them- for example, an artificial satellite relative to Earth and Moon, or relative to Earth and Sun. In the **Figure 1** are sketched two possible locations of Lagrange points L_3 relative to relative to the Earth – Sun system. Find out which of the two locations $L_3{}^1$ and $L_3{}^2$ could be the real Lagrange point relative to the system Earth – Sun; show the reason for your answer with appropriate equations and calculate the difference between one AU and Sun – L_3 distance.

You know the following data: the Earth - Sun distance $d_{ES} = 14.96 \times 10^7$ km and the Earth – Sun mass ratio $M_E/M_S = 1/332946$

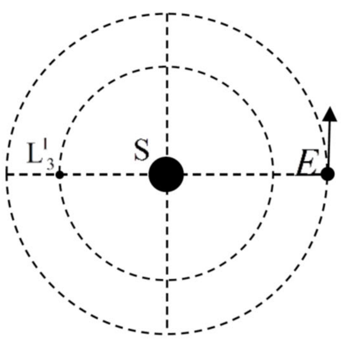


Figure 1A

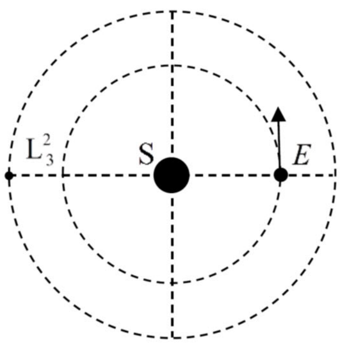


Figure 1B

Problem 2. Sun gravitational catastrophe!

In a gravitational catastrophe, the mass of the Sun mass decrease instantly to half of its actual value. If you consider that the actual Earth orbit is elliptical, its orbital period is $T_0 = 1$ *year* and the eccentricity of the Earth orbit is $e_0 = 0.0167$

Find the period of the Earth`s orbital motion, after the gravitational catastrophe, if it occurs on: a) 3rd of July (aphelion) b) 3rd of January.

Problem 3. Cosmic radiation

During studies concerning cosmic radiation, a neutral unstable particle – the π^0 meson was identified. The rest-mass of meson π^0 is much larger than the rest-mass of the electron. The studies reveal that during its flight, the meson π^0 disintegrates into 2 photons. In a particular case, one of the created photons has the maximum possible energy E_{max} and, consequently, the other one has the minimum possible energy E_{min}.

Find an expression for the initial velocity of the meson π^0, as a function of E_{max} and E_{min}. You may use as known ***c*** **-** the speed of light and the relation between the energy and momentum of any relativistic particles $E^2 = p^2c^2 + m_0{}^2c^4$

Problem 4. Sandra Bullock And George Cloony

An astronaut, with mass $M = 100$ kg, gets out of the space ship for a repairing mission. He has to repair a satellite at rest relative to the space ship, at about $d = 90$ m away from it. After he finishes his job, he realizes that the systems designed to assure his come-back to shuttle are broken. He also observes that he has air only for 3 minutes. He also notices that he possessed a sealed cylindrical can (base section $S = 30$ cm^2) firmly attached to his/her glove, with $m = 200$ g of ice inside. The can is not completely filled with ice.

Determine if the astronaut is able to return safely to the shuttle, before his air reserve is empty, if he manages to open the can in correct direction. Briefly explain your calculations. Note that he cannot throw away anything of its equipment, or touch the satellite.

You may use the following data: $T = 272$ K/ the temperature of the ice in the can, $p_s = 550$ Pa – the pressure of the saturated water vapors at the temperature $T = 272$ K; $R = 8300$ J/(kmol K) - the universal gas constant; $\mu = 18$ kg/kmol - the molar mass of the water.

Problem 5. The life –time of a main sequence star

The plot of the function $\log(L/L_S) = f(\log(M/M_S))$ for data collected from a number of stars is represented in figure 2. L and M are the luminosity and the mass of a star respectively and L_S and M_S the luminosity and the mass of the Sun respectively.

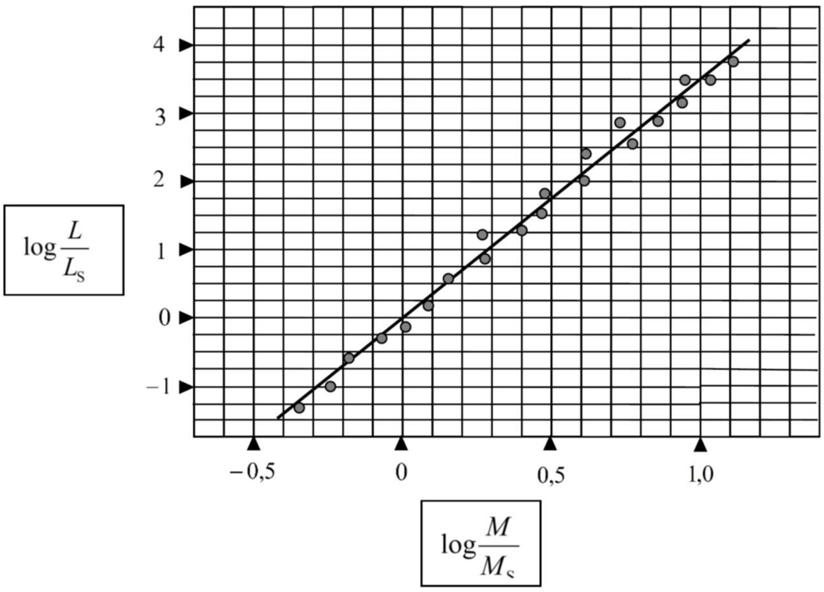


Figure 2

Find an expression for the main sequence life- time for a main sequence star from Hertzprung – Russell diagram, as a function of mass fraction converted to energy η and mass ratio to the solar mass n , Use the following assumptions: the time spent by Sun in the same Main Sequence is τ_S, for each star the mass fraction which changed into energy is η, the percent of the mass of Sun which changes into energy is η_S, the mass of each star is expressed as $n = \dfrac{M}{M_S}$ and assume that luminosity of the star remains constant, during its main sequence life time.

Problem 6. The effective temperature of a star

From the radiation emitted by a star, two radiations with wavelength values in a narrow range $\Delta\lambda \ll \lambda$ are studied, i.e. the wavelength have values between λ and $\lambda + \Delta\lambda$. According to Planck's relationship (for an absolute black body), the following relation defines, the energy emitted by star in unit time, through a unit area of its surface, per unit wavelength interval:

$$r = \frac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{k\lambda T}} - 1\right)}.$$

The spectral intensities of the radiation with wavelengths λ_1 and respectively λ_2, both within the range $\Delta\lambda$ measured on Earth are and respectively.

Find out the relation between wavelength λ_1 and λ_2 if $I_1(\lambda_1) = 2I_2(\lambda_2)$, when $hc \ll \lambda kT$.

Here: h – Planck's constant; k – Boltzmann's constant; c – speed of light in vacuum. $e^x \approx 1 + x$ *if* $x \ll 1$.

Problem 7. Pressure of light

For an observer on Earth the pressure of the radiation emitted by Sun is $p_{\text{rad,S}}$ and the pressure of the radiations emitted by a star Σ is $p_{\text{rad},\Sigma}$.

Calculate the visual apparent magnitude of the star Σ if the apparent visual magnitude of the Sun is m_S. The following assumption may be useful for solving the problem:
Generally, the pressure of the electromagnetic radiation in vacuum is equal to the volume energy density of the electromagnetic radiation $\left(p_{\text{rad}} = \frac{\Delta E}{\Delta V}\right)$.

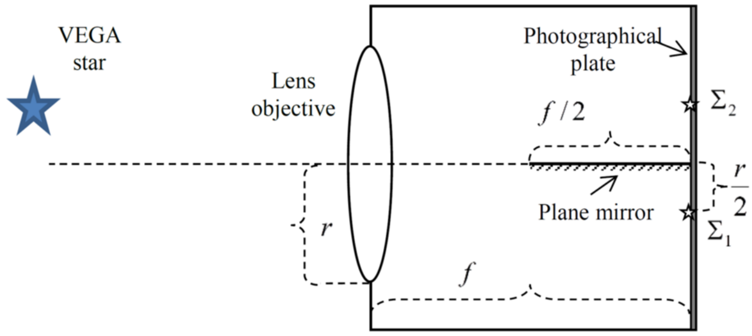
The following data are known: M_S - the mass of the Sun, R_S - the radius of Sun, G - universal gravitational constant; σ Stefan - Boltzmann's constant ; c – speed of light in vacuum.

Problem 8. Space – ship orbiting the Sun

A spherical space –ship orbits the Sun on a circular orbit, and spin around an axis of rotation that is perpendicular to the orbital plane of the space-ship. The temperature on the exterior surface of the ship is T_N. Assume the space -ship is a perfect black body and there is no activity inside it.
Find out the apparent magnitude of the Sun and the angular diameter of the Sun as seen by the astronaut on board of the space – ship. The following values are known: T_S - the effective temperature of the Sun; R_S - the radius of the Sun; d_0 - the Earth –Sun distance; m_0 - apparent magnitude of Sun measured from Earth; R_N - the radius of the space –ship.

Problem 9. The Vega star in the mirror

Inside a camera a plane mirror is placed along the optical axis of the objective (as shown in figure). The length of the mirror is half the focal length of the objective. A photographic plate is placed at the focal plane of the camera. Two images with different brightness are captured on the photographic plate (as shown in figure). The star Vega is not on the optical axis of the lens. The distance between the optical axis and the image Σ_1 is $\frac{r}{2}$. Find the difference between the apparent photographical magnitudes of the two images of the star Vega.



Figure

Problem 10. Stars with Romanian names

Two Romanian astronomers Ovidiu Tercu and Alex Dumitriu from Galati Romania, recently discovered two variable stars. The galactic coordinates of the two stars are: Galati V1(l_1 = 114.371°; b_1 = –11.35°) and Galati V2(l_2 = 113.266°; b_2 = –16.177°).

Estimate the angular distance between the stars Galati V1 and Galati V2

Problem 11. Apparent magnitude of the Moon

The apparent magnitude of the Moon as seen from the Sun is $M_M = 0.25^m$

Calculate the values of the apparent magnitudes of the Moon (as seen from the Earth) corresponding to the following Moon – phases: full-moon and the first quarter. Assume: the Moon – Earth distance – d_{ME} = 385000 km, the Earth – Sun distance – d_{ES} = 1 AU, the Moon –Sun distance, d_{MS} = 1 AU. For terrestrial observers, following phase factor must be used to correct the lunar brightness for curvature of lunar surface and phase of the moon

$$P(\Psi) = \frac{2}{3} \cdot \left[\left(1 - \frac{\Psi}{\pi}\right)\cos\Psi + \frac{1}{\pi}\sin\Psi\right],$$

where Ψ is the phase angle.

Problem 12. Absolute magnitude of a cepheid

The cepheids are variable stars, whose luminosities vary due to stellar pulsations. The period of the oscillations of a cepheid star is:

$$P = 2\pi R\sqrt{\frac{R}{GM}} \, ,$$

where: R – the mean radius of the cepheid; M – the mass of the cepheid (remains constant during oscillation), you can assume that the temperature is constant during the pulsation;

Express the mean absolute magnitude of the cepheid M_{cep}, in the following form:

$$M_{cep} = -2.5^m \cdot \log k - \left(\frac{10}{3}\right)^m \cdot \log P \, ,$$

where P is the period of cepheid's pulsation.

Long problems

1. Eagles on the Caraiman Cross!

The tallest cross built on a mountain peak is located on a plateau situated on the top of the peak called Caraiman in Romania at altitude $H = 2300$ m from the sea level. Its height, including the base-support is $h = 39.3$ m. The horizontal arms of the cross are oriented on the N-S direction. The latitude at which the Cross is located is $\varphi = 45°$.

A. On the evening of 21st of March 2014, the vernal equinox day, two eagles stop from their flight, first near the monument, and the second, on the top of the Cross as seen in figure 1. The two eagles are in the same vertical direction. The sky was very clear, so the eagles could see the horizon and observe the Sun set. Each eagle began to fly right at the moment it observed that the Sun disappeared completely.

At the same time, an astronomer is located at the sea-level, at the base of the Bucegi Mountains. Assume that he is in the same vertical direction as the two eagles.

Assuming the atmospheric refraction to be negligible, solve the following questions:

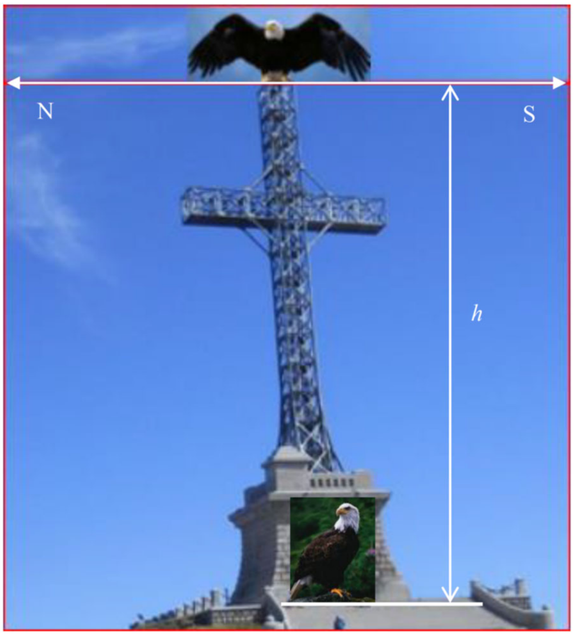


Figure 1

(1) *Calculate* the duration of the sunset, measured by the astronomer.

(2) *Calculate* the durations of sunsets measured by each of the two eagles and indicate which of the eagles leaves the Cross first. What is the time interval between the moments of the flights of the two eagles.

The following information is necessary:

The duration of the sunset measurement starts when the solar disc is tangent to the horizon line and stops when the solar disc completely disappears.

The Earth's rotational period is $T_E = 24$h, the radius of the Sun $R_S = 6.96 \times 10^5$ km, Earth – Sun distance $d_{ES} = 14.96 \times 10^7$ km, the latitude of the Heroes Cross is $\varphi = 45°$. $R_E = 6370$ km

B. At a certain moment the next day, 22nd March 2014, the two eagles come back to the

Heroes Cross. One of the eagles lands on the top of the vertical pillar of the Cross and the other one land on the horizontal plateau, just at the tip of the shadow of the vertical pillar of the Cross, at that moment of the day when the shadow length is minimum.

(1) Calculate the distance between the two eagles and the second eagle's distance from the cross.

(2) Calculate the length of the horizontal arms of the Cross l_b, if the shadow ~~on the plateau~~ of one of the arm of the cross at this moment has the length $u_b = 7$ m

C. At midnight, the astronomer visits the cross and, from its top, he identifies a bright star at the limit of the circumpolarity. He named this star "Eagles Star". Knowing that due to the atmospheric refraction the horizon lowering is $\xi = 34'$, calculate:

(1) The "Eagles star" declination;

(2) The "Eagles star" maximum height above the horizon.

2. From Romania .... to Antipod! ...with a ballistic messenger

The 8th IOAA organizers plan to send to the **antipode** (the point on the Earth's surface diametrically opposite to the launch position) the official flag using a ballistic projectile. The projectile will be launched from Romania, and the rotation of the Earth will be neglected.

(a) Calculate the coordinates of the target-point if the launch-point coordinates are: $\varphi_{\text{Romania}} = 44°$ *North*; $\lambda_{\text{Romania}} = 30°$ *East*.

(b) Determine the magnitude of the velocity and the launch angle, with respect to the horizon at launch site, in order that the projectile should hit the target.

(c) Calculate the velocity of the projectile when it hits the target.

(d) Calculate the minimum velocity of the projectile on its trajectory.

(e) Calculate the flying –time of the projectile, from the launch to the impact. You may use the value of the gravitational acceleration at Earth surface as $g_0 = 9.81$ m s^{-2}; the Earth radius $R = 6370$ km.

(f) Will it be possible that the projectile will be seen by the naked eye when it is at the maximum distance from the Earth. You will use the following values: The Moon albedo $\alpha_M = 0.12$; The Moon radius $R_M = 1738$ km; the Earth –Moon distance $r_{EM} = 385000$ km; the apparent magnitude of the full moon $m_M = -12.7^m$. You assume that the projectile is perfectly metallic sphere with radius $r_{\text{projectile}} = 400 \times 10^{-3}$ m and with perfectly reflective surface.

Problem 1 Black Hole in Milky Way

By observational facts, the scientists admit presence of a black-hole at the center of Milky Way. At the center of Milky Way, a hypothetical black-hole (Sagittarius A*) is located. A star S* is orbiting the black-hole SA*.

In the table 1 the following data is presented: the date and the angular position coordinates (α; β) of the star S* at different moments of the observation. The coordinates represent the angular distances of the projection or the star S* in the coordinates system (U, W) , centered on the SA* (see figure 1).

An angular distance of $\varphi = 1$ arcsec corresponds to linear distance in the plane of the sky $d = 41$ light days, therefore to a scale $S_0 = \dfrac{d}{\varphi} = 41 \dfrac{\text{light day}}{\text{arcsec}}$.

	Date (year)	α (arcsec)	β (arcsec)
1	1995.222	0.117	–0.166
2	1997.526	0.097	–0.189
3	1998.326	0.087	–0.192
4	1999.041	0.077	–0.193
5	2000.414	0.052	–0.183
6	2001.169	0.036	–0.167
7	2002.831	–0.000	–0.120
8	2003.584	–0.016	–0.083
9	2004.165	–0.026	–0.041
10	2004.585	–0.017	0.008
11	2004.655	–0.004	0.014
12	2004.734	0.008	0.017
13	2004.839	0.021	0.012
14	2004.936	0.037	0.009
15	2005.503	0.072	–0.024
16	2006.041	0.088	–0.050
17	2007.060	0.108	–0.091

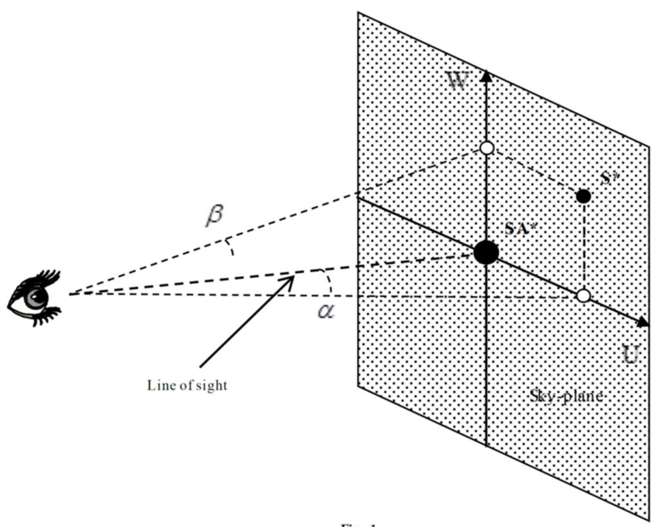


Fig. 1

By using the information provided your tasks are:

(a) Plot the projection of the trajectory of the star S* in the plane P (see figure 2). This plane is close to the observer. In this plane, $\varphi = 1$ arcsec corresponds to a linear distance $d_0 =$ 1200 mm therefore the scale is $S = \frac{d_0}{\varphi} = 1200 \frac{\text{mm}}{\text{arcsec}}$. You have to use the millimeter graph paper, carbon copy sheet of paper and the transparent sheets for an accurate plot.

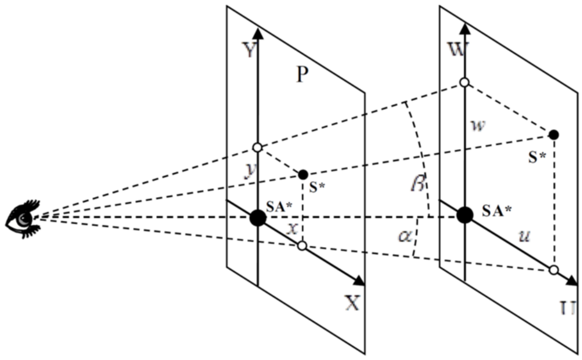


Fig. 2

(b) By using the plot prove that the line of sight is normal to the actual plane of the orbit

(c) Using your plot find out following elements of the real orbit of star S* around the black hole SA*:

I. a – semi-major axis (in light days units); b – small semi-minor axis in (in light days units); e – eccentricity;

II. $r_{\min}$ – the minimum distance between S* and SA*(in light days units); $r_{\max}$ – the maximum distance between S* and SA* (in light days units);

III. The distance from the observer to the S*;

IV. the orbiting period of star S* around SA* (obtain the best possible result by taking as many measurements as possible and by taking their arithmetic mean);

V. the total mass of the system "SA* - S*".

Presenting the intermediate and final data in tables is recommended for an accurate evaluation.

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Problem 2 Thermodynamic test

An hypotetical shuttle is launched to investigate the atmosphere (100% CO_2) of two extrasolar planets P1 and P2. The atmosphere is in static thermodynamic equilibrium. When the shuttle is near each planet, a radio probe is launched toward respective planet, in vertical direction (in the direction of the planet`s radius). When the radio probe reaches constant velocity, it starts sending values of the pressure of the atmosphere. In Fig. 3.1 is plotted the atmospheric pressure values (in arbitrary units) as function of the time of descent for the planet P_1. When the probe touches the surface of planet P_1 it sends the value of the temperature $T_0 = 700$ K and the value of the gravitational acceleration $g_0 = 10$ m s^{-2}.

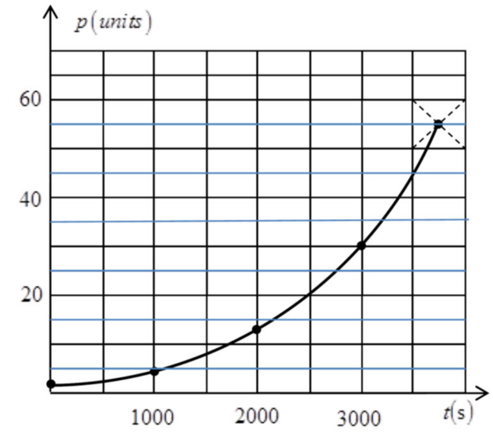


Fig. 3.1.

The gravitational acceleration on each planet is assumed to be constant during uniform descent of the radio probes.

(a) Find the altitude h_0 from where the radio probe R_1 starts the uniform descent and thus starts the transmitting information.

(b) Find the temperature of planet P_1 at the altitude $h = 39.6$ km. You know: The universal gas constant $R = 8.3$ J/mol·K; the molar mass of CO_2, $\mu = 44$ g/mol.

(c) In Fig. 3.2.was plotted the atmospheric pressure values (in arbitrary units) as a function of time of descent for the planet P_2 atmosphere. When the probe touched the surface of the planet P_2, it sends the value of the temperature $T_0 = 750$ K and respectively the value of gravitational acceleration $g_0 = 8$ m s^{-2}

Draw the following dependency graphs for $p = f(h)$ and $T = f(h)$ in the CO_2 atmosphere of the planet P_2.

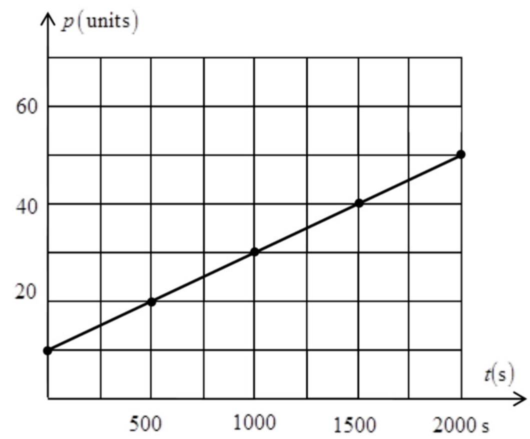


Fig. 3.2.

Problem 3 IOAA Observer on an extrasolar planet

The Sirius star, located in the constellation of Canis Major, is the brightest star in the night sky of the Earth. What the observer's eye sees as a single star is actually a binary star system. The high brightness of Sirius is a consequence of two facts: its intrinsic luminosity and its proximity to the Earth.

The Mizar multiple star system, in the constellation of Ursa Major, consists of 4 stars seen along the same line of sight from the Earth. Some of these stars form a gravitationally bound system.

Let's assume that an observer (observer A) is located on one of the planets of the Sirius system.

Determine:

(a) The magnitude of the Sun as seen by observer A ($m_{\text{Sun,Planet}}$).

(b) The magnitude of Sirius star system as seen by the observer A. ($m_{\text{SY,Planet}}$)

(c) The combined intrinsic luminosity of the Mizar system, L_{Mizar};

(d) The average distance between gravitationally bound stars of the Mizar system and Earth,

(e) The geocentric angular distance between Mizar system and Sirius, $\Delta\theta$;

(f) The physical distance between the gravitationally bound stars of the Mizar system and the observer A. ($d_{\text{Mizar–Planet}}$)

(g) The magnitude of the entire Mizar system as seen by the observer A. ($m_{\text{Mizar,Planet}}$)

Also estimate amount of errors in all your answers.

The following data may be used:

$d_{\text{Sirius,Earth}} = 2.6$ pc – the Sirius – Earth distance;

$m_{\text{Sirius,Earth}} = -1.46^{\text{m}}$ - the apparent magnitude of Sirius measured from the Earth;

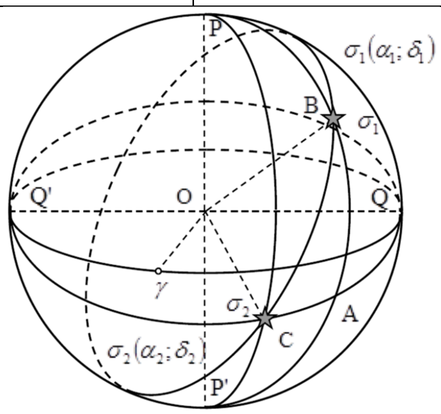
$d_{\text{Sun,Earth}} = 1$ AU - the Sun – Earth distance;

$m_{\text{Sun,Earth}} = -26.78^{\text{m}}$ - the apparent magnitude of the Sun as seen from Earth ;

$d_{\text{Sirius,Planet}} = 10$ AU - distance between Sirius and its planet where the observer A is located;

In the table below information for the stars from the Mizar system as measured from the Earth is given.

Star number	Name of the star	Apparent magnitude	Parallax (mas)
1	Alcor	3.99 ± 0.01	39.91 ± 0.13
2	Mizar A	2.23 ± 0.01	38.01 ± 1.71
3	Mizar B	3.86 ± 0.01	38.01 ± 1.71
4	Sidus Ludoviciana	7.56 ± 0.01	8 ± 4



The equatorial coordinates of Mizar system (σ_1) and respectively of Sirius (σ_2), located on the heliocentric map are:

$\alpha_{\text{Mizar}} = \alpha_1 = 13^{\text{h}}23^{\text{min}}55.5^{\text{s}}$; $\delta_{\text{Mizar}} = \delta_1 = 54°55'31''$;

$\alpha_{\text{Sirius}} = \alpha_2 = 6^{\text{h}}45^{\text{min}}$; $\delta_{\text{Sirius}} = \delta_2 = -16°43'$.

Note: $\ln(1-x) \approx -x$ for $x \ll 1$; $e^x \approx 1 + x$ for $x \ll 1$

OBSERVATIONAL
TEST
PLANETARIUM

PART I
READ CAREFULY

Inside the dome:

1) The observational round in the planetarium consists of two parts, one inside the dome and the other outside the dome.
2) The part inside the dome consists of 3 questions and takes 30 minutes.
3) When you enter the dome, you will be directed to your seat, where you will find a clipboard with your answer sheet, one data table and a flashlight. During the adaptation time the students may stand and change the position around their place, but they are not allowed to communicate with each other. During the observation you can stand and turn in order to make a comfortable observation.
4) Fill your student ID in the box on the answer sheet.
5) PAY ATTENTION TO THE ASSISTANTS, and follow their instructions.
6) The timing for the first part is as follows:
   a) **8 minutes for your eye adaptation to the darkness;**
   b) **10 minutes for the first question;**
   c) **6 minutes for the second question;**
   d) **6 minutes for the third question;**
7) Use the flash light only when you need it and point it only at your paper.
8) When you leave the dome, leave everything on your seat.
9) PLEASE WRITE ONLY ON THE PRINTED SIDE OF THE ANSWER SHEET. DON'T USE THE REVERSE SIDE. The evaluator will not take into account what is written on the reverse of the answer sheet.

GOOD LUCK!

Question 1

The sky projected in the dome corresponds to Suceava (Long 26^0 15’), at 18:00 UT, on a certain day of a certain month.

You have 8 minutes to relax and familiarize your eyes with the darkness. During this time don’t use the flash light.

Two arcs will then be projected. The arcs are segmented. Each segment represents an interval of some number of degrees. Note that this number is not the same for each arc.

10 minutes – Question 1

a. Identify each arc by circling the correct name and give the angular size of each segment (in degrees).

First arc	**Equator**	**Meridian**	**Ecliptic**	**Segment size**
Second arc	**Equator**	**Meridian**	**Ecliptic**	**Segment size**

b. Estimate the local sidereal time of the sky you see in the dome.

θ_{sidereal}	

c. Determine the month to which the projected sky would correspond at the given time. Fill in the box the number of the month (1 to 12).

Month number	

OBSERVATIONAL PLANETARIUM	PART I	STUDENT ID	
	Answer sheet		

Please write ONLY on this side of the paper

Question 2 and 3

For questions 2 and 3 the assistant will use a small red arrow pointer to point some objects in the sky. Each object will be pointed at for **2 minutes** (30 seconds arrow pointer on and 10 seconds off)**.** Please pay attention to the assistant announcements.

6 minutes – Question 2

The locations of three Messier objects will be pointed at one by one. For each Messier object pointed to, fill in the boxes with its Messier catalog number and the number which indicates its type, based on the following: **1 for galaxy, 2 for nebula, 3 for open cluster, 4 for globular cluster.**

Also, for each object, fill in the IAU abbreviation of the constellation where the star is located. Refer to **Table 1** for the abbreviations.

1st Messier object		Number which indicates the type		IAU abbreviation of the constellation	
2nd Messier object		Number which indicates the type		IAU abbreviation of the constellation	
3rd Messier object		Number which indicates the type		IAU abbreviation of the constellation	

6 minutes – Question 3

Three stars will be pointed at successively. Each star will be pointed at for 2 minutes. Fill the appropriate box with the name of the star (or the Bayer designation) and the number which indicates its type (**1 for single, 2 double**).

For each star, also fill in the IAU abbreviation of the constellation where the star is located. Refer to **Table 1** for the abbreviations.

1st Star		Number which indicates the type		IAU abbreviation of the constellation	
2nd Star		Number which indicates the type		IAU abbreviation of the constellation	
3rd Star		Number which indicates the type		IAU abbreviation of the constellation	

You have finished the first part. Verify that have written your student ID on every page. Attach the answer sheers to the clipboard and leave it on your seat along with the flashlight, and then exit the dome.

OBSERVATIONAL PLANETARIUM

PART II Star chart
Indication

Star chart:

You have 30 minutes to finish this part.

Please use only pencil to make the drawings and markings.

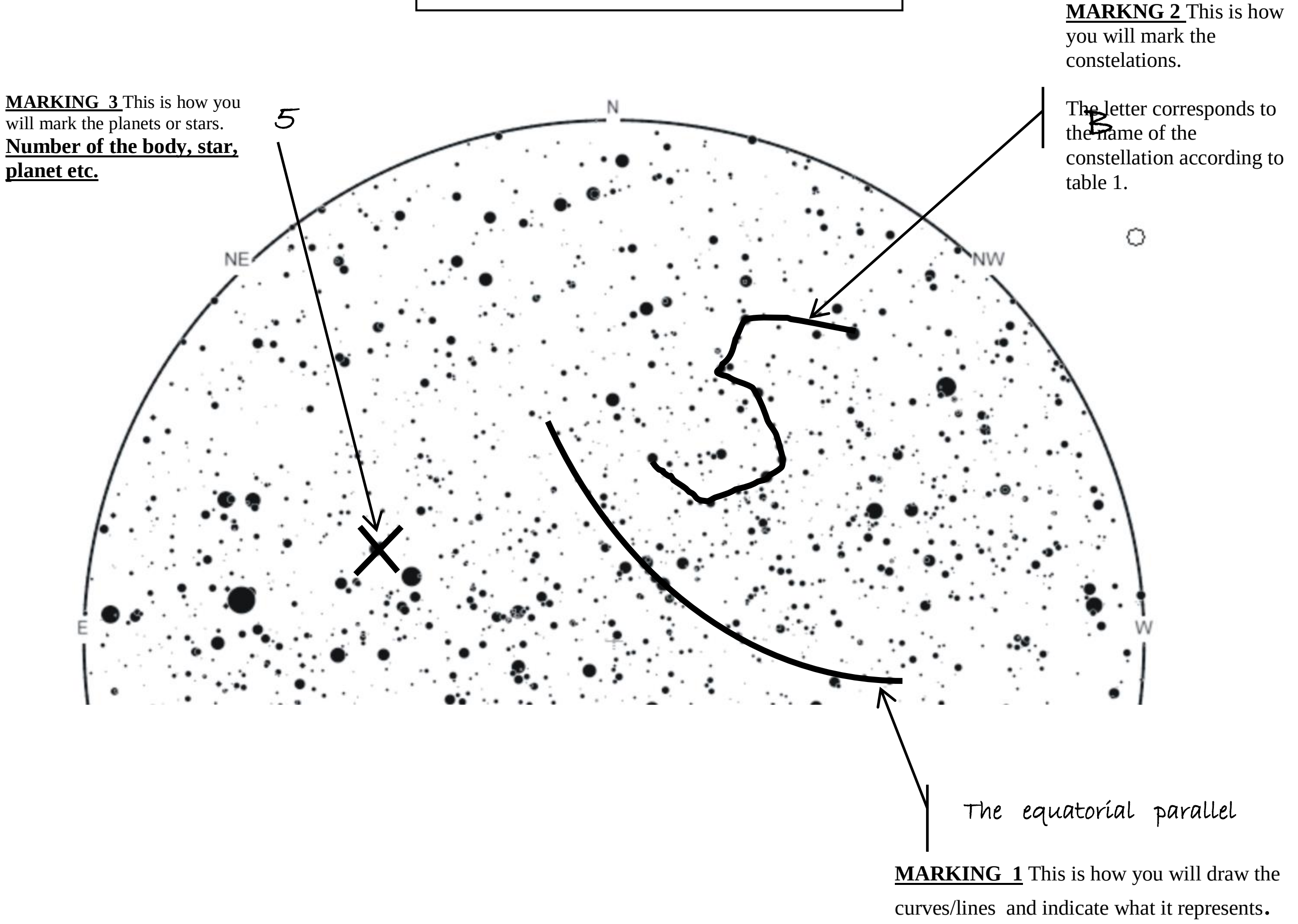
After you finish this part, write your student ID on the answer sheet as well as on the sky map.

Put the answer sheets in the folder; leave the compass, the ruler and the pencil on the table.

Thank you!

In this part you will use the sky-map found in the envelope. The map represents the sky in Suceava (Latitude 47^0 $39^{'}$ North, Longitude 26^0 15’ East) on the day of the test at 22:00 local time. The observer who made the sky-map was at a very high altitude above Suceava; the Zenith point is in the center of the chart. Please use a pencil for marking and drawing lines on the sky-map. Use the example markings 1, 2 and 3 shown below to draw lines and mark objects on the map.

HOW TO DRAW AND MARK OBJECTS ON THE SKY-MAP



OBSERVATIONAL PLANETARIUM	PART II – outside the dome **Answer sheet**	STUDENT ID	

Please write ONLY on this side of the paper

Questions

The map represents the sky in Suceava (Latitude 47^0 39' North, Longitude 26^0 15' East) at 19:00 UT on the day of test. The observer who made the sky-map was at a very high altitude above Suceava; the Zenith point is in the center of the chart. Solve question 1 to 4 on one copy of the map and questions 5 to 8 on the second copy of map.

1. Draw on the map the horizon for an observer located on the ground in Suceava. (2 points)
2. Draw the celestial equator, the ecliptic, the galactic equator and the local meridian on the map with continuous lines. (8 points)
3. Mark the cardinal points (as N for north, E for east, S for south and W for west). Mark all the planets (except Uranus and Neptune) of the Solar System on the map and number them as 1, 2, …,6 in the order of increasing orbital radius (Skip number 3 for the Earth). Note that planets are <u>not</u> currently shown on the map. (9 points)
4. Identify and mark the four brightest stars in visual band above the horizon line. Number the star starting from **1** – the brightest, and continue with the fainter ones till number **4** for the faintest. Fill in the following table the Bayer name of the four identified stars. (4 points)

Marking on the map	1	Name of the star	
	2	Name of the star	
	3	Name of the star	
	4	Name of the star	

5. Draw approximate figures of any 15 constellations which lie <u>completely above</u> the horizon on the map. Each constellation you mark should be identified on the map with the IAU abbreviation, using **Table 1**. (6 points)
6. Mark on the map the positions of the following objects: (5 points)
   a. The Messier objects: M31, M27, M13;
   b. β Cygni, δ Ursa Minoris.
7.

Estimate the sidereal time of the map; write the value in the box to the right. (10 points)	

8.

Estimate the equatorial coordinates (right ascension and declination) of the star Altair (α Aquilae). Write your answer in the box. (6 points)	α= δ=

OBSERVATIONAL PLANETARIUM	PART II – outside the dome **Answer sheet**	STUDENT ID	

Please write ONLY on this side of the paper

OBSERVATIONAL TEST OUTDOOR

Indications
READ CAREFULY

Outside the dome:

1) The observational round in the field should take a maximum of 30 minutes;
2) Please pay attention to the instructions by assistants.
3) You will be directed to a designated telescope. Here you will find attached to the clipboard the answersheet.
4) For the observational test outdoor, we are using a Newtonian telescope on equatorial mount EQ5 (D=200 mm, F=1000 mm).

Note: the telescope is already aligned, but not necessarily calibrated – do not change the position of the tripod!

5) Fill your student ID in the box.
6) **Please write the time at the start of the observation test on the top of next page!**
7) PLEASE WRITE ONLY ON THE PRINTED SIDE OF THE ANSWER SHEET. DON'T USE THE REVERSE SIDE. The evaluator will not take into account what is written on the reverse of the answersheet.

GOOD LUCK!

OBSERVATIONAL TEST OUTDOOR

Time of start

STUDENT ID	

Questions	Answers	Space designated for the evaluator
1. Name any five constellations which will be at the meridian 2 hours from the start of your observation test?		
2. Point the telescope to M39. When you finish, ask your assistant to verify. Write in the box the number which corresponds to the object type (1 - globular cluster, 2 - double cluster, 3 - open cluster, 4 - galaxy, 5 - nebula).	Number which corresponds to the object type	
3. The right ascension and the declination for β Aql (Alshain) are α=19h55m and δ=6°26'. By using the telescope find out the right ascension and the declination for δ Cep. Write down the values in the appropriate boxes.	Right ascension (α) Declination (δ)	
4. Point the telescope to the coordinate $\propto$=2h22m and δ=57°10'. When you finish, ask your assistant to verify. Write in the box the number which corresponds to the object type (1 - globular cluster, 2 - double cluster, 3 - open cluster, 4 - galaxy, 5 - nebula).	Number which corresponds to the object type	
5. Estimate UT when the meridian, the ecliptic and the equator are intersecting at the same point in this night. You may use the telescope or any other method.	Value of time	
6. Estimate the galactic latitude of ξ Dra (Grumium).		
7. Estimate the ecliptic latitude of ε Cyg (Gienah).		

8th International Olympiad on Astronomy and Astrophysics
Suceava - Gura Humorului - August 2014

TEAM COMPETITION

For this test you have to use the materials you found in the box: milimetric paper, plasticine, a ruler, wire, knife, scissors, auto adhesive paper, adhesive band, pins and a support plate.

Defensing the Earth with plasticine, a wire, a pen and scissors

A giant asteroid, potentially dangerous for Earth flight towards the Earth. The collision is inevitable so from an artificial Earth's satellite a nuclear missile will be launched in order to brake into parts the asteroid. You are in a control post, with no computers, only with materials from the box you receive. In one hour and half you have to send the box with the solution of the following problems in order to plan the defensive strategy for our planet.

Real facts
The asteroid 2013 UX11 (Galați – Romania 2013)

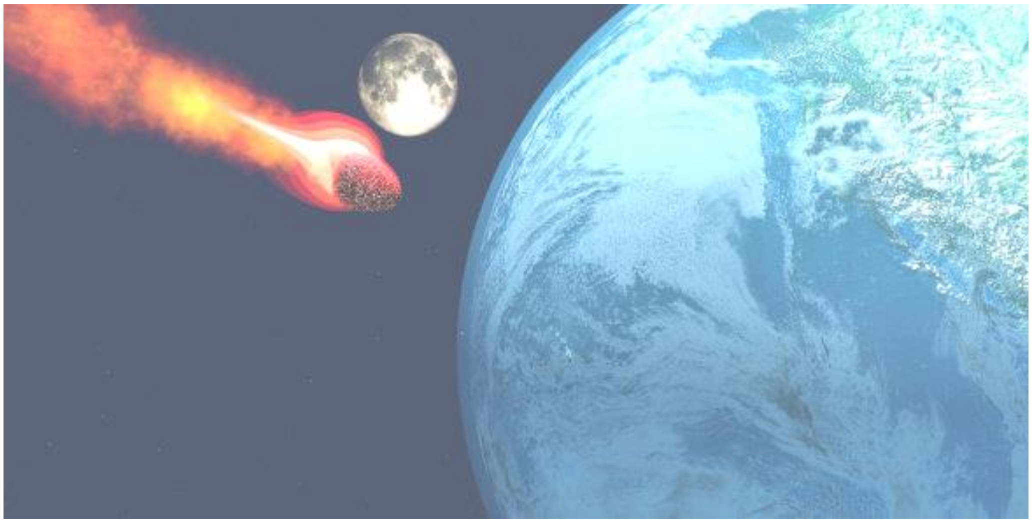
During the night between 29th and 30th of October 2013, two astronomers Ovidiu Tercu and Alex Dumitriu from the Astronomic Observatory from Galati, Romania discovered an asteroid in the Torro constellation. The asteroid named 2013 UX11 has a diameter of $D = 2{,}5\,\text{km}$. This was the first time that an asteroid was discovered by Romanian astronomer in order to make this problem for IOAA. After analyzing the data in the "Minor Planet Center" the following communication was received: ***2013 UX11*** is an asteroid from the Main Belt of Asteroids, orbiting between the orbits of Mars an Jupiter with period $T = 4{,}2\,\textit{years}$ with an eccentricity $e = 0{,}15$.

a) *Find out* the characteristics of orbit of the ***2013*** UX11 regrdless to the Sun- $(a, b, c, r_{\min}, r_{\max})$. You know the mass of the Sun $M = 9{,}1 \cdot 10^{30}\,\text{kg}$ and the gravitational constant $K = 6{,}67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.

b) *Find out the maximum and respectively* the minimum of the temperatures of the surface of the asteroid. You know : Temperature of the surface of the Sun $T_S = 6000\,\text{K}$; the radius of the Sun $R_S \approx 7 \cdot 10^5\ \text{km}$; the albedo of the surface of the asteroid $\alpha = 0{,}2$.

The plan

You have received a scaled sketch of the positions of the Earth (P), satellite (S) and the point where the missile have to hit the asteroid. The distance between asteroid and the Earth is $r = 30000\,\text{km}$.



You have to analyze the problem of saving the Earth only by using the materials from the box in the following conditions:

A. For a given speed of the missile v_0, the problem have only one solution i.e. there is only one trajectory form S to A, available for the missile in order to hit the asteroid. You have to determine by using the materials :

a) the elements of the missile trajectory;

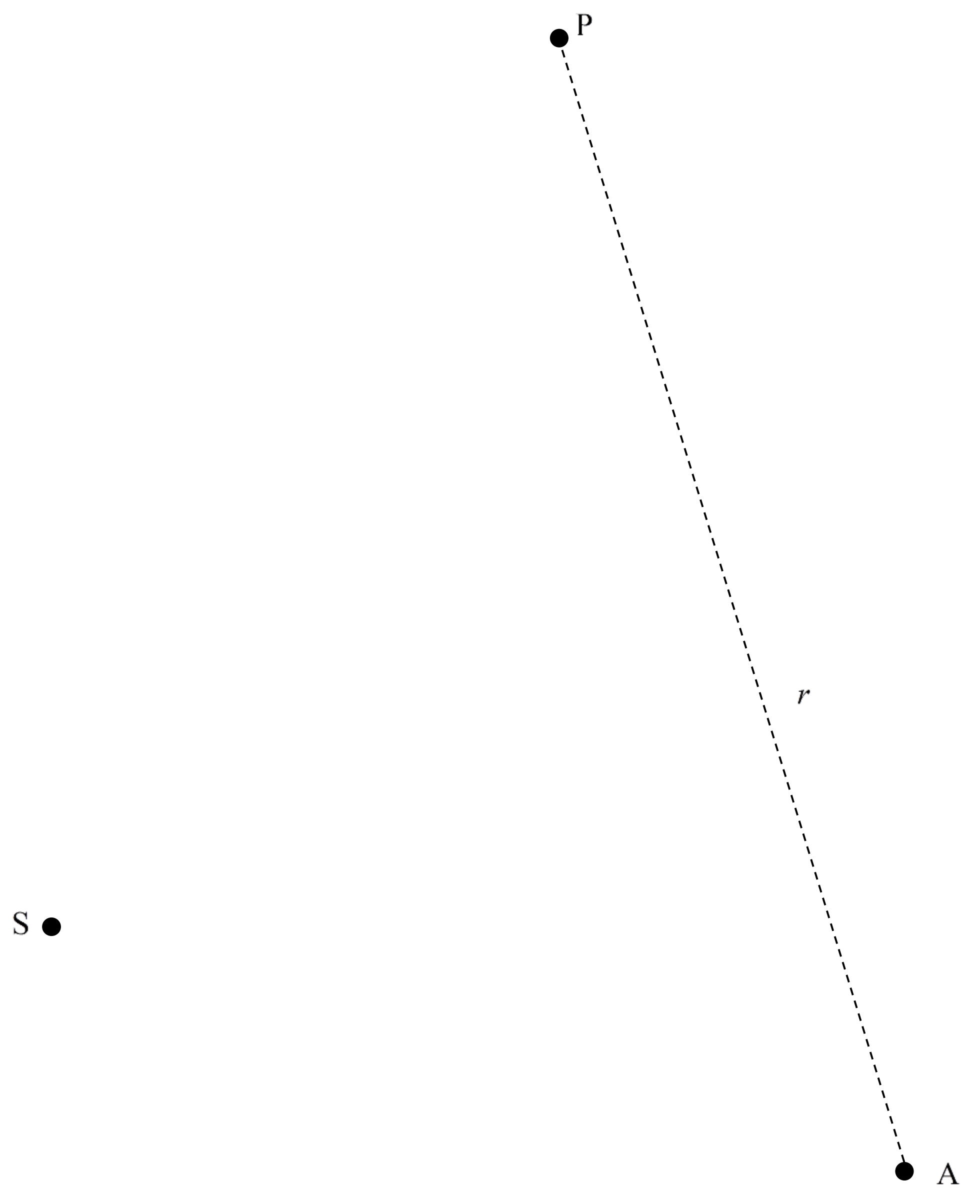
b) the direction of the initial speed of the missile $\vec{v}_0$ in order to be able to hit the asteroid. Indicate on the paper sheet the angle you measured.

c) Calculate the launch velocity of the missile v_0 if you know the mass of the Earth $M = 6 \cdot 10^{24}$ kg and the gravitational constant $K = 6{,}67 \cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

d) Using the plasticine imagine a very simple device in order to calculate the duration of missile flight from launch untill the impact with the asteroid.

e) Using the wire, measure the distance covered by the missile from the launch-point to the impact-point.

A



B. Two solutions . To be sure that the asteroid will be destroyed from another satellite orbiting the Earth at another altitude will be launched in the same moment of time, a second missile in order to hit the asteroid simultaneously with the first missile. Notations P – center of the Earth; S – launch position of the new missile, A – the impact point at distance $r = 30000\,\text{km}$ from the Earth.

In this configuration you know that there are two possible trajectories, form **S** to **A** for the second missile. For that a mysterious point X is marked on the sketch.

You have to *find out*:

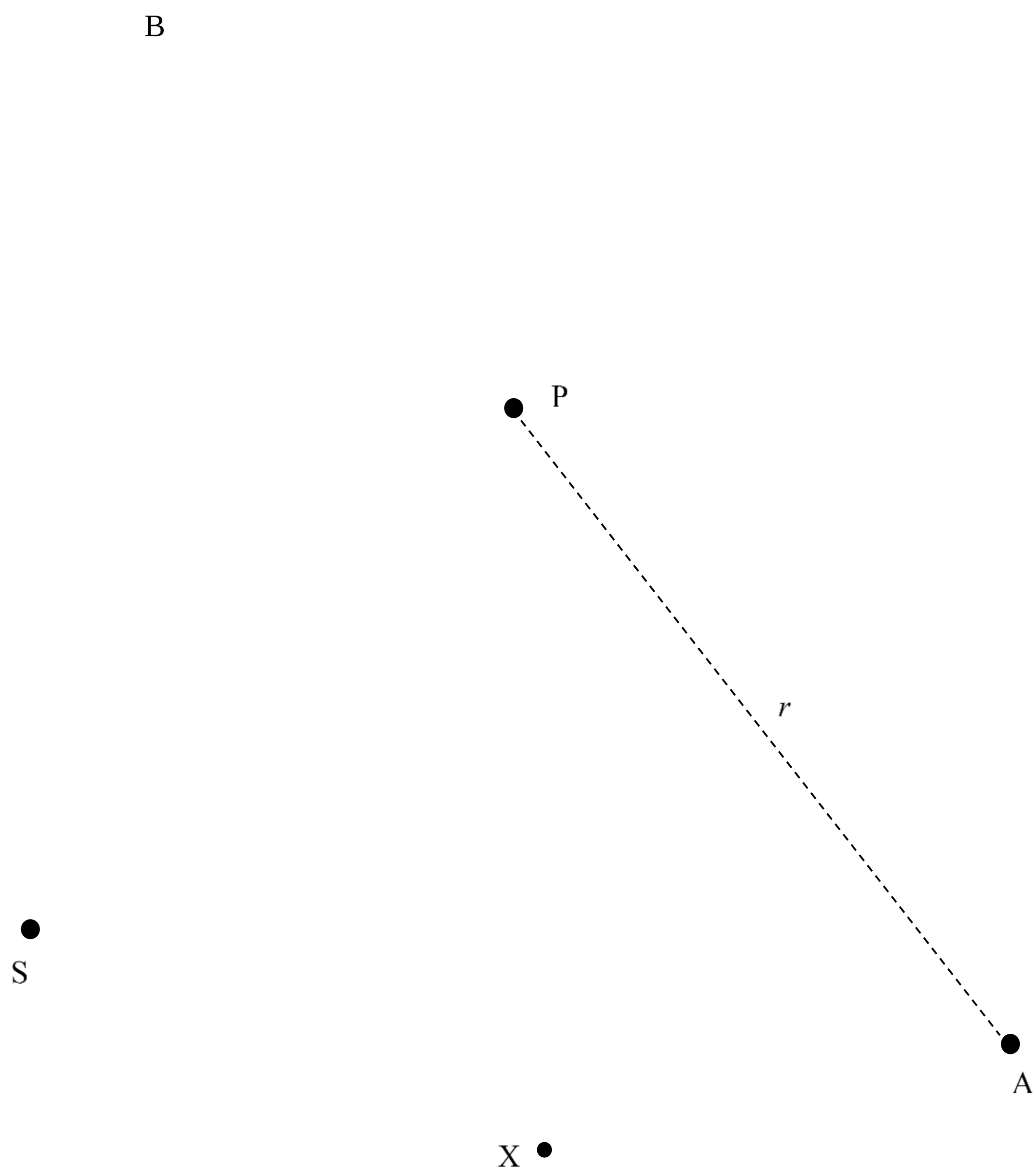
a) the elements of the two possible trajectories.

b) For each trajectory determine the directions of the launch-speeds, at the same velocity which allows the missile to hit the target;

c) The launch- velocity v_0, if you know the maa of the Earth $M = 6 \cdot 10^{24}$ kg and the gravitational constant $G = 6.67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$;

d) Using the plasticine imagine a very simple device in order to calculate the durations of missile flight from launch till the impact with the asteroid for the both trajectories

e) Using the wire, measure the distances covered by the missile from launch-point to the impact-point.


B
P
r
S
A
X

C. Security zone

You have to delimitate around the Earth a security zone. From the point A of this problem you know that there is a unique possible trajectory for the defensive missile. This means that the pointA on the sketch is locate on a curve which delimitates a security yone for the Earth.

a) If you know that the triangle defined by the points **P-** the Earth, **S** – Sun, **A** – impact point remains unchanged in time establish the shape of the curve which delimitates the security zone
b) Draw on the paper this curve and determine its parameters
c) Where have to be situated the impact – point **A** in order to hit the asteroid simultaneously with to missiles
d) Use your plasticine device to fiind out the time spent by an fragment of the asteroid which flies after the impact with the missile, on the curve which delimitates the security zone

IOAA 2015 – Magelang, Indonesia

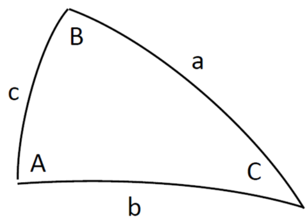
The 9^{th} IOAA was held from 26^{th} July to 4^{th} August 2015. Total 46 teams from 41 countries participated in the event. These included first time teams from Estonia, Georgia, Kyrgyzstan and United Kingdom. In addition, observers from Sweden and Qatar attended the IOAA.

The IOAA 2015 logo concept is based on the mythology of Indonesian cultural wisdom that represents astrological phenomenon in Indonesia. Metaphored as *rambutnya bak mayang terurai*, the movements of comet or meteor and galaxies are represented in this logo. The movements from star-studded and visual styles also reflects modern, futuristic, and fun concept. Implementation of complementary colors represents the spirit of cultural diversity, national unity in sciences and high passion of younger generation through IOAA 2015 event.

Astronomical and Physical Constants

Astronomical unit (AU)	1.4960×10^{11} m
Light year (ly)	9.4605×10^{15} m = 63 240 au
Parsec(pc)	3.0860×10^{16} m = 206 265 au
Jansky (Jy)	10^{-26} W m^{-2} Hz^{-1}
1 Å	10^{-10} m
1 erg	10^{-7} J
1 dyne	10^{-5} N
Mass of Earth	5.9736×10^{24} kg
Mean radius of Earth	6.371×10^{6} m
Equatorial radius of Earth	6.378×10^{6} m
Mass of Moon	7.3490×10^{22} kg
Radius of Moon	1.737×10^{6} m
Mass of Jupiter	1.89813×10^{27} kg
Mean Earth -Moon distance	3.844×10^{8} m
Mass of Sun	1.98892×10^{30} kg
Radius of Sun	6.96×10^{8} m
Effective temperature of the Sun	5780 K
Luminosity of the Sun	3.96×10^{26} J s^{-1}
Solar constant	1366 W m^{-2}
Angular diameter of the Sun	$30'$
Speed of light in vacuum (c)	2.9979×10^{8} m s^{-1}
Gravitational constant (G)	6.6738×10^{-11} N m^{2} kg^{-2}
Boltzmann constant (k)	1.381×10^{-23} m kg s^{-2} K^{-1}
Universal gas constant(R)	8.31 J K^{-1} mol^{-1}
Stefan-Boltzmann constant (σ)	5.6704×10^{-8} kg s^{-3} K^{-4}
Planck constant (h)	6.6261×10^{-34} J s
electron charge (e)	1.602×10^{-19} C
Mass of hydrogen atom	1.67×10^{-27} kg
Current inclination of the ecliptic (ε)	$23°26.3'$
Coordinates of the northern ecliptic pole for epoch 2000.0 (α_{E}, δ_{E})	$18^{\mathrm{h}}00^{\mathrm{m}}00^{\mathrm{s}}$, $+66°33.6'$
Coordinates of the northern galactic pole for epoch 2000.0 (α_{G}, δ_{G})	$12^{\mathrm{h}}51^{\mathrm{m}}$, $+27°08'$

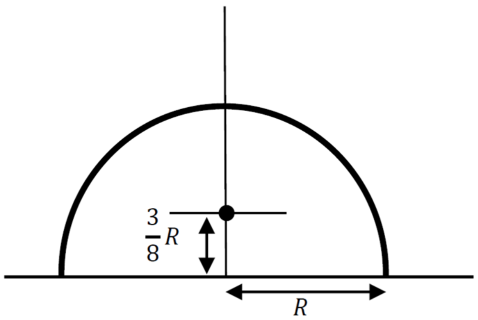
Basic equations of spherical trigonometry



$$\begin{aligned} \sin a \sin B &= \sin b \sin A \\ \sin a \cos B &= \cos b \sin c - \sin b \cos c \cos A \\ \cos a &= \cos b \cos c + \sin b \sin c \cos A \end{aligned}$$

Rayleigh-Jeans formula (long wavelength approximation of Planck's Law):

$$B_\nu(T) = \frac{2kT\nu^2}{c^2}$$



Center of mass of a solid hemisphere of a radius R is a distance of $\frac{3}{8}R$ from the center of the flat side.

Short Problems

1. Several exoplanets have been observed in the Gliese 876 system ($M_G = 0.33 \pm 0.03 M_\odot$) as given in the following table,

Gliese System	Mass	Semi Major Axis (AU)
Gliese 876 b	$2.276 M_J$	0.2083
Gliese 876 c	$0.714 M_J$	0.1296
Gliese 876 d	$6.8 M_\oplus$	0.0208
Gliese 876 e	$15 M_\oplus$	0.334

where $M_\odot$ is mass of Sun, M_J is mass of Jupiter ($M_J = 1.89813 \times 10^{27}$ kg), and $M_\oplus$ is mass of Earth. Assume that all these planets revolve around Gliese 876 in the same direction. Two planets are said to be in resonant orbits if the synodic period of one planet with respect to the other planet is an integer multiple of the orbital period of the second planet. Find if any of the exoplanets of Gliese 876 system may have resonant orbits.

2. One satellite of a planet has an orbital period of 7 days, 3 hours, 43 minutes, and the semi major axis is 15.3 times the mean radius of the planet. The Moon has an orbital period of 27 days, 7 hours, 43 minutes and the semi major axis is 60.3 times the Earth's mean radius. Assume that the mass of the moon and the satellite is negligible compared to the mass of the planet. Calculate the ratio of the planet's mean density to that of the Earth.

3. On 27 May 2015 at 02:18:49, the occultation of the star HIP 89931 (δ–Sgr) by the asteroid 1285 Julietta was observed from Borobudur temple, which was located at the center of the asteroid shadow path. It lasted for only 6.201 s. Assume that Earth's orbit is circular and the orbit of Julietta is on the ecliptic plane and revolves in the same direction as Earth. At the occultation, Julietta is near its aphelion. At the time of the occultation the distances of Julietta from the Sun and the Earth are 3.076 AU and 2.156 AU respectively. Find the approximate diameter of asteroid Julietta, if the semi major axis of Julietta is $a = 2.9914$ AU.

4. Let us assume that an observer using a hypothetical, far-infrared, Earth-sized telescope (wavelength range 20 to 640 μm) found a static and neutral supermassive black hole with a mass of $2.1 \times 10^{10} M_\odot$. Determine the maximum distance at which this black hole can be resolved by the observer.

5. An observer is trying to determine an approximate value of the orbital eccentricity of a man-made satellite. When the satellite was at apogee, it was observed to have moved by $\Delta\theta_1 = 2'44''$ in a short time. When the radius vector connecting Earth and the satellite is perpendicular to the major axis (true anomaly is equal to 90°), within the same duration of time, it was observed to have moved by $\Delta\theta_2 = 21'17''$. Assume that the observer is located at the center of the Earth. Find an approximate value of the eccentricity of the satellite's orbit.

6. At the start of every observation, a radio telescope is pointed at a point-source calibrator that has a known flux density of 21.86 Jy outside the Earth's atmosphere. However, on a certain date, the measured flux density of the calibrator source was 14.27 Jy. If the calibrator source was at an altitude of 35 degrees, estimate the zenith atmospheric optical depth, τ_Z.

7. A galaxy at the boundary of a galaxy cluster of radius 10 Mpc is expected to escape from the cluster if it has an initial velocity of at least 700 km/s relative to the center of the cluster. Calculate the density of the cluster.

8. A strong continuum radio signal from a celestial body has been observed as a burst with a very short duration of 700 μs. The observed flux density at a frequency of 1660 MHz is measured to be 0.35 kJy. If the distance from the source is known to be 2.3 kpc, estimate the brightness temperature of this source.

9. Assume that the Sun is a perfect blackbody. Venus is also assumed to be a blackbody, with temperature T_V, and it is in thermal equilibrium (i.e. it is radiating about as much energy as it receives from the Sun) at its orbital distance of 0.72 AU. Suppose that at closest approach to Earth, Venus has an angular diameter of about 66 arcsec. What is the flux density of Venus at the closest approach to Earth as observed by a radio telescope at an observing frequency of 5 GHz?

10. A molecular hydrogen cloud is known to have a temperature T = 115 K. The hydrogen atoms (assumed spherical) have (covalent) radius $r_H = 0.37 \times 10^{-10}$ m and the separation centre-to-centre distance between the two atoms is $d_{H_2} = 0.74 \times 10^{-10}$ m. Assume that the molecules are in thermal equilibrium. Estimate the frequency at which they will radiate due to molecular rotational excitation.

11. The mass density of an object is inversely proportional to the radial distance from the center of the object with a factor of proportionality $\alpha = 5.0 \times 10^{13}$ kg/m^2. If the escape velocity at the surface of the object is $v_0 = 1.5 \times 10^4$ m/s, calculate the total mass of the object.

12. A proton with a kinetic energy of 1 GeV propagates out from the surface of the Sun towards the Earth. Neglecting the magnetic field of the Sun, calculate the travel time of the proton as seen from the Earth.

13. Volcanic activity on Io, whose rotation period synchronizes with its orbital period, was proposed to be the result of tidal heating mainly from Jupiter. The resultant tidal force on a body is the difference in gravitational force experienced by the near and far sides of that body due to another body. Measurements of the surface distortion of Io via satellite radar altimeter mapping indicate that the surface rises and falls by up to 100 m during one-half orbit. Only the surface layers will move by this amount. Interior layers within Io will move by a smaller amount, and thus we assume that on average the entire mass of Io is moved through 50 m. Assume that Io is considered as two hemispheres each treated as a point mass. Calculate the average power of the tidal heating on Io.

Hint: you can use the following approximation $(1 + x)^n = 1 + nx$ for small x.

The mass of Io is $m_{\mathrm{Io}} = 8.931938 \times 10^{22}$ kg
The perijove distance is $r_{\mathrm{peri}} = 420000$ km
The apojove distance is $r_{\mathrm{apo}} = 423400$ km
The orbital period of Io is 152853 s
The radius of Io is $R_{\mathrm{Io}} = 1821.6$ km.

14. Suppose we live in a static and infinitely large universe where the average density of stars is $n = 10^9$ Mpc^{-3} and the average stellar radius is equal to the solar radius. Assume that standard Euclidean geometry holds true in this universe. How far, on average, could you see in any direction before your line of sight strikes a star? Please write your answer in Mpc.

15. An airplane was flying from Lima, capital of Peru (12°2′S and 77°1′W) to Yogyakarta (7°47′S and 110°26′E), near the venue of the 9$^{\text{th}}$ IOAA. The airplane chooses the shortest flight path from Lima to Yogyakarta. Find the latitude of the southernmost point of the flight path.

Long Problems

1. A moon is orbiting a planet such that the plane of its orbit is perpendicular to the surface of the planet where an observer is standing. After some necessary scaling, suppose the orbit satisfies the following equation:

$$9\left(\frac{x}{2}+\frac{\sqrt{3}\,y}{2}-4\right)^2+25\left(-\frac{\sqrt{3}\,x}{2}+\frac{y}{2}\right)^2=225$$

Consider Cartesian coordinates where x is on the horizontal plane and y is on the zenith of the observer. Let r be the radius of the moon. Assume that the period of rotation of the planet is much larger than the orbital period of the moon. Ignore the atmospheric refraction.

a. Calculate the semimajor and semiminor axis of the ellipse.

b. Calculate the zenith angle of perigee.

c. Determine $\tan\dfrac{\theta}{2}$ where θ is the elevation angle (altitude of the upper tangent of the moon) when the moon looks largest to the observer.

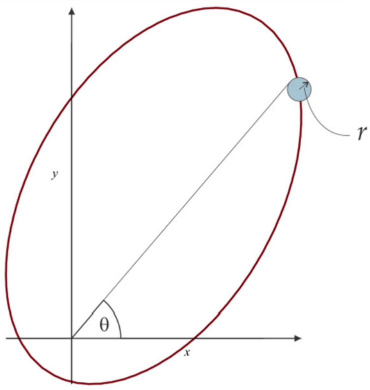


Figure 1

2. Two massive stars A and B with masses m_A and m_B are separated by a distance d. Both stars orbit around their center of mass under gravitational force. Assume their orbits are circular and lie on the X–Y plane whose origin is at the stars' center of mass (see Figure 2)

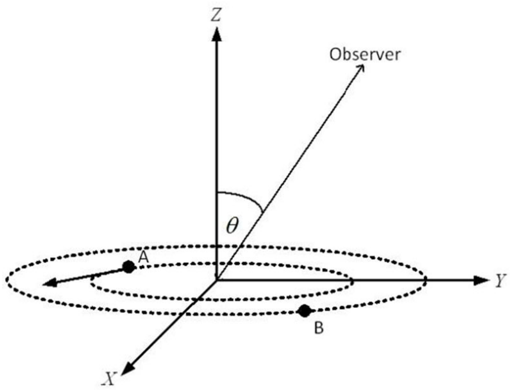


Figure 2

a. Find the expressions for the tangential and angular speeds of star A.

An observer standing on the Y–Z plane (see Figure 2) sees the stars from a large distance with an angle θ relatively to the Z–axis. He measures that the velocity component of star A to his line of sight has the form $K\cos(\omega t+\varepsilon)$, where K and ε are positive.

b. Express $K^3/\omega G$ in terms of m_A, m_B, and θ where G is the universal gravitational constant.

Assume that the observer then identifies that star A has mass equal to $30M_S$ where M_S is the Sun's mass. In addition, he observes that star B produces X-rays and then realizes that it could be a neutron star or a black hole. This conclusion would depend on m_B, i.e.: i) If $m_B < 2M_S$, then B is a neutron star; ii) If $m_B > 2M_S$, then B is a black hole.

c. A measurement by the observer shows that $\dfrac{K^3}{\omega G} = \dfrac{1}{250} M_S$. In practice, the value of θ is usually not known. What is the condition on θ for star B to be a black hole?

3. Suppose a static spherical star consists of N neutral particles with radius R (see Figure 3).

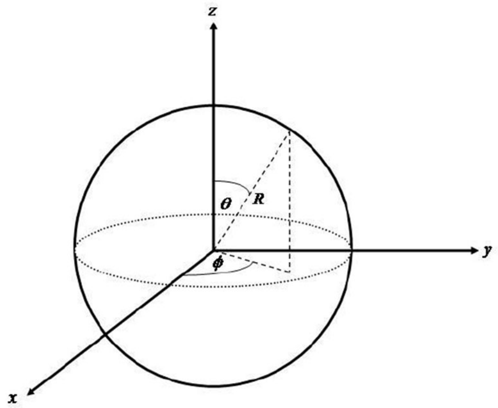


Figure 3

with $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$, satisfying the following equation of states

$$PV = Nk\frac{T_R - T_0}{\ln(T_R/T_0)} \tag{1}$$

where P and V are the pressure inside the star and the volume of the star respectively, k is the Boltzmann constant. T_R and T_0 are the temperatures at the surface $r = R$ and the temperature at the center $r = 0$ respectively. Assume that $T_R \le T_0$.

a. Simplify the stellar equation of state (1) if $\Delta T = T_R - T_0 \approx 0$ (this is called ideal star) (Hint: Use the approximation $\ln(1 + x) \approx x$ for small x)

Suppose the star undergoes a quasi-static process, in which it may slightly contract or expand, such that the above stellar equation of state (1) still holds.

The star satisfies first law of thermodynamics

$$Q = \Delta Mc^2 + W \tag{2}$$

where Q, M, and W are heat, mass of the star, and work respectively, while c is the light speed in the vacuum and $\Delta M = M_{\text{final}} - M_{\text{initial}}$.

In the following we assume T_0 to be constant, while $T_R \equiv T$ varies.

b. Find the heat capacity of the star at constant volume C_v in term of M and at constant pressure C_p expressed in C_v and T (Hint: Use the approximation $(1 + x)^n = 1 + nx$ for small x)

Assuming that C_v is constant and the gas undergo the isobaric process so the star produces the heat and radiates it outside to the space.

c. Find the heat produced by the isobaric process if the initial temperature and the final temperature are T_i and T_f, respectively.

d. (this part is removed)

For the next parts, assume the star is the Sun.

e. If the sunlight is monochromatic with frequency 5×10^{14} Hz, estimate the number of photons radiated by the Sun per second.

f. Calculate the heat capacity C_v of the Sun assuming its surface temperature varies from 5500 K to 6000 K in one second.

Instruction

Data Analysis Round

1. Write your answer on the answer-sheet only. You can use more than one sheet for one problem. Please ask the organizer for the additional answer-sheets.
2. Different problems should be done on different sheets.
3. You can use your own calculator without program and graphic facility.
4. Draw the graphic in the milimeter-block provided.
5. In this exam, you don't need to calculate errors

Data Analysis Problem

Problem 1

Photometry and radial velocity data for the Cepheid type star HV2257 are given in Table 1-3, based on observations by Gieren (MNRAS vol265, 1993). The pulsation period of the star is P = 39.294 days. A reference graph for the temperature–color relation and the bolometric correction tables are given in Figure 1 (Houdashelt et al., 2000) and Table 4 (http://xoomer.virgilio.it/hrtrace/Straizvs. htm). Given that the solar luminosity is $L_{\odot} = 3.96 \times 10^{26}$ J s^{-1} and its bolometric magnitude $M_{\odot bol} = 4.72$. Please do not use period-luminosity relation from the second question for this question.

a. Plot the light curve based on Table 1, between phases 0.6 and 1.
b. Plot the color in Table 2, between phases 0.6 and 1.
c. Plot the Radial velocity curve from Table 3, between phases 0.6 and 1.
d. Calculate the average radial velocity of the star.
e. Calculate the distance to this pulsating star using the observed data and supplementary data given in Table 4 and Figure 1. Assume that there is no extinction in this direction.

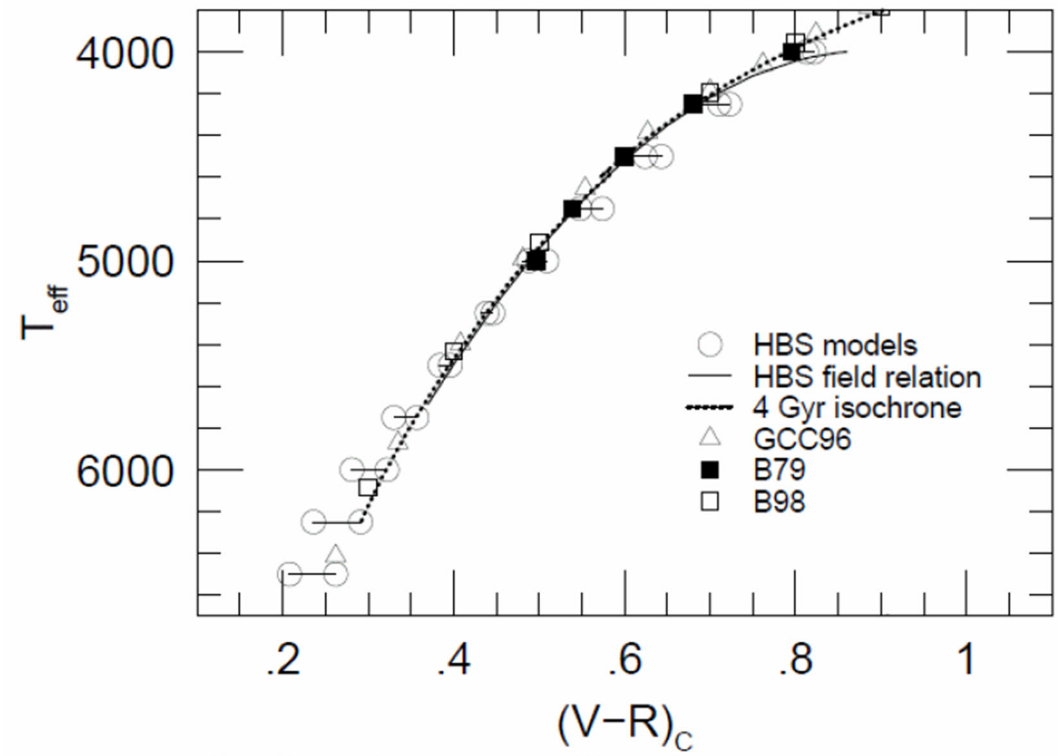


Fig. 1 The V-R color and temperature relation. Different symbols correspond to different authors.

Table 1

Phase	V mag
0.11	12.81
0.13	12.84
0.14	12.87
0.16	12.88
0.19	12.90
0.19	12.94
0.24	12.99
0.43	13.32
0.46	13.31
0.46	13.32
0.51	13.36
0.54	13.41
0.54	13.45
0.56	13.46
0.59	13.53
0.59	13.52
0.61	13.55
0.64	13.60
0.64	13.62
0.72	13.68
0.74	13.61
0.77	13.45
0.79	13.18
0.80	13.12
0.80	13.07
0.82	12.80
0.82	12.78
0.82	12.73
0.84	12.57
0.85	12.54
0.85	12.53
0.87	12.48
0.87	12.47
0.89	12.49
0.90	12.51
0.92	12.51

Table 2

Phase	V – R
0.22	0.71
0.24	0.73
0.25	0.74
0.27	0.75
0.29	0.75
0.29	0.75
0.34	0.77
0.51	0.87
0.53	0.85
0.53	0.87
0.57	0.85
0.60	0.87
0.60	0.88
0.62	0.87
0.64	0.90
0.64	0.90
0.66	0.88
0.68	0.91
0.69	0.90
0.76	0.88
0.78	0.82
0.80	0.79
0.82	0.70
0.82	0.70
0.82	0.68
0.84	0.60
0.84	0.59
0.84	0.58
0.86	0.53
0.86	0.51
0.87	0.52
0.88	0.51
0.89	0.51
0.90	0.55
0.91	0.53
0.93	0.56

Table 3

Phase	RadVel (km/s)
0.03	232
0.05	234
0.08	234
0.08	237
0.13	242
0.13	246
0.18	243
0.20	249
0.23	250
0.28	254
0.33	259
0.35	261
0.36	260
0.38	266
0.40	265
0.44	266
0.46	272
0.46	265
0.49	270
0.51	270
0.54	272
0.54	273
0.56	274
0.59	274
0.61	273
0.62	274
0.64	274
0.67	276
0.67	274
0.69	274
0.71	274
0.72	276
0.74	278
0.77	271
0.77	264
0.79	253
0.80	259
0.82	242
0.85	230
0.87	228
0.90	224
0.92	224
0.92	225
0.95	228
0.96	228

Table 4. Bolometric correction

T_{eff}, K	BC, mag
9600	-0.25
9400	-0.16
9150	-0.10
8900	-0.03
8400	0.05
8000	0.09
7300	0.13
7100	0.11
6500	0.08
6150	0.03
5950	0.00
5800	-0.05
5500	-0.13
5250	-0.22
5050	-0.29
4950	-0.35
4850	-0.42
4700	-0.57
4600	-0.75
4400	-1.17
3900	-1.25
3750	-1.40
3550	-1.60
3400	-2.00

Problem 2

BVRUHKLMN photometry of 2 stars from the constellation Cassiopeia is given in Table 5. For both stars it is believed that their light is affected by extinction by diffuse Interstellar Medium (ISM) only. Assuming that the observation is done from outside the atmosphere.

a) Using the data given in Tables 5 to 9, plot E_{X-V}/E_{B-V} as a function of $1/\lambda_X$ for filters B,V,R, I, J, H, K, L, M, N for both stars. Fit approximate curves by eye (in particular, note that $E_{X-V}/E_{B-V} \sim$ const. as $1/\lambda_X \to 0$). X is each band in the photometric system. E_{B-V} is the colour excess.

b) Using the graphs obtained in a), estimate R_V and R_R for each star.

$R_V = \dfrac{A_V}{E_{B-V}}$ and $R_R = \dfrac{A_R}{E_{R-I}}$

(A_V is the absorption in V).

Now apply these results in order to derive a distance estimate for IC 342, a spiral galaxy in Cassiopeia obscured by Milky Way. You should assume that the properties of the ISM in IC 342 are similar to those of the ISM in our Galaxy.

c) Using the period-magnitude diagrams for 20 Cepheids from IC 342 (Figures 2 and 3) and assuming the period-luminosity relations:

$$\langle M_R \rangle = -2.91\left(\lg\left(\frac{P}{\text{day}}\right) - 1\right) - 4.04 \quad \text{and} \quad \langle M_I \rangle = -3.00\left(\lg\left(\frac{P}{\text{day}}\right) - 1\right) - 4.06$$

where $\langle M_R \rangle$ and $\langle M_I \rangle$ are the mean absolute magnitudes in filters R and I, find A_R for objects in IC 342. Find the distance to IC 342.

Table 5 BVRIJHKLMN photometry of two stars in Cassiopeia

Star	MK class	$\frac{B}{\text{mag}}$	$\frac{V}{\text{mag}}$	$\frac{R}{\text{mag}}$	$\frac{I}{\text{mag}}$	$\frac{J}{\text{mag}}$	$\frac{H}{\text{mag}}$	$\frac{K}{\text{mag}}$	$\frac{L}{\text{mag}}$	$\frac{M}{\text{mag}}$	$\frac{N}{\text{mag}}$
HD 4817	K3Iab	8.08	6.18	4.73	3.64	2.76	1.86	1.54	1.32	1.59	-
HD 11092	K4II	8.66	6.57	-	-	3.10	2.14	1.63	1.41	1.65	1.44

Table 6 $(B-V)_0$ intrinsic colours for selected sp. types and luminosity classes

	$\frac{(B-V)_0}{\text{mag}}$	
	II	Iab / Ia
F0	-	0.15
G0	0.73	0.82
K0	1.06	1.18
K3	1.40	1.42
K4	1.42	1.50

Table 7 Infrared intrinsic colours for selected sp. types of supergiant stars

	$\frac{(V-R)_0}{\text{mag}}$	$\frac{(V-I)_0}{\text{mag}}$	$\frac{(V-J)_0}{\text{mag}}$	$\frac{(V-H)_0}{\text{mag}}$	$\frac{(V-K)_0}{\text{mag}}$	$\frac{(V-L)_0}{\text{mag}}$	$\frac{(V-M)_0}{\text{mag}}$	$\frac{(V-N)_0}{\text{mag}}$
F0	0.20	0.31	0.36	0.51	0.60	0.64	0.65	0.82
G0	0.55	0.90	1.14	1.52	1.71	1.72	1.72	1.98
K0	0.95	1.59	2.01	2.64	2.80	2.87	2.79	3.14
K3	1.13	1.96	2.41	3.14	3.25	3.39	3.25	3.63
K4	1.20	2.13	2.59	3.37	3.44	3.62	3.46	3.84

Table 8 Infrared intrinsic colours for selected sp. types of giant stars

	$\frac{(V-R)_0}{\text{mag}}$	$\frac{(V-I)_0}{\text{mag}}$	$\frac{(V-J)_0}{\text{mag}}$	$\frac{(V-H)_0}{\text{mag}}$	$\frac{(V-K)_0}{\text{mag}}$	$\frac{(V-L)_0}{\text{mag}}$	$\frac{(V-M)_0}{\text{mag}}$	$\frac{(V-N)_0}{\text{mag}}$
K0	0.60	1.03	1.23	1.72	1.94	1.97	1.90	1.92
K3	0.86	1.39	1.84	2.40	2.69	2.82	2.70	2.73
K4	0.96	1.61	2.16	2.77	3.05	3.22	3.08	3.02

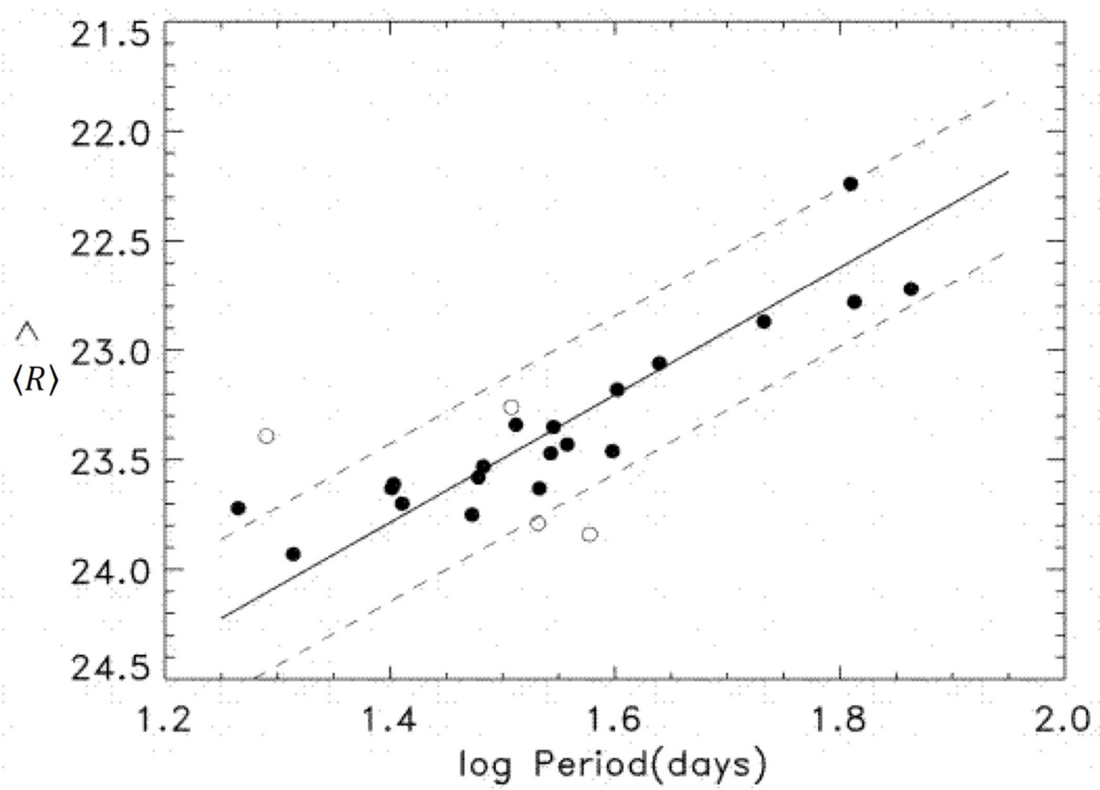


Fig. 2 $\langle R \rangle$ is the mean apparent magnitude in filter R

Table 9 Effective wavelengths of selected photometric filters

Filter	B	V	R	I	J	H	K	L	M	N
λ_F/nm	450	555	670	870	1200	1620	2200	3500	5000	9000

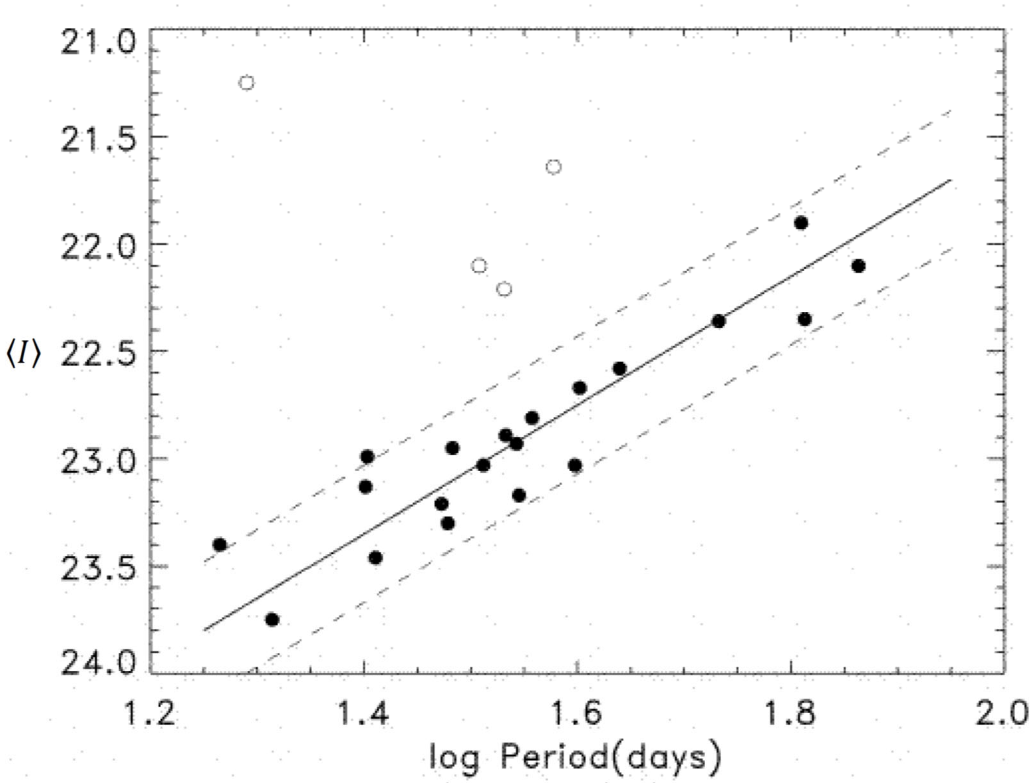


Fig. 3 $\langle I \rangle$ is the mean apparent magnitude in filter I

1. During observation test using smal1 telescope, you will receive:

- Problem sheet
- Answer sheet
- Additional sheet
- Flashlight

2. During naked eye test, you will receive:

- Problem sheet
- Answer sheet
- Additional sheet
- Flashlight

3. During observation test using main telescope, you will receive:

- Problem sheet
- Answer sheet
- Additional sheet
- Headlamp
- Clip Board

4. During observation test using planetarium (in case of rainy or cloudy), you will receive:

- Problem sheet
- Answer sheet
- Additional sheet
- Flashlight
- Clip Board

5. Please write your student ID in every answer sheet:

6. The total duration for naked eye + small telescope observation is 60 minutes, main telescope 20 minutes, planetarium 15 minutes

OBSERVATIONAL QUESTIONS

Naked Eye

1. In the blank square given in the answer sheet, you should assume that Nova Sagittarii 2 is in the center. (marked by green star)
   RA(J2000): 18h 36m 56.84s
   Dec(J2000): $-28^\circ\ 55'\ 39.8''$
   Draw Declination and Right Ascension lines passing through Nova Sagittari. Also draw parallel gridlines at convenient interval and label the accordingly. (The field of view is 45°)(10 points)

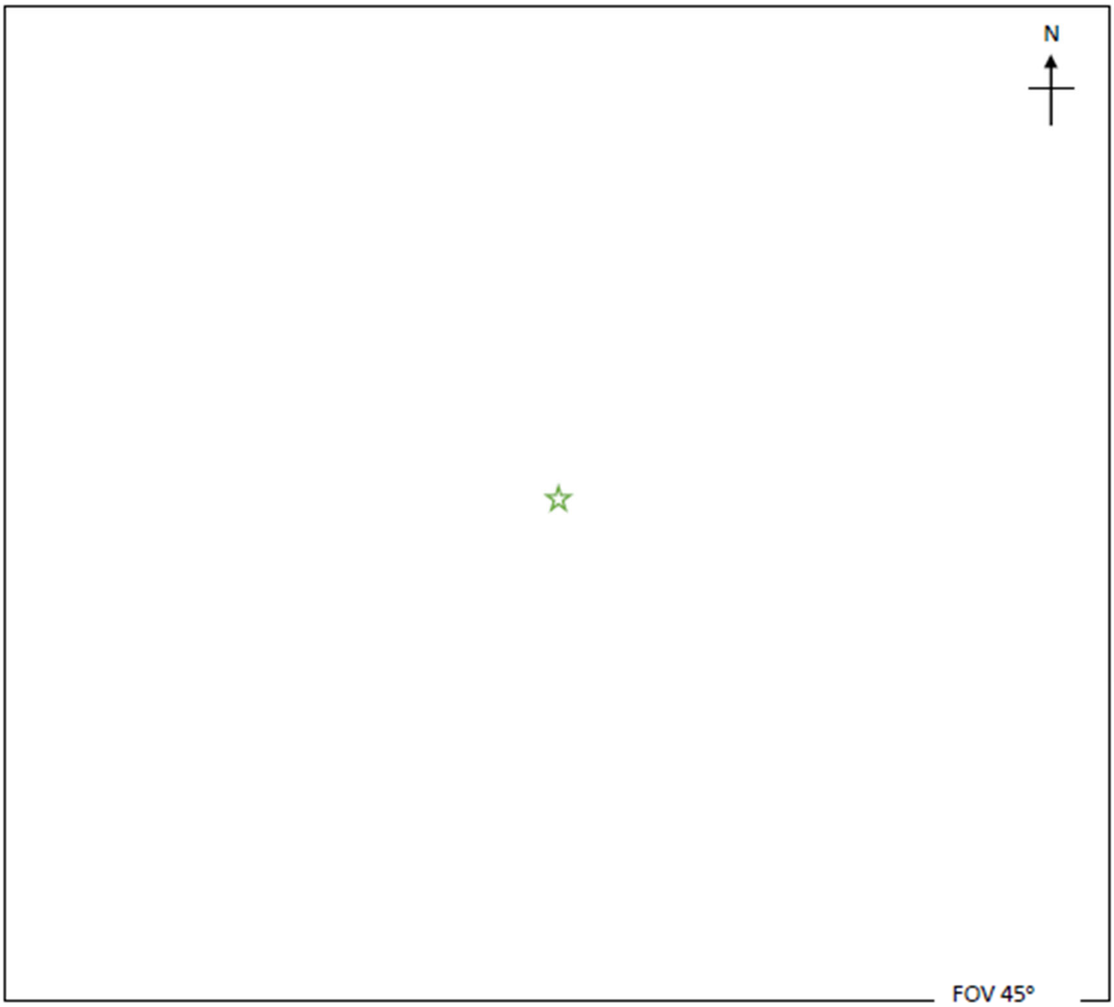
2. Mark with filled circles, the five brightest stars in the field of view on the answer sheet along with their names. You can add several other stars for your orientation. (25 points)

3. Draw an unfilled circle representing the Moon's position tomorrow night (on **29 July 2015** at about the same time with the observation time). (10 points)

4. Mark the position of any five Messier objects in the field of view by cross (x), and write down their names beside their marks. (25 points)

5. Draw a line representing the galactic plane. (10 points)

6. Mark the positions of Nunki (σSgr) and Shaula (λSco) by star (*). Draw a line connecting the two stars. Estimate the angular distance between these stars and write it on the line connecting both stars. (20 points)


N
FOV 45°

OBSERVATIONAL QUESTIONS

Small Telescope

In this round you are going to observe the Moon using the telescope you have already made. Set up your telescope on the table provided for you. You have been given a map of the Moon. Please answer these questions on the given map.

1. sketch the terminator of the Moon. (25 points)
2. Write down the celestial cardinal directions on the map (N, S, E, W) (25 points)
3. Mark the position of the south pole of the Moon with a cross (x). (25 points)
4. Mark the position of the famous Tycho crater with a circle (the size of the circle should be comparable to the size of the crater). (25 points)

OBSERVATIONAL QUESTIONS

Small Telescope

In this round you are going to observe the Moon using the telescope you made. Set up your telescope in the table provided for you. Beside this question paper, you must get a map of the moon and an answer sheet. Please write or draw the answer of these questions in the answer sheet.

1. Point your telescope to the moon, sketch termination line of the moon on the chart given. (30 points)
2. Write down the cardinal directions in the chart (N, S, E, W) (30 points)
3. Mark the position of the south pole of the moon. (20 points)
4. Mark the position of the famous Tycho crater. (20 points)

WORKING INSTRUCTIONS

1. Participants get 4 sheets of paper containing three questions (1l sheet) and 3 pieces of answer sheets.

2. Participants are allowed to work on the problems when problems appear.

3. After the time is up, participants have to put the answers and question sheets back to the envelope.

WORK RULES

1. Participants are prohibited from communicating with other participants when work on the problems.

2. Participants have to stop for doing anything after the time has run out

3. Participants are forbidden to leave the seat when work on the problems, but allowed to look around.

4. Participants can't leave the planetarium mini before the time's up

PLANETARIUM QUESTIONS

(Total Duration: 15 minutes)

Please pay attention to the North direction marked in the answer sheets.

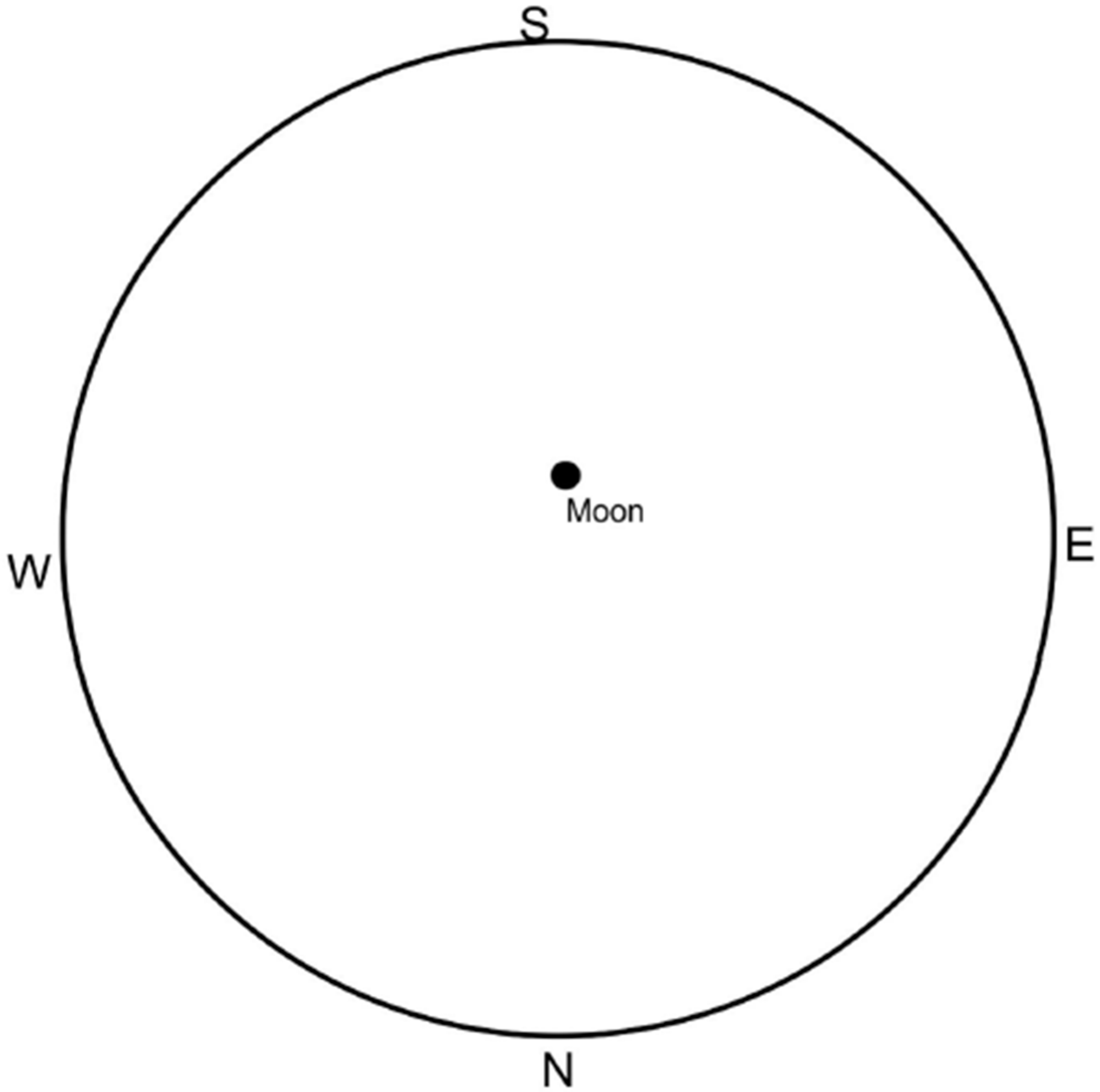
1. The projected image in the planetarium is a simulated image of the Magelang sky for **July 28, 2015** at **21.00 (UTC+7)**. There are three stars missing in the image. Please mark the position of the three missing bright stars in Answer Sheet 1 by a cross (x) and write their names beside the cross marks using their IAU designations. For guidance you can also include a few stars that you see in the planetarium and mark them with circles (o). The list of the bright stars which possibly missing are attached. (Duration: 6 minutes) (60 points)

2. Assume that a bright supernova appears in the sky on **September 24, 2015** and you watch it at **22.00 (UTC+7)**. The planetarium will now show the sky for this date with the supernova. Mark the position of the supernova by a cross(x) in Answer Sheet 2. You can mark other stars by circle (o) for guidance. (Duration: 3 minutes) (50 points)

3. The planetarium will now display the Magelang sky for **January 3, 2016**, at **23.00 (UTC+7)**. The examiner will point to a star with an arrow. Estimate the times at which this star will rise and set from Magelang on this date up to 20 minutes accuracy. (Duration: 6 minutes) (40 points)

ANSWER SHEET 1

Answer of question 1:

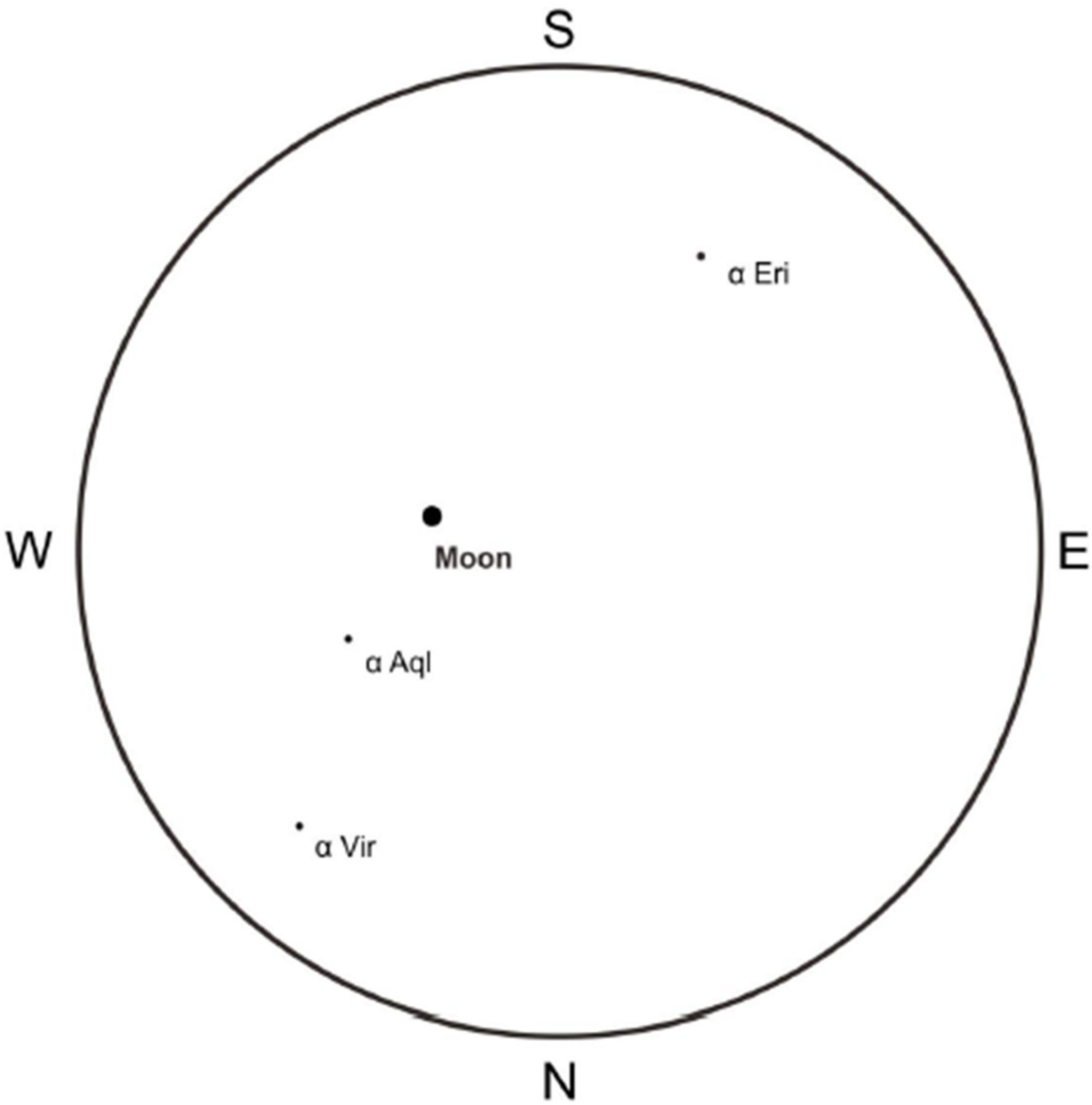


Note: the circumference indicates the horizon.

α Pav	α Lyr	α Aql
α Vir	α CrB	α Gru
α Ser	α Cen	α Cyg
α Oph	α Cru	α Boo
α PsA	α Sco	β Aqr
β Cru	β Oph	β Peg
γ Sco	γ Cyg	δ Pav
ε Peg	ε Boo	

ANSWER SHEET 2

Answer of question 2



Answer of Question 3:

Rise time:

Set time:

Observational Problem

Main Telescope (Total Duration: 20 minutes)

1. Point your telescope to NGC 6475 (M7)
   RA (J2000): 17h 53m 54s
   Dec (J2000): –34°49′
   Show the object to your telescope assistant. If you fail to find the object in the first 10minutes, ask the assistant to help, but you will lose the mark. (23 points)

2. Write down the cardinal directions (North, South, East and West) on Chart 1 in the answer sheet. (10 points)

3. Three stars of M7 are missing from the Chart 1. Mark the position of the missing stars with crosses (x). (21 points)

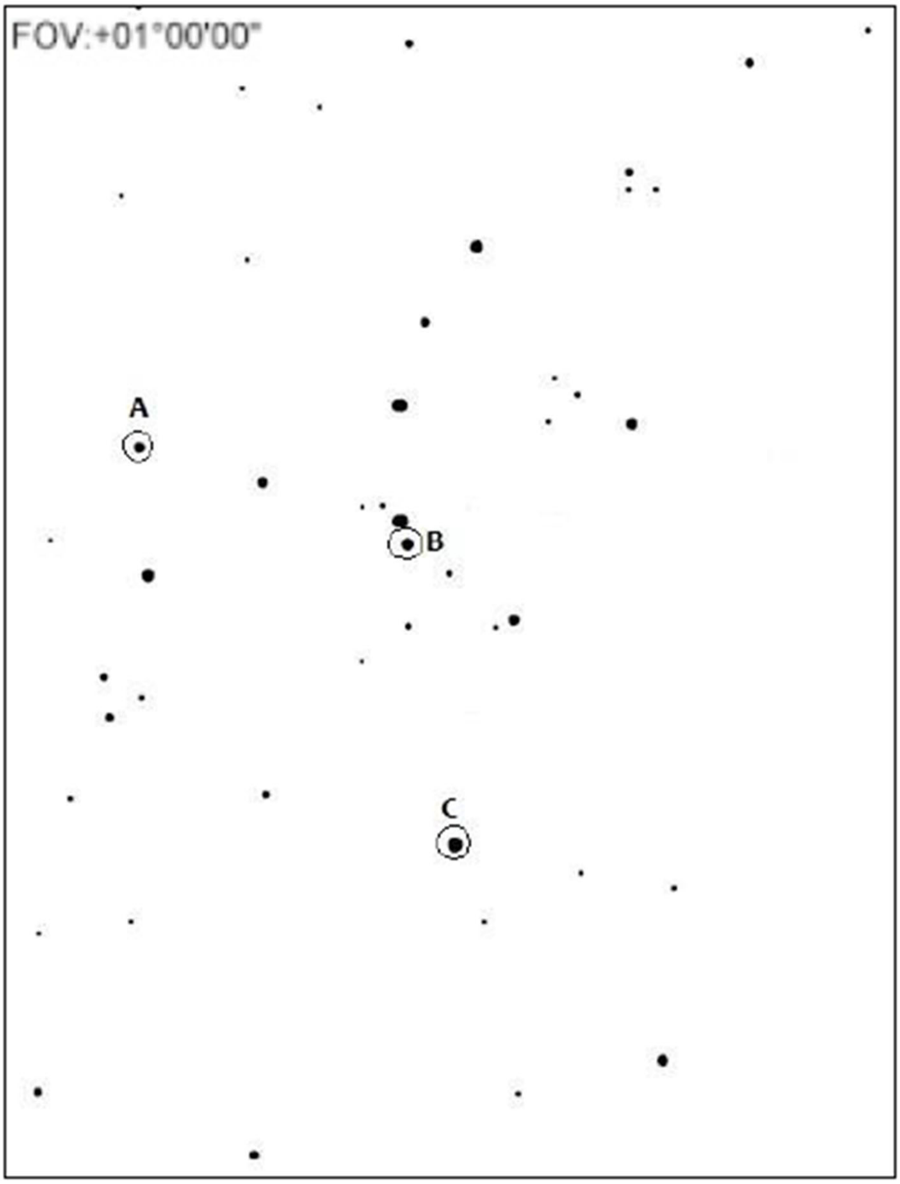
4. The apparent magnitudes of comparison stars A, B and c are 7.6, 7.2, and 5.6 respectively. Estimate and write down the apparent magnitudes of the missing stars beside respective cross marks. (21 points)

5. Estimate the FOV of the instrument, using the given stopwatch. show your calculation in the answer sheet. (25 points)

Main Telescope Answer Sheet

Chart 1 (M7)

Chart size: 1° × 1.2°



TEAM COMPETITION PROBLEM

The Borobudur Temple, built in the 9th century, is the biggest Buddhist temple in the world built of stone. It is considered to be one of the wonders of the ancient world. It has nine stacked platforms. The first six platforms have a square form and beautifully decorated with numerous reliefs. The upper three platforms are circular, with 72 small bell-shaped stupas, surrounding one large central stupa (main stupa). The altitude of the ground close to the entrance is 265 m above mean sea level. Note that the height of the temple is 35 m (measured from the top of the main stupa to the ground).

We believe that the constructors of this temple had quite a good knowledge of astronomy. In search of the possible existence of the astronomical alignment between Borobudur and any celestial bodies, we will make some measurements. observers are distributed around the temple, mainly on the eastern and western sides. The observing station number card (placed on the table) specifies the exact latitude and longitude for that station. Example coordinates for some stations are given in the table below.

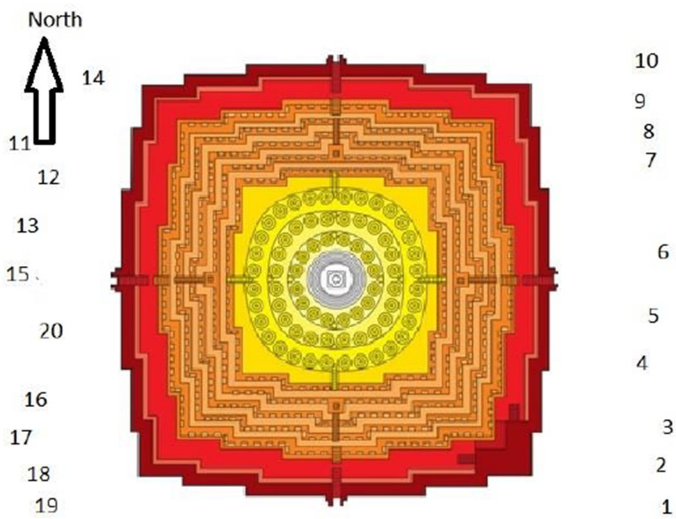


Table 1: Example Coordinates of observer's positions

Position	longitude	latitude	Altitude (m)
1	$110°12'16.52''$	$-7°36'30.10''$	264
2	$110°12'16.69''$	$-7°36'29.80''$	264
3	$110°12'16.82''$	$-7°36'29.62''$	264
4	$110°12'16.65''$	$-7°36'28.85''$	265
5	$110°12'16.54''$	$-7°36'28.26''$	264

From your position, if you observe the sky in the direction of the top of the main stupa (not of the top of the lightning rod), determine:

a. If you observed at 18:00 **(UTC+7)**, what constellation would align with the main stupa?

b. Draw the constellations you expect to see from your position in the direction of the stupa at 18:00 and 19:00 **(UTC+7)**.

Note: assume compass points to the true north

You are provided with:
- Planisphere
- Magnetic compass
- Protractor, string, and a weight

Describe your answer thoroughly.

IOAA 2016 – Bhubneswar, India

The 10^{th} IOAA was held from 09^{th} December to 19^{th} December 2016. Total 48 teams from 41 countries participated in the event.

The IOAA 2016 logo is designed to bring out India's rich cultural heritage as well it's commitment to modern science. The radio antenna in place of letter "I" represents Giant Metrewave Radio Telescope (G.M.R.T.), which is located in western India, 90km from the city of Pune. GMRT is the world's largest telescope at metre wavelengths. It consists of 30 dishes, spread in a circle of 17 km in radius, with an aperture of 45m each. The spoked wheel in place of letter "O" represents the famous Sun temple of Konark. This temple is a world heritage site located about 50km away from Bhubaneswar and is built in shape of a 24 wheeled chariot of the Sun.

(T1) **True or False**

Determine if each of the following statements is True or False. In the Summary Answersheet, tick the correct answer (TRUE / FALSE) for each statement. No justifications are necessary for this question.

(T1.1) In a photograph of the clear sky on a Full Moon night with a sufficiently long exposure, the colour of the sky would appear blue as in daytime. **2**

(T1.2) An astronomer at Bhubaneswar marks the position of the Sun on the sky at 05: 00 UT every day of the year. If the Earth's axis were perpendicular to its orbital plane, these positions would trace an arc of a great circle. **2**

(T1.3) If the orbital period of a certain minor body around the Sun in the ecliptic plane is less than the orbital period of Uranus, then its orbit must necessarily be fully inside the orbit of Uranus. **2**

(T1.4) The centre of mass of the solar system is inside the Sun at all times. **2**

(T1.5) A photon is moving in free space. As the Universe expands, its momentum decreases. **2**

(T2) **Gases on Titan** **10**

Gas particles in a planetary atmosphere have a wide distribution of speeds. If the r.m.s. (root mean square) thermal speed of particles of a particular gas exceeds $1/6$ of the escape speed, then most of that gas will escape from the planet. What is the minimum atomic weight (relative atomic mass), $A_{\rm min}$, of an ideal monatomic gas so that it remains in the atmosphere of Titan?

Given, mass of Titan $M_{\rm T} = 1.23 \times 10^{23}$ kg, radius of Titan $R_{\rm T} = 2575$ km, surface temperature of Titan $T_{\rm T} = 93.7$ K.

(T3) **Early Universe**

Cosmological models indicate that radiation energy density, $\rho_{\rm r}$, in the Universe is proportional to $(1+z)^4$, and the matter energy density, $\rho_{\rm m}$, is proportional to $(1+z)^3$, where z is the redshift. The dimensionless density parameter, Ω, is given as $\Omega = \rho/\rho_{\rm c}$, where $\rho_{\rm c}$ is the critical energy density of the Universe. In the present Universe, the density parameters corresponding to radiation and matter, are $\Omega_{\rm r_0} = 10^{-4}$ and $\Omega_{\rm m_0} = 0.3$, respectively.

(T3.1) Calculate the redshift, $z_{\rm e}$, at which radiation and matter energy densities were equal. **3**

(T3.2) Assuming that the radiation from the early Universe has a blackbody spectrum with a temperature of 2.732 K, estimate the temperature, $T_{\rm e}$, of the radiation at redshift $z_{\rm e}$. **4**

(T3.3) Estimate the typical photon energy, E_ν (in eV), of the radiation as emitted at redshift $z_{\rm e}$. **3**

(T4) **Shadows** **10**

An observer in the northern hemisphere noticed that the length of the shortest shadow of a 1.000 m vertical stick on a day was 1.732 m. On the same day, the length of the longest shadow of the same vertical stick was measured to be 5.671 m.

Find the latitude, ϕ, of the observer and declination of the Sun, $\delta_\odot$, on that day. Assume the Sun to be a point source and ignore atmospheric refraction.

(T5) **GMRT beam transit** **10**

Giant Metrewave Radio Telescope (GMRT), one of the world's largest radio telescopes at metre wavelengths, is located in western India (latitude: $19°6'$ N, longitude: $74°3'$ E). GMRT consists of 30 dish antennas, each with a diameter of 45.0 m. A single dish of GMRT was held fixed with its axis pointing at a zenith angle of $39°42'$ along the northern meridian such that a radio point source would pass along a diameter of the beam, when it is transiting the meridian.

What is the duration $T_{\rm transit}$ for which this source would be within the FWHM (full width at half maximum) of the beam of a single GMRT dish observing at 200 MHz?

Hint: The FWHM size of the beam of a radio dish operating at a given frequency corresponds to the angular resolution of the dish. Assume uniform illumination.

(T6) **Cepheid Pulsations**

The star β-Doradus is a Cepheid variable star with a pulsation period of 9.84 days. We make a simplifying assumption that the star is brightest when it is most contracted (radius being R_1) and it is faintest when it is most expanded (radius being R_2). For simplicity, assume that the star maintains its spherical shape and behaves as a perfect black body at every instant during the entire cycle. The bolometric magnitude of the star varies from 3.46 to 4.08. From Doppler measurements, we know that during pulsation the stellar surface expands or contracts at an average radial speed of 12.8 km s^{-1}. Over the period of pulsation, the peak of thermal radiation (intrinsic) of the star varies from 531.0 nm to 649.1 nm.

(T6.1) Find the ratio of radii of the star in its most contracted and most expanded states (R_1/R_2). **7**

(T6.2) Find the radii of the star (in metres) in its most contracted and most expanded states (R_1 and R_2). **3**

(T6.3) Calculate the flux of the star, F_2, when it is in its most expanded state. **5**

(T6.4) Find the distance to the star, D_{star}, in parsecs. **5**

(T7) **Telescope optics**

In a particular ideal refracting telescope of focal ratio $f/5$, the focal length of the objective lens is 100 cm and that of the eyepiece is 1 cm.

(T7.1) What is the angular magnification, m_0, of the telescope? What is the length of the telescope, L_0, i.e. the distance between its objective and eyepiece? **4**

An introduction of a concave lens (Barlow lens) between the objective lens and the prime focus is a common way to increase the magnification without a large increase in the length of the telescope. A Barlow lens of focal length 1 cm is now introduced between the objective and the eyepiece to double the magnification.

(T7.2) At what distance, d_{B}, from the prime focus must the Barlow lens be kept in order to obtain this desired double magnification? **6**

(T7.3) What is the increase, ΔL, in the length of the telescope? **4**

A telescope is now constructed with the same objective lens and a CCD detector placed at the prime focus (without any Barlow lens or eyepiece). The size of each pixel of the CCD detector is 10 μm.

(T7.4) What will be the distance in pixels between the centroids of the images of the two stars , n_{p}, on the CCD, if they are $20''$ apart on the sky? **6**

(T8) **U-Band photometry**

A star has an apparent magnitude $m_{\text{U}} = 15.0$ in the *U*-band. The *U*-band filter is ideal, i.e., it has perfect (100%) transmission within the band and is completely opaque (0% transmission) outside the band. The filter is centered at 360 nm, and has a width of 80 nm. It is assumed that the star also has a flat energy spectrum with respect to frequency. The conversion between magnitude, m, in any band and flux density, f, of a star in Jansky (1 Jy $= 1 \times 10^{-26}$ W Hz^{-1} m^{-2}) is given by

$$f = 3631 \times 10^{-0.4m} \text{Jy}$$

(T8.1) Approximately how many *U*-band photons, N_0, from this star will be incident normally on a $1\ \text{m}^2$ area at the top of the Earth's atmosphere every second? **8**

This star is being observed in the *U*-band using a ground based telescope, whose primary mirror has a diameter of 2.0 m. Atmospheric extinction in *U*-band during the observation is 50%. You may assume that the seeing is diffraction limited. Average surface brightness of night sky in *U*-band was measured to be 22.0 mag/arcsec^2.

(T8.2) What is the ratio, R, of number of photons received per second from the star to that received from the sky, when measured over a circular aperture of diameter $2''$? **8**

(T8.3) In practice, only 20% of U-band photons falling on the primary mirror are detected. How many photons, N_{t}, from the star are detected per second? **4**

(T9) **Mars Orbiter Mission**

India's Mars Orbiter Mission (MOM) was launched using the Polar Satellite Launch Vehicle (PSLV) on 5 November 2013. The dry mass of MOM (body + instruments) was 500 kg and it carried fuel of mass 852 kg. It was initially placed in an elliptical orbit around the Earth with perigee at a height of 264.1 km and apogee at a height of 23903.6 km, above the surface of the Earth. After raising the orbit six times, MOM was transferred to a trans-Mars injection orbit (Hohmann orbit).

The first such orbit-raising was performed by firing the engines for a very short time near the perigee. The engines were fired to change the orbit without changing the plane of the orbit and without changing its perigee. This gave a net impulse of 1.73×10^5 kg m s^{-1} to the satellite. Ignore the change in mass due to burning of fuel.

(T9.1) What is the height of the new apogee, $h_{\rm a}$ above the surface of the Earth, after this engine burn? **14**

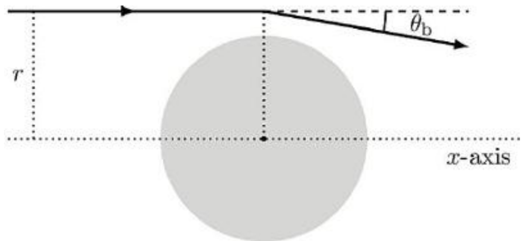
(T9.2) Find the eccentricity (e) of the new orbit after the burn and the new orbital period (P) of MOM in hours. **6**

(T10) **Gravitational Lensing Telescope**

Einstein's General Theory of Relativity predicts bending of light around massive bodies. For simplicity, we assume that the bending of light happens at a single point for each light ray, as shown in the figure. The angle of bending, $\theta_{\rm b}$, is given by

$$\theta_{\rm b} = \frac{2R_{\rm sch}}{r}$$

where $R_{\rm sch}$ is the Schwarzschild radius associated with that gravitational body. We call r, the distance of the incoming light ray from the parallel x-axis passing through the centre of the body, as the "impact parameter".



A massive body thus behaves somewhat like a focusing lens. The light rays coming from infinite distance beyond a massive body, and having the same impact parameter r, converge at a point along the axis, at a distance f_r from the centre of the massive body. An observer at that point will benefit from huge amplification due to this gravitational focusing. The massive body in this case is being used as a Gravitational Lensing Telescope for amplification of distant signals.

(T10.1) Consider the possibility of our Sun as a gravitational lensing telescope. Calculate the shortest distance, $f_{\rm min}$, from the centre of the Sun (in A.U.) at which the light rays can get focused. **6**

(T10.2) Consider a small circular detector of radius a, kept at a distance $f_{\rm min}$ centered on the x-axis and perpendicular to it. Note that only the light rays which pass within a certain annulus (ring) of width h (where $h \ll R_\odot$) around the Sun would encounter the detector. The amplification factor at the detector is defined as the ratio of the intensity of the light incident on the detector in the presence of the Sun and the intensity in the absence of the Sun. **8**

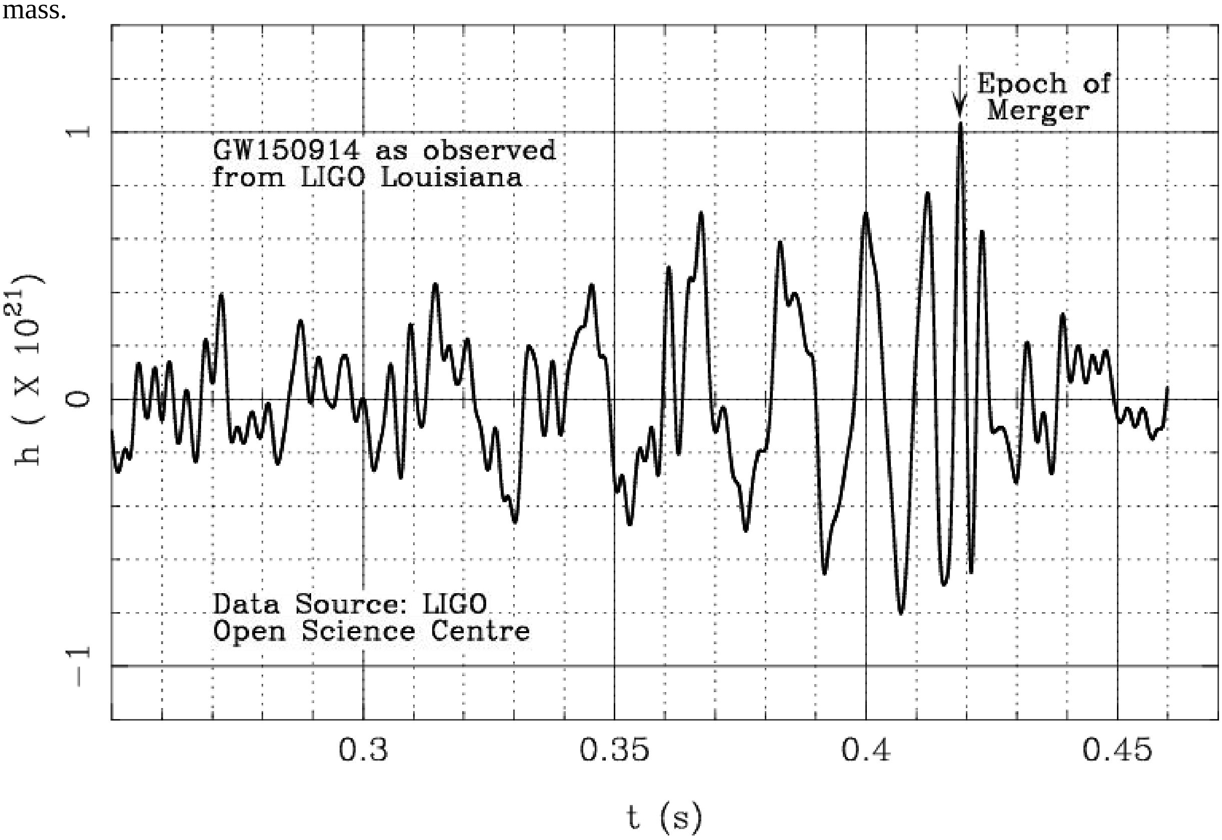
Express the amplification factor, $A_{\rm m}$, at the detector in terms of $R_\odot$ and a.

(T10.3) Consider a spherical mass distribution, such as dark matter in a galaxy cluster, through which light rays can pass while undergoing gravitational bending. Assume for simplicity that for the gravitational bending with impact parameter, r, only the mass $M(r)$ enclosed inside the radius r is relevant. **6**

What should be the mass distribution, $M(r)$, such that the gravitational lens behaves like an ideal optical convex lens?

(T11) **Gravitational Waves**

The first signal of gravitational waves was observed by two advanced LIGO detectors at Hanford and Livingston, USA in September 2015. One of these measurements (strain vs time in seconds) is shown in the accompanying figure. In this problem, we will interpret this signal in terms of a small test mass m orbiting around a large mass M (i.e., $m \ll M$), by considering several models for the nature of the central mass.



The test mass loses energy due to the emission of gravitational waves. As a result the orbit keeps on shrinking, until the test mass reaches the surface of the object, or in the case of a black hole, the innermost stable circular orbit – ISCO – which is given by $R_{\text{ISCO}} = 3R_{\text{sch}}$, where R_{sch} is the Schwarzschild radius of the black hole. This is the "epoch of merger". At this point, the amplitude of the gravitational wave is maximum, and so is its frequency, which is always twice the orbital frequency. In this problem, we will only focus on the gravitational waves before the merger, when Kepler's laws are assumed to be valid. After the merger, the form of gravitational waves will drastically change.

(T11.1) Consider the observed gravitational waves shown in the figure above. Estimate the time period, T_0, and hence calculate the frequency, f_0, of gravitational waves just before the epoch of merger. **3**

(T11.2) For any main sequence (MS) star, the radius of the star, R_{MS}, and its mass, M_{MS}, are related by a power law given as, **10**

$$R_{\text{MS}} \propto (M_{\text{MS}})^{\alpha}$$

$$\text{where } \alpha = 0.8 \quad \text{for } M_{\odot} < M_{\text{MS}}$$

$$= 1.0 \quad \text{for } 0.08M_{\odot} \le M_{\text{MS}} \le M_{\odot}$$

If the central object were a main sequence star, write an expression for the maximum frequency of gravitational waves, f_{MS}, in terms of mass of the star in units of solar masses ($M_{\text{MS}}/M_{\odot}$) and α.

(T11.3) Using the above result, determine the appropriate value of α that will give the maximum possible frequency of gravitational waves, $f_{\text{MS,max}}$ for any main sequence star. Evaluate this frequency. **9**

(T11.4) White dwarf (WD) stars have a maximum mass of 1.44 $M_{\odot}$ (known as the Chandrasekhar limit) and obey the mass-radius relation $R \propto M^{-1/3}$. The radius of a solar mass white dwarf is equal **8** to 6000 km. Find the highest frequency of emitted gravitational waves, $f_{\mathrm{WD,max}}$, if the test mass is orbiting a white dwarf.

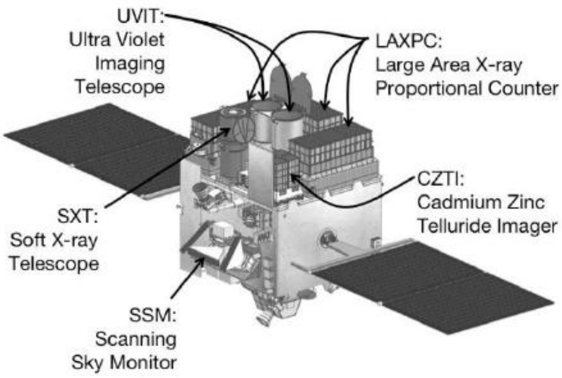
(T11.5) Neutron stars (NS) are a peculiar type of compact objects which have masses between 1 and $3M_{\odot}$ and radii in the range $10-15$ km. Find the range of frequencies of emitted gravitational waves, $f_{\mathrm{NS,min}}$ and $f_{\mathrm{NS,max}}$, if the test mass is orbiting a neutron star at a distance close to the neutron star radius. **8**

(T11.6) If the test mass is orbiting a black hole (BH), write the expression for the frequency of emitted gravitational waves, f_{BH}, in terms of mass of the black hole, M_{BH}, and the solar mass $M_{\odot}$. **7**

(T11.7) Based only on the time period (or frequency) of gravitational waves before the epoch of merger, determine whether the central object can be a main sequence star (MS), a white dwarf (WD), a neutron star (NS), or a black hole (BH). Tick the correct option in the Summary Answersheet. Estimate the mass of this object, M_{obj}, in units of $M_{\odot}$. **5**

(T12) **AstroSat (discarded)**

India astronomy satellite, AstroSat, launched in September 2015, has five different instruments.



In this question, we will discuss three of these instruments (SXT, LAXPC, CZTI), which point in the same direction and observe in X-ray wavelengths. The details of these instruments are given in the table below.

Instrument	Band [keV]	Collecting Area [m^2]	Effective Photon Detection Efficiency	Saturation level [counts]	No. of Pixels
SXT	0.3 – 80	0.067	60%	15000 (total)	512 x 512
LAXPC	3 – 80	1.5	40%	50000 (in any one counter) or 200000 (total)	---
CZTI	10 – 150	0.09	50%	---	4 x 4096

You should note that LAXPC energy range is divided into 8 different energy band counters of equal bandwidth with no overlap.

(T12.1) Some X-ray sources like Cas A have a prominent emission line at 0.01825 nm corresponding to a radioactive transition of ^{44}Ti. Suppose there exists a source which emits only one bright emission line corresponding to this transition. What should be the minimum relative velocity (v) of the source, which will make the observed peak of this line to get registered in a different energy band counter of LAXPC as compared to a source at rest? **13**

These instruments were used to observe an X-ray source (assumed to be a point source), whose energy spectrum followed the power law,

$$F(E) = KE^{-2/3} \qquad \text{[in units of counts/keV/m}^2\text{/s]}$$

where E is the energy in keV, K is a constant and $F(E)$ is photon flux density at that energy. Photon flux density, by definition, is given for per unit collecting area (m^2) per unit bandwidth (keV) and per unit time (seconds). From prior observations, we know that the source has a flux density of 10 $\mathrm{counts/keV/m^2/s}$ at 1 keV, when measured by a detector with 100% photon detection efficiency. The "counts" here mean the number of photons reported by the detector.

As the source flux follows the power law given above, we know that for a given energy range from E_1 (lower energy) to E_2 (higher energy) the total photon flux (F_T) will be given by

$$F_{\mathrm{T}} = 3K\,(E_2^{1/3} - E_1^{1/3}) \qquad [\text{in units of counts/m}^2\text{/s}]$$

(T12.2) Estimate the incident flux density from the source at 1 keV, 5 keV, 40 keV and 100 keV. Also estimate what will be the total count per unit bandwidth recorded by each of the instruments at these energies for an exposure time of 200 seconds. **8**

(T12.3) For this source, calculate the maximum exposure time (t_S), without suffering from saturation, for the CCD of SXT. **4**

(T12.4) If the source became 3500 times brighter, calculate the expected counts per second in LAXPC counter 1, counter 8 as well as total counts across the entire energy range. If we observe for longer period, will the counter saturate due to any individual counter or due to the total count? Tick the appropriate box in the Summary Answersheet. **8**

(T12.5) Assume that the counts reported by CZTI due to random fluctuations in electronics are about 0.00014 counts per pixel per keV per second at all energy levels. Any source is considered as "detected" when the SNR (signal to noise ratio) is at least 3. What is minimum exposure time, t_c, needed for the source above to be detected in CZTI? **10**

Note that the "noise" in a detector is equal to the square root of the counts due to random fluctuations.

(T12.6) Let us consider the situation where the source shows variability in number flux, so that the factor K increases by 20%. AstroSat observed this source for 1 second before the change and 1 second after this change in brightness. Calculate the counts measured by SXT, LAXPC and CZTI in both the observations. Which instrument is best suited to detect this change? Tick the appropriate box in the Summary Answersheet. **7**

*(T12) **Exoplanets**

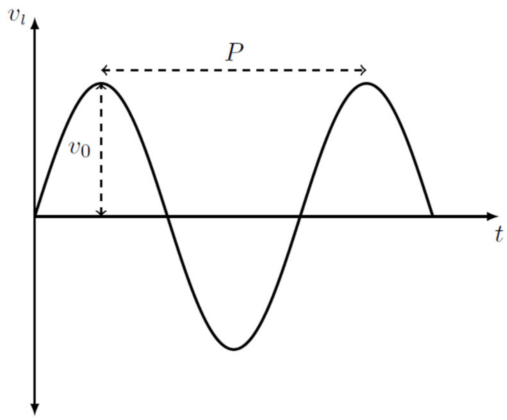
Two major methods of detection of exoplanets (planets around stars other than the Sun) are the radial velocity (or so-called "wobble") method and the transit method. In this problem, we find out how a combination of the results of these two methods can reveal a lot of information about an orbiting exoplanet and its host star.

Throughout this problem, we consider the case of a planet of mass M_p and radius R_p moving in a circular orbit of radius a around a star of mass M_s ($M_s \gg M_p$) and radius R_s. The normal to the orbital plane of the planet is inclined at angle i with respect to the line of sight ($i = 90°$ would mean "edge on" orbit). We assume that there is no other planet orbiting the star and $R_s \ll a$.

"Wobble" Method:

When a planet and a star orbit each other around their barycentre, the star is seen to move slightly, or "wobble", since the centre of mass of the star is not coincident with the barycentre of the star-planet system. As a result, the light received from the star undergoes a small Doppler shift related to the velocity of this wobble.

The line of sight velocity, v_l, of the star can be determined from the Doppler shift of a known spectral line, and its periodic variation with time, t, is shown in the schematic diagram below. In the diagram, the two measurable quantities in this method, namely, the orbital period P and maximum line of sight velocity v_0 are shown.



(T12.1) Derive expressions for the orbital radius (a) and orbital speed (v_p) of the planet in terms of M_s and P.

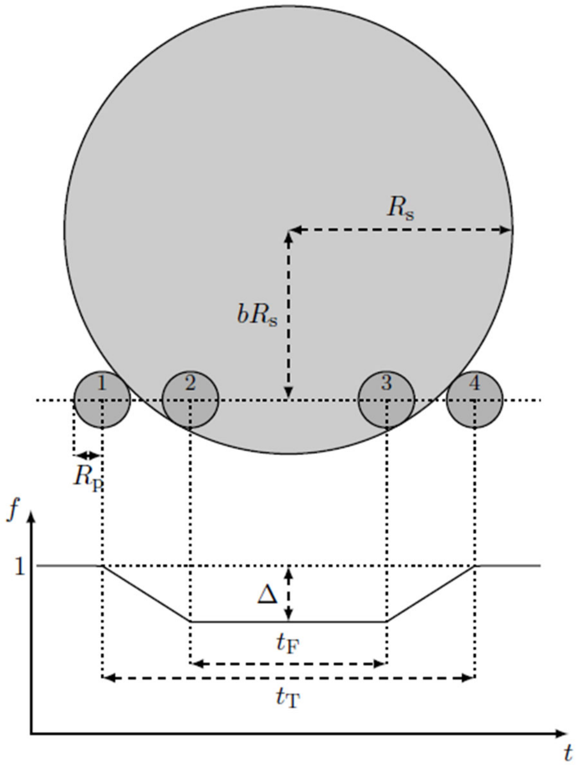
(T12.2) Obtain a lower limit on the mass of the planet, $M_{p,\min}$ in terms of M_s, v_0 and v_p.

Transit Method:

As a planet orbits its host star, for orientations of the orbital plane that are close to "edge-on"($i \approx 90°$), it will pass periodically, or "transit", in front of the stellar disc as seen by the observer. This would cause a tiny decrease in the observed stellar flux which can be measured. The schematic diagram below (NOT drawn to scale) shows the

* The original problem (AstroSat) was discarded.

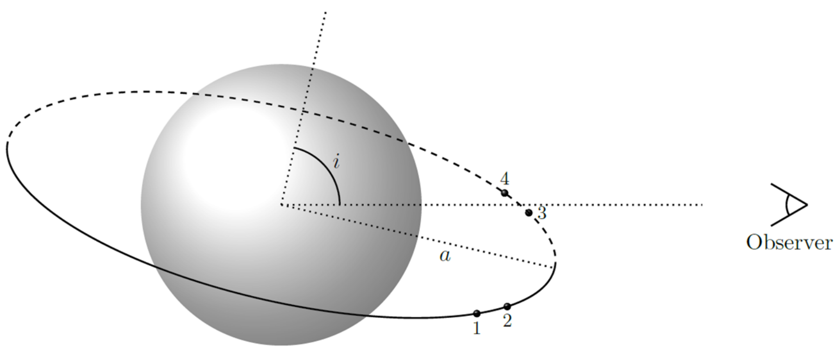
situation from the observer's perspective and the resulting transit light curve (normalised flux, f, vs time, t) for a uniformly bright stellar disc.



If the inclination angle i is exactly $90°$, the planet would be seen to cross the stellar disc along a diameter. For other values of i, the transit occurs along a chord, whose centre lies at a distance bR_s from the centre of the stellar disc, as shown. The no-transit flux is normalised to 1 and the maximum dip during the transit is given by Δ.

The four significant points in the transit are the first, second, third and fourth contacts, marked by the positions 1 to 4, respectively, in the figure above. The time interval during the second and third contacts is denoted by t_F, when the disc of the planet overlaps the stellar disc fully. Thetime interval between the first and fourth contacts is denoted by t_T.

These points are also marked in the schematic diagram below showing a "side-on" view of the orbit (NOT drawn to scale).



The measurable quantities in the transit method are P, t_T, t_F and Δ.

(T12.3) Find the constraint on I in terms of R_s and a for the transit to be visible at all to the distant observer.

(T12.4) Express Δ in terms of R_s and R_p.

(T12.5) Express t_T and t_F in terms of R_s, R_p, a, P and b.

(T12.6) In the approximation of an orbit much larger than the stellar radius, show that the parameter b is given by

$$b = \left[1 + \Delta - 2\sqrt{\Delta}\,\frac{1 + \left(\frac{t_F}{t_T}\right)^2}{1 - \left(\frac{t_F}{t_T}\right)^2}\right]^{1/2}$$

(T12.7) Use the result of part (T12.6) to obtain an expression for the ratio a/R_s in terms of measurable transit parameters, using a suitable approximation.

(T12.8) Combine the results of the wobble method and the transit method to determine the stellar mean density $\rho_s \equiv \dfrac{M_s}{4\pi R_s^3/3}$ in terms of t_T, t_F, Δ and P.

Rocky or gaseous:

Let us consider an edge-on ($i = 90°$) star-planet system (circular orbit for the planet), as seen from the Earth. It is known that the host star is of mass $1.00 M_\odot$. Transits are observed with a period (P) of 50.0 days and total transit duration (t_T) of 1.00 hour. The transit depth (Δ) is 0.0064. The same system is also observed in the wobble method to have a maximum line of sight velocity of 0.400 m s^{-1}.

(T12.9) Find the orbital radius a of the planet in units of AU and in metres.

(T12.10) Find the ratio t_F/t_T of the system.

(T12.11) Obtain the mass M_p and radius R_p of the planet in terms of the mass ($M_\oplus$) and radius ($R_\oplus$) of the Earth respectively. Is the composition of the planet likely to be rocky or gaseous? Tick the box for ROCKY or GASEOUS in the Summary Answersheet.

Transit light curves with starspots and limb darkening:

(T12.12) Consider a planetary transit with $i = 90°$ around a star which has a starspot on its equator, comparable to the size of the planet, R_p. The rotation period of the star is $2P$. Draw schematic diagrams of the transit light curve for five successive transits of the planet (in the templates provided in the Summary Answersheet). The no-transit flux for each transit may be normalised to unity independently. Assume that the planet does not encounter the starspot on the first transit but does in the second.

(T12.13) Throughout the problem we have considered a uniformly bright stellar disc. However, real stellar discs have limb darkening. Draw a schematic transit light curve when limb darkening is present in the host star.

(D1) **Binary Pulsar**

Through systematic searches during the past decades, astronomers have found a large number of millisecond pulsars (spin period < 10 ms). Majority of these pulsars are found in binaries, with nearly circular orbits.

For a pulsar in a binary orbit, the measured pulsar spin period (P) and the measured line-of-sight acceleration (a) both vary systematically due to orbital motion. For circular orbits, this variation can be described mathematically in terms of orbital phase ϕ $(0 \le \phi \le 2\pi)$ as,

$$P(\phi) = P_0 + P_{\rm t}\, cos\phi \qquad \text{where } P_t = \frac{2\pi P_0 r}{cP_{\rm B}}$$

$$a(\phi) = -a_{\rm t}\, sin\phi \qquad \text{where } a_{\rm t} = \frac{4\pi^2 r}{P_{\rm B}^2}$$

where $P_{\rm B}$ is the orbital period of the binary, P_0 is the intrinsic spin period of the pulsar and r is the radius of the orbit.

The following table gives one such set of measurements of P and a at different heliocentric epochs, T, expressed in truncated Modified Julian Days (tMJD), i.e. number of days since MJD $= 2{,}440{,}000$.

No.	T (tMJD)	P (μs)	a (m s^{-2})
1	5740.654	7587.8889	- 0.92 ± 0.08
2	5740.703	7587.8334	- 0.24 ± 0.08
3	5746.100	7588.4100	- 1.68 ± 0.04
4	5746.675	7588.5810	+ 1.67 ± 0.06
5	5981.811	7587.8836	+ 0.72 ± 0.06
6	5983.932	7587.8552	- 0.44 ± 0.08
7	6005.893	7589.1029	+ 0.52 ± 0.08
8	6040.857	7589.1350	+ 0.00 ± 0.04
9	6335.904	7589.1358	+ 0.00 ± 0.02

By plotting $a(\phi)$ as a function of $P(\phi)$, we can obtain a parametric curve. As evident from the relations above, this curve in the period-acceleration plane is an ellipse.

In this problem, we estimate the intrinsic spin period, P_0, the orbital period, $P_{\rm B}$, and the orbital radius, r, by an analysis of this data set, assuming a circular orbit.

(D1.1) Plot the data, including error bars, in the period-acceleration plane (mark your graph as "D1.1"). **7**

(D1.2) Draw an ellipse that appears to be a best fit to the data (on the same graph "D1.1"). **2**

(D1.3) From the plot, estimate P_0, $P_{\rm t}$ and $a_{\rm t}$, including error margins. **7**

(D1.4) Write expressions for $P_{\rm B}$ and r in terms of P_0, $P_{\rm t}$, $a_{\rm t}$. **4**

(D1.5) Calculate approximate value of $P_{\rm B}$ and r based on your estimations made in (D1.3), including error margins. **6**

(D1.6) Calculate orbital phase, ϕ, corresponding to the epochs of the following five observations in the above table: data rows 1, 4, 6, 8, 9. **4**

(D1.7) Refine the estimate of the orbital period, $P_{\rm B}$, using the results in part (D1.6) in the following way:

(D1.7a) First determine the initial epoch, T_0, which corresponds to the nearest epoch of zero orbital phase before the first observation. **2**

(D1.7b) The expected time, $T_{\rm calc}$, of the estimated orbital phase angle of each observation is given by, **7**

$$T_{\rm calc} = T_0 + \left(n + \frac{\phi}{360°}\right) P_{\rm B},$$

where n is the number of full cycle of orbital phases that may have elapsed between T_0 and T (or $T_{\rm calc}$). Estimate n and $T_{\rm calc}$ for each of the five observations in part (D1.6). Note down difference $T_{\rm O-C}$ between observed T and $T_{\rm calc}$. Enter these calculations in the table given in the Summary Answersheet.

(D1.7c) Plot $T_{\rm O-C}$ against n (mark your graph as "D1.7"). **4**

(D1.7d) Determine the refined values of the initial epoch, $T_{0,\rm r}$, and the orbital period, $P_{\rm B,r}$. **7**

(D2) **Distance to the Moon**

Geocentric ephemerides of the Moon for September 2015 are given in the form of a table. Each reading was taken at 00:00 UT.

Date	R.A. (α)			Dec. (δ)			Angular Size (θ)	Phase (ϕ)	Elongation
	h	m	s	°	'	"	"		Of Moon
Sep 01	0	36	46.02	3	6	16.8	1991.2	0.927	148.6° W
Sep 02	1	33	51.34	7	32	26.1	1974.0	0.852	134.7° W
Sep 03	2	30	45.03	11	25	31.1	1950.7	0.759	121.1° W
Sep 04	3	27	28.48	14	32	4.3	1923.9	0.655	107.9° W
Sep 05	4	23	52.28	16	43	18.2	1896.3	0.546	95.2° W
Sep 06	5	19	37.25	17	55	4.4	1869.8	0.438	82.8° W
Sep 07	6	14	19.23	18	7	26.6	1845.5	0.336	70.7° W
Sep 08	7	7	35.58	17	23	55.6	1824.3	0.243	59.0° W
Sep 09	7	59	11.04	15	50	33.0	1806.5	0.163	47.5° W
Sep 10	8	49	0.93	13	34	55.6	1792.0	0.097	36.2° W
Sep 11	9	37	11.42	10	45	27.7	1780.6	0.047	25.1° W
Sep 12	10	23	57.77	7	30	47.7	1772.2	0.015	14.1° W
Sep 13	11	9	41.86	3	59	28.8	1766.5	0.001	3.3° W
Sep 14	11	54	49.80	0	19	50.2	1763.7	0.005	7.8° E
Sep 15	12	39	50.01	-3	20	3.7	1763.8	0.026	18.6° E
Sep 16	13	25	11.64	-6	52	18.8	1767.0	0.065	29.5° E
Sep 17	14	11	23.13	-10	9	4.4	1773.8	0.120	40.4° E
Sep 18	14	58	50.47	-13	2	24.7	1784.6	0.189	51.4° E
Sep 19	15	47	54.94	-15	24	14.6	1799.6	0.270	62.5° E
Sep 20	16	38	50.31	-17	6	22.8	1819.1	0.363	73.9° E
Sep 21	17	31	40.04	-18	0	52.3	1843.0	0.463	85.6° E
Sep 22	18	26	15.63	-18	0	41.7	1870.6	0.567	97.6° E
Sep 23	19	22	17.51	-17	0	50.6	1900.9	0.672	110.0° E
Sep 24	20	19	19.45	-14	59	38.0	1931.9	0.772	122.8° E
Sep 25	21	16	55.43	-11	59	59.6	1961.1	0.861	136.2° E
Sep 26	22	14	46.33	-8	10	18.3	1985.5	0.933	150.0° E
Sep 27	23	12	43.63	-3	44	28.7	2002.0	0.981	164.0° E
Sep 28	0	10	48.32	0	58	58.2	2008.3	1.000	178.3° E
Sep 29	1	9	5.89	5	38	54.3	2003.6	0.988	167.4° W
Sep 30	2	7	39.02	9	54	16.1	1988.4	0.947	153.2° W

The composite graphic[1] below shows multiple snapshots of the Moon taken at different times during the total lunar eclipse, which occurred in this month. For each shot, the centre of frame was coinciding with the central north-south line of umbra.

For this problem, assume that the observer is at the centre of the Earth and angular size refers to angular diameter of the object / shadow.

[1] Credit: NASA's Scientific Visualization Studio

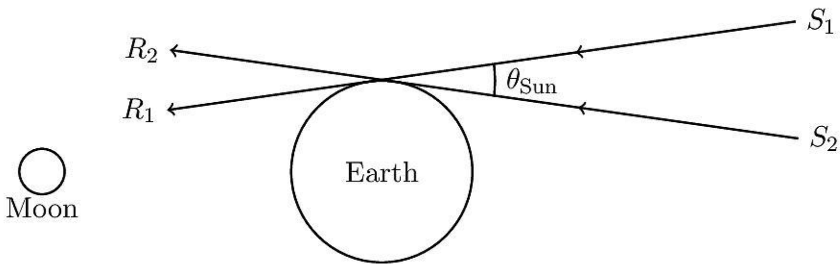
(D2.1) In September 2015, apogee of the lunar orbit is closest to **[3]**
New Moon / First Quarter / Full Moon / Third Quarter.
Tick the correct answer in the Summary Answersheet. No justification for your answer is necessary.

(D2.2) In September 2015, the ascending node of lunar orbit with respect to the ecliptic is closest to **[4]**
New Moon / First Quarter / Full Moon / Third Quarter.
Tick the correct answer in the Summary Answersheet. No justification for your answer is necessary.

(D2.3) Estimate the eccentricity, e, of the lunar orbit from the given data. **[4]**

(D2.4) Estimate the angular size of the umbra, θ_{umbra}, in terms of the angular size of the Moon, θ_{Moon}. Show your working on the image given on the backside of the Summary Answersheet. **[8]**

(D2.5) The angle subtended by the Sun at Earth on the day of the lunar eclipse is known to be $\theta_{\text{Sun}} = 1915.0''$. In the figure below, S_1R_1 and S_2R_2 are rays coming from diametrically opposite ends of the solar disk. The figure is not to scale. **[9]**



Calculate the angular size of the penumbra, θ_{penumbra}, in terms of θ_{Moon}. Assume the observer to be at the centre of the Earth.

(D2.6) Let θ_{Earth} be angular size of the Earth as seen from the centre of the Moon. Calculate the angular size of the Moon, θ_{Moon}, as would be seen from the centre of the Earth on the eclipse day in terms of θ_{Earth}. **[5]**

(D2.7) Estimate the radius of the Moon, R_{Moon}, in km from the results above. **[3]**

(D2.8) Estimate the shortest distance, r_{perigee}, and the farthest distance, r_{apogee}, to the Moon. **[4]**

(D2.9) Use appropriate data from September 10 to estimate the distance, d_{Sun}, to the Sun from the Earth. **[10]**

(D3) **Type IA Supernovae**

Supernovae of type Ia are considered very important for the measurements of large extragalactic distances. The brightening and subsequent dimming of these explosions follow a characteristic light curve, which helps in identifying these as supernovae of type Ia.

Light curves of all type Ia supernovae can be fit to the same model light curve, when they are scaled appropriately. In order to achieve this, we first have to express the light curves in the reference frame of the host galaxy by taking care of the cosmological stretching/dilation of all observed time intervals, Δt_{obs}, by a factor of $(1 + z)$. The time interval in the rest frame of the host galaxy is denoted by Δt_{gal}.

The rest frame light curve of a supernova changes by two magnitudes compared to the peak in a time interval Δt_0 after the peak. If we further scale the time intervals by a factor of s (i.e. $\Delta t_s = s\Delta t_{\rm gal}$) such that the scaled value of Δt_0 is the same for all supernovae, the light curves turn out to have the same shape. It also turns out that s is related linearly to the absolute magnitude, $M_{\rm peak}$, at the peak luminosity for the supernova. That is, we can write

$$s = a + bM_{\rm peak} ,$$

where a and b are constants. Knowing the scaling factor, one can determine absolute magnitudes of supernovae at unknown distances from the above linear equation.

The table below contains data for three supernovae, including their distance moduli, μ (for the first two), their recession speed, cz, and their apparent magnitudes, $m_{\rm obs}$, at different times. The time $\Delta t_{\rm obs} \equiv t - t_{\rm peak}$ shows number of days from the date at which the respective supernova reached peak brightness. The observed magnitudes have already been corrected for interstellar as well as atmospheric extinction.

Name	**SN2006TD**	**SN2006IS**	**SN2005LZ**
μ (mag)	**34.27**	**35.64**	
cz (km s^{-1})	**4515**	**9426**	**12060**
$\Delta t_{\rm obs}$ (days)	$m_{\rm obs}$ (mag)	$m_{\rm obs}$ (mag)	$m_{\rm obs}$ (mag)
-15.00	19.41	18.35	20.18
-10.00	17.48	17.26	18.79
-5.00	16.12	16.42	17.85
0.00	15.74	16.17	17.58
5.00	16.06	16.41	17.72
10.00	16.72	16.82	18.24
15.00	17.53	17.37	18.98
20.00	18.08	17.91	19.62
25.00	18.43	18.39	20.16
30.00	18.64	18.73	20.48

(D3.1) Compute $\Delta t_{\rm gal}$ values for all three supernovae, and fill them in the given blank boxes in the data tables on the BACK side of the Summary Answersheet. On a graph paper, plot the points and draw the three light curves in the rest frame (mark your graph as "D3.1"). **15**

(D3.2) Take the scaling factor, s_2, for the supernova SN2006IS to be 1.00. Calculate the scaling factors, s_1 and s_3, for the other two supernovae SN2006TD and SN2005LZ, respectively, by calculating Δt_0 for them. **5**

(D3.3) Compute the scaled time differences, $\Delta t_{\rm s}$, for all three supernovae. Write the values for $\Delta t_{\rm s}$ in the same data tables on the Summary Answersheet. On another graph paper, plot the points and draw the 3 light curves to verify that they now have an identical profile (mark your graph as "D3.3"). **14**

(D3.4) Calculate the absolute magnitudes at peak brightness, $M_{\rm peak,1}$, for SN2006TD and $M_{\rm peak,2}$, for SN2006IS. Use these values to calculate a and b. **6**

(D3.5) Calculate the absolute magnitude at peak brightness, $M_{\rm peak,3}$, and distance modulus, μ_3, for SN2005LZ. **4**

(D3.6) Use the distance modulus μ_3 to estimate the value of Hubble's constant, H_0. Further, estimate the characteristic age of the universe, $T_{\rm H}$. **6**

Observational Examination (OT)

When you arrive at your observing station, **DO NOT** disturb the telescope before attempting the first question (OT1).

(OT1) The telescope is already set to a deep sky object. Identify the object and tick the correct box in the Summary Answersheet. **10**

Note: You can use any technique to identify the object. However, if you disturb the telescope, you will **NOT** be helped to bring it back to the original position.

(OT2)

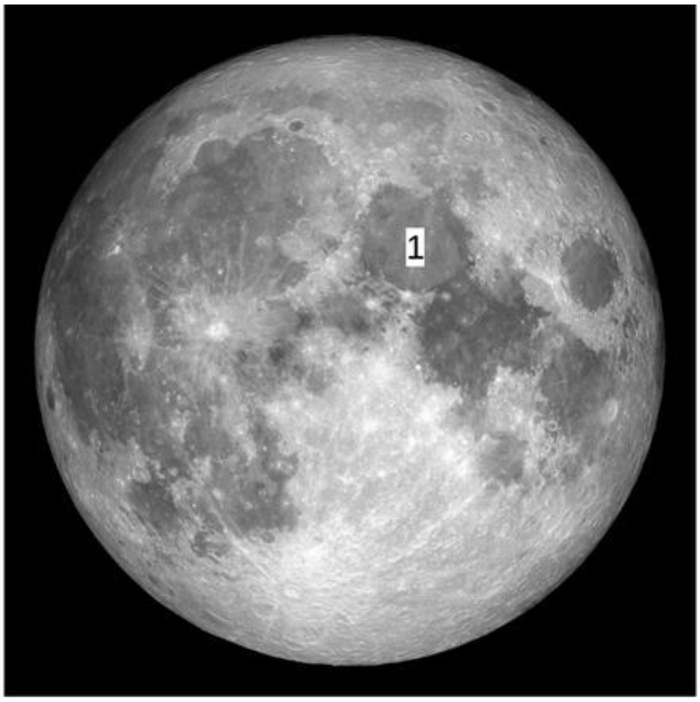
(OT2.1) Point the telescope to M45. Show the object to the examiner. **5**

Note: 1. After 5 minutes, 1 mark will be deducted for a delay of every minute (or part thereof) in pointing the telescope.

2. You have a single chance to be evaluated. If your pointing is incorrect the examiner will change the pointing to M45 for the next part of the question.

(OT2.2) Your Summary Answersheet shows telescopic field of M45. In the image, seven (7) brightest stars of the cluster are replaced by '+' sign. Compare the image with the field you see in the telescope and number the '+' marks from 1 to 7 in the order of decreasing brightness (brightest is 1 and faintest is 7) of the corresponding stars. **15**

(OT3) The examiner will give you a moon filter, an eyepiece with a cross-wire and a stopwatch. Point the telescope towards the Moon. Attach the filter to the telescope. On the surface of the Moon, you will see several "seas" (maria) which are nearly circular in shape. Estimate the diameter of Mare Serenitatis, D_{MSr}, labelled as "1" in the figure below, as a fraction of the lunar diameter, D_{Moon}, by measuring the telescope drift times, t_{Moon} and t_{MSr}, for the Moon and the mare, respectively. **20**



Observational Examination (OP)

(OP1) Eight well known historical supernovae will appear in the projected sky one at a time (not necessarily in chronological order). You have to identify the appropriate map (Map 1 / Map 2) where a particular supernova belongs and mark it in the corresponding map with '+' sign and write codes 'S1' to 'S8' besides it. **40**

Each supernova code will be projected on dome for 10 seconds, followed by appearance of supernova for 60 seconds and then 20 seconds for you to mark the answers.

(OP1.1) For S1, S2, S3, S4 and S5, the projected sky corresponds to the sky as seen from Rio de Janeiro on the midnight of 21st May.

(OP1.2) For S6, S7 and S8, the projected sky corresponds to the sky as seen from Beijing on the midnight of 20th November. There will be a gap of two minute after S5 for change over and adaptation to new sky.

(OP2) We are now projecting sky of another planet. The sky will be slowly rotated for 5 minutes . Identify the visible celestial pole of this planet and mark it with a '+' sign and label it as 'P' on the appropriate map (Map 1 / Map 2). **10**

Observational Examination (OM)

(OM1) Mark any 5 (five) of the following stars on the map by putting a circle (O) around the appropriate star and writing its code next to it. If you mark more than 5 stars, only the first 5 in serial order will be considered. **20**

Code	Name	Bayer Name
S1	Caph	β Cas
S2	Asellus Australis	δ Cnc
S3	Acrux	α Cru
S4	Alphard	α Hya

Code	Name	Bayer Name
S5	Sheliak	β Lyr
S6	Albireo	β Cyg
S7	Rasalhague	α Oph
S8	Kaus Australis	ϵ Sgr

(OM2) Mark location of any 3 (three) of the following galaxies on the map by putting a '+' sign at appropriate place in the map and writing its code next to it. If you mark more than 3 galaxies, only the first 3 in serial order will be considered. **15**

Code	Name	M number
G1	Triangulum Galaxy	M 33
G2	Whirlpool Galaxy	M 51
G3	Southern Pinwheel Galaxy	M 83

Code	Name	M number
G4	Virgo A	M 87
G5	Sombrero Galaxy	M 104

(OM3) Draw ecliptic on the map and label it as 'E'. **5**

(OM4) Show position of Autumnal Equinox (descending node of the ecliptic) on the map by a '+' sign and label it as 'A'. **5**

(OM5) Draw local meridian for Bhubaneswar on Winter Solstice day (22nd December) at local midnight and label it as 'M'. **5**

Group Examination

(G1) A spacecraft of mass m and velocity $\vec{v}$ approaches a massive planet of mass M and orbital velocity $\vec{u}$, as measured by an inertial observer. We consider a special case, where the incoming trajectory of the spacecraft is designed in a way such that velocity vector of the planet does not change direction due to the gravitational boost given to the spacecraft. In this case, the gravitational boost to the velocity the spacecraft can be estimated using conservation laws by measuring asymptotic velocity of the spacecraft before and after the interaction and angle of approach of the spacecraft.

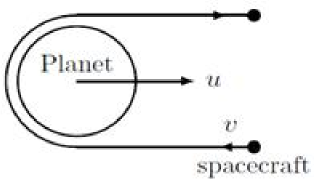


Figure 1

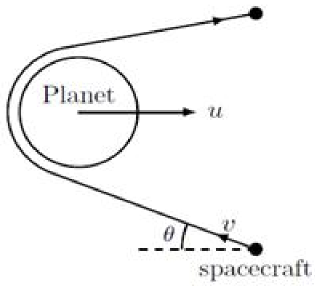


Figure 2

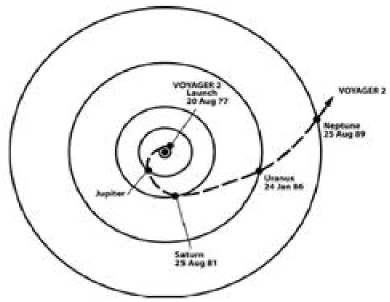


Figure 3

(G1.1) What will be the final velocity ($\vec{v_f}$) of the spacecraft, if $\vec{v}$ and $\vec{u}$ are exactly anti-parallel (see Figure 1). **[3]**

(G1.2) Simplify the expression for the case where $m << M$. **[1]**

(G1.3) If angle between $\vec{v}$ and $-\vec{u}$ is θ and $m << M$ (see Figure 2), use results above to write expression for the magnitude of final velocity (v_f). **[3]**

(G1.4) Table on the last page gives data of Voyager-2 spacecraft for a few months in the year 1979 as it passed close to Jupiter. Assume that the observer is located at the centre of the Sun. The distance from the observer is given in AU and λ is heliocentric ecliptic longitude in degrees. Assume all objects to be in the ecliptic plane. Assume that the orbit of the Earth to be circular. Plot appropriate column against the date of observation to find the date at which the spacecraft was closest to the Jupiter, and label the graph as G1.4. **[8]**

(G1.5) Find the Earth-Jupiter distance, (d_{E-J}) on the day of the encounter. **[4]**

(G1.6) On the day of the encounter, around what standard time (t_{std}) had the Jupiter transited the meridian in the sky of Bhubaneswar ($20.27°$ N; $85.84°$E; UT $+$ $05:30$)? **[6]**

(G1.7) Speed of the spacecraft (in km s^{-1}) as measured by the same observer on some dates before the encounter and some dates after the encounter are given below. Here day n is the date of encounter. Use these data to find the orbital speed of Jupiter (u) on the date of encounter and angle θ. **[12]**

date	n-45	n-35	n-25	n-15	n-5	n
$\mathbf{v_{tot}}$	10.1408	10.0187	9.9078	9.8389	10.2516	25.5150
date	n+5	n+15	n+25	n+35	n+45	
$\mathbf{v_{tot}}$	21.8636	21.7022	21.5580	21.3812	21.2365	

(G1.8) Find eccentricity, e_J, of Jupiter's orbit. **[8]**

(G1.9) Find heliocentric ecliptic longitude, λ_p, of Jupiter's perihelion point. **[5]**

Month	Date	λ (∘)	Distance (AU)
June	1	135.8870	5.1589731906
June	2	135.9339	5.1629499712
June	3	135.9806	5.1669246607
June	4	136.0272	5.1708975373
June	5	136.0736	5.1748689006
June	6	136.1200	5.1788390741
June	7	136.1662	5.1828084082
June	8	136.2122	5.1867772826
June	9	136.2582	5.1907461105
June	10	136.3040	5.1947153428
June	11	136.3496	5.1986854723
June	12	136.3951	5.2026570402
June	13	136.4405	5.2066306418
June	14	136.4857	5.2106069354
June	15	136.5307	5.2145866506
June	16	136.5756	5.2185705999
June	17	136.6202	5.2225596924
June	18	136.6647	5.2265549493
June	19	136.7090	5.2305575243
June	20	136.7532	5.2345687280
June	21	136.7970	5.2385900582
June	22	136.8407	5.2426232385
June	23	136.8841	5.2466702671
June	24	136.9273	5.2507334797
June	25	136.9702	5.2548156324
June	26	137.0127	5.2589200110
June	27	137.0550	5.2630505798
June	28	137.0969	5.2672121872
June	29	137.1384	5.2714108557
June	30	137.1795	5.2756542053
July	1	137.2200	5.2799520895
July	2	137.2600	5.2843175880
July	3	137.2993	5.2887686308
July	4	137.3378	5.2933308160
July	5	137.3754	5.2980426654
July	6	137.4118	5.3029664212
July	7	137.4467	5.3082133835
July	8	137.4798	5.3140161793
July	9	137.5116	5.3210070441
July	10	137.5628	5.3312091210
July	11	137.6898	5.3405592121
July	12	137.8266	5.3466522674
July	13	137.9599	5.3516661563
July	14	138.0903	5.3561848203
July	15	138.2186	5.3604205657
July	16	138.3453	5.3644742164

Month	Date	λ (∘)	Distance (AU)
July	17	138.4707	5.3684017790
July	18	138.5949	5.3722377051
July	19	138.7183	5.3760047603
July	20	138.8409	5.3797188059
July	21	138.9628	5.3833913528
July	22	139.0841	5.3870310297
July	23	139.2048	5.3906444770
July	24	139.3250	5.3942369174
July	25	139.4448	5.3978125344
July	26	139.5641	5.4013747321
July	27	139.6831	5.4049263181
July	28	139.8016	5.4084696349
July	29	139.9198	5.4120066575
July	30	140.0377	5.4155390662
July	31	140.1553	5.4190683021
August	1	140.2725	5.4225956100
August	2	140.3895	5.4261220723
August	3	140.5062	5.4296486357
August	4	140.6225	5.4331761326
August	5	140.7387	5.4367052982
August	6	140.8546	5.4402367851
August	7	140.9702	5.4437711745
August	8	141.0856	5.4473089863
August	9	141.2007	5.4508506867
August	10	141.3157	5.4543966955
August	11	141.4303	5.4579473912
August	12	141.5448	5.4615031166
August	13	141.6591	5.4650641822
August	14	141.7731	5.4686308707
August	15	141.8869	5.4722034391
August	16	142.0006	5.4757821220
August	17	142.1140	5.4793671340
August	18	142.2272	5.4829586711
August	19	142.3402	5.4865569133
August	20	142.4530	5.4901620256
August	21	142.5657	5.4937741595
August	22	142.6781	5.4973934544
August	23	142.7904	5.5010200385
August	24	142.9024	5.5046540300
August	25	143.0143	5.5082955377
August	26	143.1260	5.5119446617
August	27	143.2375	5.5156014948
August	28	143.3488	5.5192661222
August	29	143.4599	5.5229386226
August	30	143.5709	5.5266190687
August	31	143.6817	5.5303075275

IOAA 2017 – Phuket, Thailand

The 11$^{\text{th}}$ IOAA was held from 12$^{\text{th}}$ to 21$^{\text{st}}$ November 2017. Total of 44 countries participated in the event.

Logo concepts: Coconut tree representing the pleasantness of beaches; Earth, star, and curve indicating the orbit of star; Mountains and beaches, the main tourist sites in Phuket, Thailand.

Instructions

1. The theoretical examination lasts for 5 hours and is worth a total of 300 marks.

2. Dedicated IOAA **Summary Answer Sheets** are provided for writing your answers. Enter the final answers into the appropriate boxes in the corresponding **Summary Answer Sheet**. On each **Summary Answer Sheet**, please fill in
   - Student Code (Country Code and 1 digit)

3. There are **Answer Sheets** for carrying out detailed work/rough work. On each **Answer Sheet**, please fill in
   - Student Code (Country Code and 1 digit)
   - Question no.
   - Page no. and total number of pages.

4. Start each problem on a separate Answer Sheet. <u>Please write only on the printed side of the sheet. Do not use the reverse side.</u> If you have written something on any sheet which you do not want to be evaluated, cross it out.

5. Use as many mathematical expressions as you think may help the evaluator to better understand your solutions. The evaluator may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible in English).

6. You are not allowed to leave your work desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, etc.), please draw the attention of the proctor using the Help card.

7. The beginning and end of the examination will be indicated by a long sound signal. Additionally, there will be a buzzer sound, fifteen minutes before the end of the examination (before the final sound signal).

8. At the end of the examination you must stop writing immediately. Sort and put your sheets in separate stacks,
   a) Stack 1: Summary Answer Sheets, Answer Sheets of part 1
   b) Stack 2: Summary Answer Sheets, Answer Sheets of part 2
   c) Stack 3: Summary Answer Sheets, Answer Sheets of part 3
   d) Stack 4: question papers and paper sheets you do not want to be graded.

9. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the examination room.

10. A list of constants and a table of the mark distribution for this exam are given on the next two pages.

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity (g)	$9.8\ \mathrm{m \cdot s^{-1}}$	
Obliquity of Ecliptic	23°27’	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass($M_☾$)	7.35×10^{22} kg	**Moon**
Radius ($R_☾$)	1.74×10^{6} m	
Mean distance from Earth	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	-12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	**Sun**
Radius ($R_\odot$)	6.96×10^{8} m	
Luminosity ($L_\odot$)	3.83×10^{26} W	
Absolute Magnitude ($\mathscr{M}_\odot$)	4.80 mag	
Angular diameter	0.5 degrees	
Rotational velocity in the Galaxy	$220\ \mathrm{km\ s^{-1}}$	
Distance from Galactic centre	8.5 kpc	
Mass	1.89×10^{27} kg	**Jupiter**
Orbital semi-major axis	5.20 au	
Orbital period	11.86 year	
Mass	5.68×10^{26} kg	**Saturn**
Orbital semi-major axis	9.58 au	
Orbital period	29.45 year	
1 au	1.50×10^{11} m	**Physical constants**
1 pc	206 265 au	
Gravitational constant (G)	$6.67 \times 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}}$	
Planck constant (h)	$6.62 \times 10^{-34}\ \mathrm{J \cdot s}$	
Boltzmann constant (k_B)	$1.38 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$	
Stefan-Boltzmann constant (σ)	$5.67 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$	
Hubble constant (H_0)	$67.8\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$	
Speed of light in vacuum (c)	$299\,792\,458\ \mathrm{m \cdot s^{-1}}$	
Proton mass	$938.27\ \mathrm{MeV \cdot c^{-2}}$	
Deuterium mass	$1875.60\ \mathrm{MeV \cdot c^{-2}}$	
Neutron mass	$939.56\ \mathrm{MeV \cdot c^{-2}}$	
Helium-3 mass	$2808.30\ \mathrm{MeV \cdot c^{-2}}$	
Helium-4 mass	$3727.40\ \mathrm{MeV \cdot c^{-2}}$	

Mark distribution of this exam

Problem number	**Marks**
T1	10
T2	10
T3	10
T4	10
T5	10
T6	15
T7	20
T8	20
T9	20
T10	25
T11	50
T12	40
T13	60
Total	300

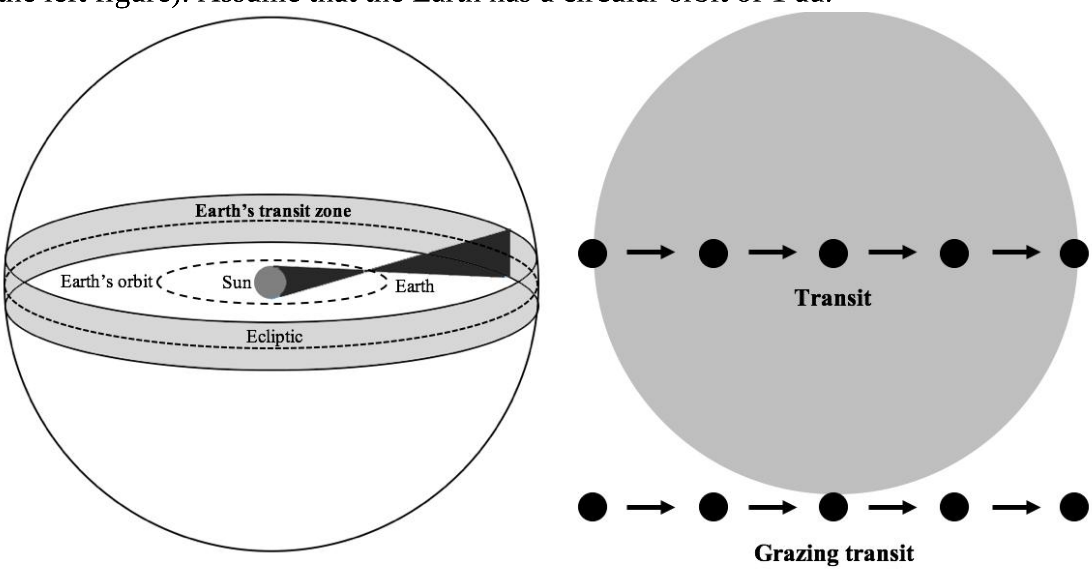
Part 1

(T1) The Large Magellanic Cloud in Phuket **[10 marks]**

The coordinates of the Large Magellanic Cloud (LMC) are R.A. = 5h 24min and Dec = $-70^\circ 00'$. The latitude and longitude of Phuket are $7^\circ 53'$ N and $98^\circ 24'$ E, respectively. What is the date when the LMC culminates at 9pm as seen from Phuket in the same year? You may note that the Greenwich Sidereal Time, GST, at 00h UT 1st January is about 6h 43min, and Phuket is in the UT+7 time zone. [10]

(T2) Earth's Transit Zone **[10 marks]**

Earth's transit zone is an area where extrasolar observers (located far away from the Solar System) can detect the Earth transiting across the Sun. For observers on the Earth, this area is the projection of a band around the Earth's ecliptic onto the celestial plane (light grey area in the left figure). Assume that the Earth has a circular orbit of 1 au.



a) Find the angular width of that part of the Earth's transit zone in degrees, in which the extrasolar observers can detect Earth's total transit (when the whole of the Earth's disk passes in front of the Sun). [5]

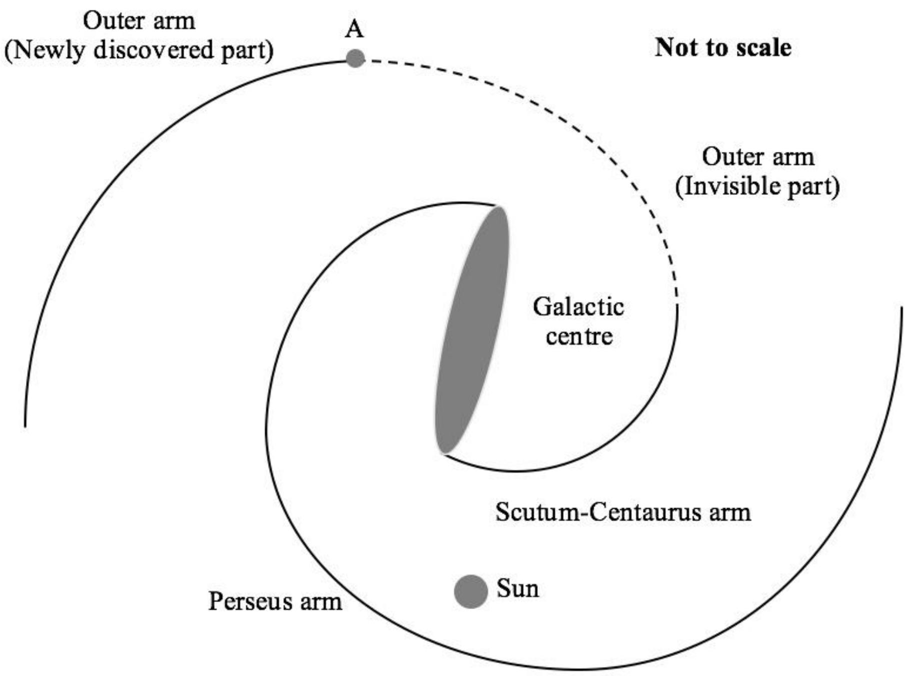
b) Find the angular width of that part of the Earth's transit zone in degrees, where the extrasolar observers can detect at least Earth's grazing transit (when any part of the Earth's disk passes in front of the Sun). [5]

(T3) The Milky Way's Distant Outer Arm **[10 marks]**

In 2011, Dame and Thaddeus found a new part of the outer arm of the Milky Way by studying the CO line using the CfA 1.2m telescope. They found that the CO line was detected at galactic longitude $\ell = 13.25^\circ$ (marked **A** in the figure) where it had a radial velocity of 20.9 km s^{-1} towards the Sun. Assume that the galactic rotation curve is flat beyond 5 kpc from the Galactic centre. The distance between the Sun and the Galactic centre is 8.5 kpc. The velocity of the Sun around the Galactic centre is 220 km s^{-1}.

a) Find the distance from the start of the arm (point **A**) to the Galactic centre. [7]

b) Find the distance from the start of the arm (point **A**) to the Sun. [3]



(T4) 21-cm HI galaxy survey **[10 marks]**

A radio telescope is equipped with a receiver which can observe in a frequency range from 1.32 to 1.52 GHz. Its detection limit is 0.5 mJy per beam for a 1-minute integration time. In a galaxy survey, the luminosity of the HI spectral line of a typical target galaxy is 10^{28} W with a linewidth of 1 MHz. For a large beam, the HI emitting region from a far-away galaxy can be approximated as a point source. The HI spin-flip spectral line has a rest-frame frequency of 1.42 GHz.

What is the highest redshift, z, of a typical HI galaxy that can be detected by a survey carried out with this radio telescope, using 1-minute integration time? You may assume in your calculation that the redshift is small and the non-relativistic approximation can be used. Note that 1 Jy = 10^{-26} W m^{-2} Hz^{-1}. [10]

(T5) A Synchronous Satellite **[10 marks]**

A synchronous satellite is a satellite which orbits the Earth with its period exactly equal to the period of rotation of the Earth. The height of these satellites is 35786 km above the surface of the Earth. A satellite is put in an inclined synchronous orbit with an inclination of $\theta = 6.69°$ to the equatorial plane. Calculate the precise value of the maximum possible altitude of the satellite for an observer at latitude of $\phi = 51.49°$. Ignore the effect of refraction due to the Earth's atmosphere. [10]

Part 2

(T6) Supernova 1987A **[15 marks]**

Supernova SN 1987A was at its brightest with an apparent magnitude of +3 on about 15th May 1987 and then faded, finally becoming invisible to the naked eye by 4th February 1988. It is assumed that brightness B varied with time t as an exponential decline, $B = B_0 e^{-t/\tau}$, where B_0 and τ are constant. The maximum apparent magnitude which can be seen by the naked eye is +6.

a) Determine the value of τ in days. [5]

b) Find the last day that observers could have seen the supernova if they had a 6-inch (15.24-cm) telescope with transmission efficiency $T = 70\%$. Assume that the average diameter of the human pupil is 0.6 cm. [10]

(T7) Life on Other Planets **[20 marks]**

One place to search for life is on planets orbiting main sequence stars. A good starting point is the planets that have an Earth-like temperature range and a small temperature fluctuation. Assume that for a main sequence star, the relation between the luminosity L and the mass M is given by

$$L \propto M^{3.5}.$$

You may assume that the total energy E released over the lifetime of the star is proportional to the mass M of the star. For the Sun, it will have a main sequence lifetime of about 10 billion years. The stellar spectral types are given in the table below. Assume that the spectral subclasses of stars (0-9) are assigned on a scale that is linear in $\log M$.

Spectral Class	O5V	B0V	A0V	F0V	G0V	K0V	M0V
Mass ($M_\odot$)	60	17.5	2.9	1.6	1.05	0.79	0.51

a) If it takes at least 4×10^9 years for an intelligent life form to evolve, what is the spectral type (accurate to the subclass level) of the most massive star in the main sequence around which astronomers should look for intelligent life? [6]

b) Assume that the target planet has the same emissivity ε and albedo a as the Earth. In order to have the same temperature as the Earth, express the distance d, in au, of the planet to its parent main sequence star, of mass M. [6]

c) The existence of a planet around a star can be shown by the variation in the radial velocity of the star about the star-planet system centre of mass. If the smallest Doppler shift in the wavelength detectable by the observer is $(\Delta\lambda / \lambda) = 10^{-10}$, calculate the lowest mass of such a planet in b), in units of Earth masses, that can be detected by this method, around the main sequence star in a). [8]

(T8) The Star of Bethlehem **[20 marks]**

A great conjunction is a conjunction of Jupiter and Saturn for observers on Earth. Assume that Jupiter and Saturn have circular orbits in the ecliptic plane.

The time between successive conjunctions may vary slightly as viewed from the Earth. However, the average time period of the great conjunctions is the same as that of an observer at the centre of the Solar system.

a) Find the average great conjunction period (in years) and average heliocentric angle between two successive great conjunctions (in degrees). [6]

b) The next great conjunction will be on 21st December 2020 with an elongation of 30.3° East of the Sun. Suggest the constellation in which the conjunction on 21st December 2020 will occur. (Give the IAU Latin name or IAU three-letter abbreviation of the constellation, i.e. Ursa Major or UMa) [2]

In 1606, Johannes Kepler determined that in some years the great conjunction can be happen three times in the same year due to the retrograde motions of the planets. He also determined that such an event happened in the year 7 BC, which could have been the event commonly known as "The Star of Bethlehem". For the calculations below you may ignore the precession of the axis of the Earth.

c) Suggest the constellation in which the great conjunctions in 7 BC occurred. (Give the IAU Latin name or IAU three-letter abbreviation of the constellation, i.e. Ursa Major or UMa) [8]

d) During the second conjunction of the series of three conjunctions in 7 BC, suggest the constellation the Sun was in as viewed by the observer on Earth. (Give the IAU Latin name or IAU three-letter abbreviation of the constellation, i.e. Ursa Major or UMa) [4]

(T9) Galactic Outflow **[20 marks]**

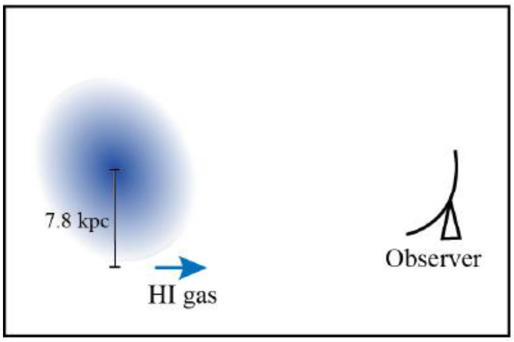
Cannon et al. (2004) conducted an HI observation of a disk starburst galaxy, IRAS 0833+6517, with the Very Large Array (VLA). The galaxy is located at a distance of 80.2 Mpc with an approximate inclination angle of 23 degrees. According to the HI velocity map, IRAS 0833+6517 appears to be undergoing regular rotation with an observed radial velocity of the HI gas of roughly 5850 km s^{-1} at a distance of 7.8 kpc from the centre (the left panel of the figure below).

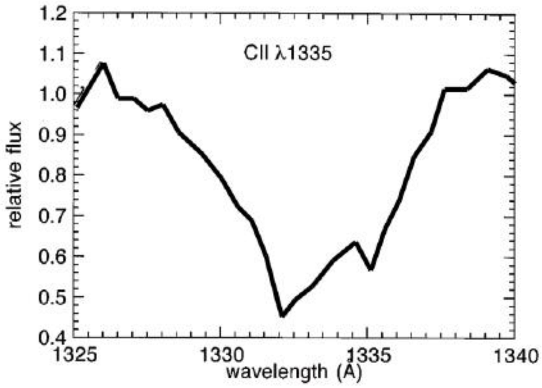
Gas outflow from IRAS 0833+6517 is traced by using the blueshifted interstellar absorption lines observed against the backlight of the stellar continuum (the right panel of the figure). Assuming that this galaxy is gravitationally stable and all the stars are moving in circular orbits,

a) Determine the rotational velocity (v_{rot}) of IRAS 0833+6517 at the observed radius of HI gas. [5]

b) Calculate the escape velocity for a test particle in the gas outflow at the radius of 7.8 kpc. [9]

c) Determine if the outflowing gas can escape from the galaxy at this radius by considering the velocity offset of the C II λ1335 absorption line, which is already corrected for the cosmological recessional velocity. (The central rest-frame wavelength of the CII absorption line is 1335 Å.) (YES / NO) [6]





(T10) GOTO **[25 marks]**

The Gravitational-Wave Optical Transient Observer (GOTO) aims to carry out searches of optical counterparts of any Gravitational Wave (GW) sources within an hour of their detection by the LIGO and VIRGO experiments. The survey needs to cover a big area on the sky in a short time to search all possible regions constrained by the GW experiments before the optical burst signal, if any, fades away. The GOTO telescope array is composed of 4 identical reflective telescopes, each with 40-cm diameter aperture and f-ratio of 2.5, working together to image large regions of the sky. For simplicity, we assume that the telescopes' fields-of-view (FoV) do not overlap with one another.

a) Calculate the projected angular size per mm at the focal plane, i.e. plate scale, of each telescope. [6]

b) If the zero-point magnitude (i.e. the magnitude at which the count rate detected by the detector is 1 count per second) of the telescope system is 18.5 mag, calculate the minimum time needed to reach 21 mag at Signal-to-Noise Ratio (SNR) = 5 for a point source. We first assume that the noise is dominated by both the Read-Out Noise (RON) at 10 counts/pixel and the CCD dark (thermal) noise (DN) rate of 1 count/pix/minute. The CCDs used with the GOTO have a 6-micron pixel size and gain (conversion factor between photo-electron and data count) of 1. The typical seeing at the observatory site is around 1.0 arcsec. [8]

The Signal-to-Noise Ratio is defined as

$$\mathrm{SNR} \equiv \frac{\text{Total Source Count}}{\sqrt{\Sigma_i \mathrm{Noise}_i^2}} = \frac{\text{Total Source Count}}{\sqrt{\sigma_{\mathrm{RON}}^2 + \sigma_{\mathrm{DN}}^2 + \ldots}},$$

$$\sigma_{\mathrm{RON}} = \sqrt{N_{\mathrm{pix}} \cdot \mathrm{RON}^2}, \quad \sigma_{\mathrm{DN}} = \sqrt{N_{\mathrm{pix}} \cdot \mathrm{DN} \cdot t},$$

where t is the exposure time.

c) Normally when the exposure time is long and the source count is high then Poisson noise from the source is also significant. Determine the relation between SNR and exposure time in the case that the noise is dominated by Poisson noise of the source. Recalculate the minimum exposure time required to reach 21 mag with SNR = 5 from part b) if Poisson noise is also taken into consideration. The Poisson noise (standard deviation) of the source is given by $\sigma_{\mathrm{source}} = \sqrt{\text{Source Count}}$. In reality, there is also the sky background which can be important source of Poisson noise. For our purpose here, please ignore any sky background in the calculation. [6]

d) The typical localisation uncertainty of the GW detector is about 100 square-degrees and we would like to cover the entire possible location of any candidate within an hour after the GW is detected. Estimate the minimum side length of the square CCD needed for each telescope in terms of the number of pixels. You may assume that the time taken for the CCD read-out and the pointing change are negligible. [5]

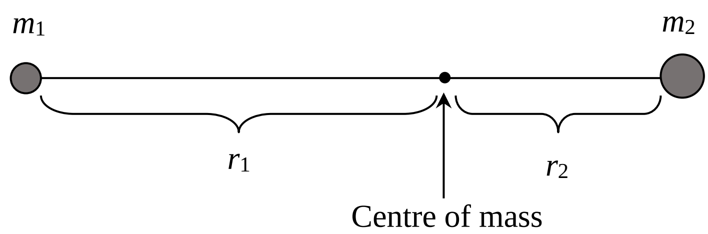
Part 3

(T11) Mass of the Local Group **[50 marks]**

The dynamics of M31 (Andromeda) and the Milky Way (MW) can be used to estimate the total mass of the Local Group (LG). The basic idea is that galaxies currently in a binary system were at approximately the same point in space shortly after the Big Bang. To a reasonable approximation, the mass of the local group is dominated by the masses of the MW and M31. Via Doppler shifts of the spectral lines, it was found that M31 is moving towards the MW with a speed of 118 km s^{-1}. This may be surprising, given that most galaxies are moving away from each other with the general Hubble flow. The fact that M31 is moving towards the MW is presumably because their mutual gravitational attraction has eventually reversed their initial velocities. In principle, if the pair of galaxies is well-represented by isolated point masses, their total mass may be determined by measuring their separation, relative velocity and the time since the universe began. Kahn and Woltjer (1959) used this argument to estimate the mass in the LG.

In this problem we will follow this argument through our calculation as follows.

a) Consider an isolated system with negligible angular momentum of two gravitating point masses m_1 and m_2 (as observed by an inertial observer at the centre of mass).



Write down the expression of the total mechanical energy (E) of this system in mathematical form connecting m_1, m_2, r_1, r_2, v_1, v_2, and the universal gravitational constant G, where v_1 and v_2 are the radial velocities of m_1 and m_2, respectively.

[5]

b) Re-write the equation in a) in terms of r, v, μ, M, and G, where $r \equiv r_1 + r_2$ is the separation distance between m_1 and m_2, v is the changing rate of the separation distance, $\mu \equiv \dfrac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the system, and $M \equiv m_1 + m_2$ is the total mass of the system.

[10]

c) Show that the equation in b) yields

$$v^2 = (2GM)\left(\frac{1}{r} - \frac{1}{r_0}\right),\text{ where } r_0 \text{ is a new constant.}$$

Find r_0 in terms of μ, M, G and E.

[5]

The solution of the equation in b) is given below in parametric form, under the initial condition $r = 0$ at $t = 0$:

$$r(\theta) = \frac{r_0}{2}(1 - \cos\theta),$$

$$t(\theta) = \left(\frac{r_0^3}{8GM}\right)^{\frac{1}{2}} (\theta - \sin\theta),$$

where θ is in radians.

d) From the above parametric equations, show that an expression for $\frac{vt}{r}$ is

$\frac{vt}{r} = \frac{(\sin\theta)(\theta - \sin\theta)}{(1-\cos\theta)^2}$ [10]

e) Now we consider m_1 and m_2 as the MW and M31, respectively, such that the current values of v and r are $v = -118$ km s^{-1} and $r = 710$ kpc, and t may be taken to be the age of the Universe (13700 million years). Find θ using numerical iteration. [10]

f) Use the value of θ from e) to calculate the maximum distance between M31 and the MW, $r_{\max}$, and hence also obtain the value of M in solar masses. [10]

(T12) Shipwreck [40 marks]

You are shipwrecked on an island. Fortunately, you are still wearing a watch that is set to Bangkok time, and you also have a compass, an atlas and a calculator. You are initially unconscious, but wake up to find it has recently become dark. Unfortunately, it is cloudy. An hour or so later you see Orion through a gap in the clouds. You estimate that the star "Rigel" is about 52.5° above the horizon and with your compass you find that it has an astronomical azimuth of 109°. Your watch says it is currently 01:00 on the 21st November 2017. You happen to remember from your astronomy class that Greenwich Sidereal Time (GST) at 00h UT 1st January 2017 is about 6h 43min and that the R.A. and Dec of Rigel are 5h 15min and $-8^\circ\ 11'$, respectively. Bangkok is in the UT+7 time zone.

a) Find the Local Hour Angle (LHA) of Rigel. [10]
b) Find the current Greenwich sidereal time (GST). [10]
c) Find the longitude of the island. [5]
d) Find, to the nearest arcminute, the Latitude of the island. [15]

(T13) Exomoon **[60 marks]**

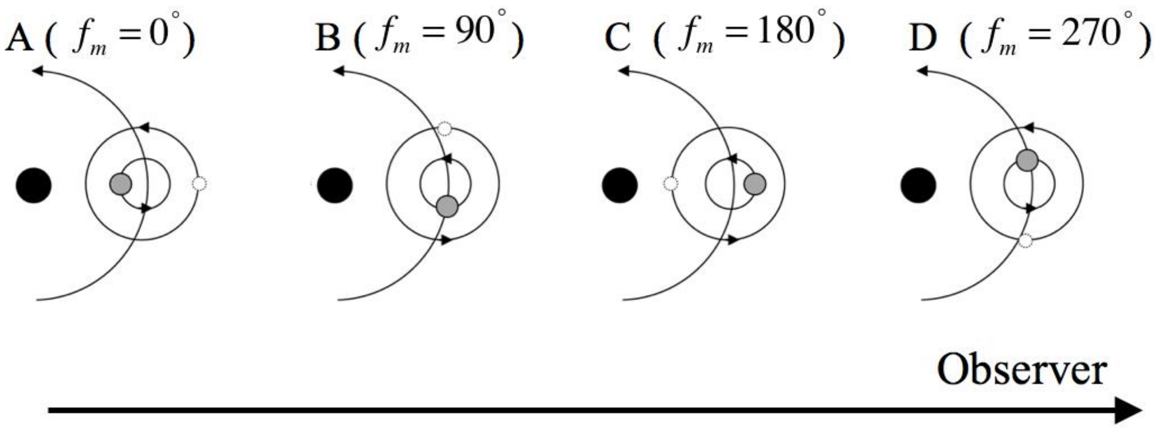
Exomoons are natural satellites of exoplanets. The gravitational influence of such a moon will affect the position of the planet relative to the planet-moon barycentre, resulting in Transit Timing Variations (σ_{TTV}, TTVs) as the observed transit of the planet occurs earlier or later than the predicted time of transit for a planet without a moon.

The motion of the planet around the planet-moon barycentre will also induce Transit Duration Variations (σ_{TDV}, TDVs) as the observed transit duration is shorter or longer than the predicted transit duration for a planet without a moon.

We will consider edge-on circular orbits with the following parameters

- M_P is the planet mass
- M_m is the moon mass
- P_P is the planet-moon barycentre's period around the host star
- P_m is the moon's period around the planet
- a_p is the distance of the planet-moon barycentre to the star
- a_m is the distance of the moon to the planet-moon barycentre
- f_m is the moon phase, $f_m = 0^\circ$ when the moon is in opposition to the star
- τ is the mean transit duration of the planet (as if it has no moon)

We will only consider the orbit of a prograde moon orbiting in the same plane as the planet's orbit. Example phases of the moon, as observed by distant observers, are shown in the figure below.



Phase of the moon.

Black, grey and white circles represent the star, planet and moon, respectively.

a) We define $\sigma_{TTV} \equiv t_m - t$ where t is the predicted transit time without the moon, and t_m is the observed transit time with the moon. Show that

$$\sigma_{TTV} = \left[\frac{a_m M_m P_p}{2\pi a_p M_p}\right]\sin(f_m)$$

A positive value of σ_{TTV} indicates that the transit occurs later than the predicted time of transit for a planet without a moon. [10]

b) Similarly, we define $\sigma_{TDV} \equiv \tau_m - \tau$ where τ is the predicted transit duration without the moon, and τ_m is the observed transit duration with the moon. We can assume that the planet's velocity around the star is much bigger than the moon's velocity around the planet-moon barycentre, and also the moon does not change phase during the transit. Show that

$$\sigma_{TDV} = \tau\left[\frac{P_p M_m a_m}{P_m M_p a_p}\right]\cos(f_m)$$

A positive value of σ_{TDV} indicates that the transit duration is longer than the predicted transit duration without a moon. [13]

An exoplanet is observed transiting a main-sequence solar-type star (1 $M_\odot$, 1 $R_\odot$, spectral class: G2V). The planet has an edge-on circular orbit with a period of 3.50 days. From the observational data, the planet has a mass of 120 $M_\oplus$ and a radius of 12 $R_\oplus$. The observed relation between σ^2_{TTV} and σ^2_{TDV} can be written as

$$\sigma^2_{TDV} = -0.7432\sigma^2_{TTV} + 1.933\times10^{-8}\ \text{days}^2$$

c) Assume that the moon's mass is much smaller than the planet's mass. Find the mean transit duration of the planet (τ) in days. [6]

d) Find the moon's period (P_m) in days [7]

e) Estimate the distance of the moon to the planet-moon barycentre (a_m) in units of Earth radii. Also find the moon mass (M_m) in units of Earth mass. [7]

f) The Hill sphere is a region around a planet within which the planet's gravity dominates. The radius of the Hill sphere can be written as

$$R_h = a_p\sqrt[3]{\frac{M_p}{xM_*}}$$

where M_* is the host star mass.

Find the value of the constant x (Hint: for a massive host star, the radius of the Hill sphere of the system is approximately equal to the distance between the planet and the Lagrange point L_1 or L_2). Hence, find the radius of the Hill sphere of this planetary system in units of Earth radii. [11]

g) The Roche limit is the minimum orbital radius at which a satellite can orbit without being torn apart by tidal forces. Take the Roche limit as

$$R_r = 1.26 R_p \sqrt[3]{\frac{\rho_p}{\rho_m}}$$

where ρ_p and ρ_m are the density of the planet and moon, respectively and R_p is the planet's radius. Assuming that the moon is a rocky moon with the same density as the Earth, find the Roche limit of the system. [3]

h) Does the moon have a stable orbit? (YES / NO) [3]

Instructions

1. The data analysis examination lasts for 4 hours and is worth a total of 150 marks.

2. Dedicated IOAA **Summary Answer Sheets** are provided for writing your answers. Enter the final answers into the appropriate boxes in the corresponding **Summary Answer Sheet**. On each Answer Sheet, please fill in
    - Student Code (Country Code and 1 digit)

3. **Graph Papers** are required for your solutions. On each Graph Paper, please fill in
    - Student Code (Country Code and 1 digit)
    - Question no.
    - Graph no. and total number of graph papers used.

4. There are **Answer Sheets** for carrying out detailed work/rough work. On each Answer Sheet, please fill in
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10. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the examination area.

11. A list of constants is given on the next page.

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity (g)	9.8 m s^{-2}	
Obliquity of Ecliptic	23°27’	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass($M_☾$)	7.35×10^{22} kg	**Moon**
Radius ($R_☾$)	1.74×10^{6} m	
Mean distance from Earth	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	-12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	**Sun**
Radius ($R_\odot$)	6.96×10^{8} m	
Luminosity ($L_\odot$)	3.83×10^{26} W	
Absolute Magnitude ($\mathscr{M}_\odot$)	4.80 mag	
Angular diameter	0.5 degrees	
1 au	1.50×10^{11} m	**Physical constants**
1 pc	206,265 au	
Distance from Sun to Barnard’s Star	1.83 pc	
Gravitational constant (G)	6.67×10^{-11} N · m^2 · kg^{-2}	
Planck constant (h)	6.62×10^{-34} J · s	
Boltzmann constant (k_B)	1.38×10^{-23} J · K^{-1}	
Stefan-Boltzmann constant (σ)	5.67×10^{-8} W · m^{-2} · K^{-4}	
Hubble constant (H_0)	72 km s^{-1} Mpc^{-1}	
Speed of light in vacuum (c)	299,792,458 m s^{-1}	
Proton mass	938.27 MeV · c^{-2}	
Deuterium mass	1875.60 MeV · c^{-2}	
Neutron mass	939.56 MeV · c^{-2}	
Helium-3 mass	2808.30 MeV · c^{-2}	
Helium-4 mass	3727.40 MeV · c^{-2}	

(D1) Calibrating the distance ladder to the LMC [75 marks]

An accurate trigonometric parallax calibration for Galactic Cepheids has long been sought, but is very difficult to achieve in practice. All known classical (Galactic) Cepheids are more than 250 pc away, therefore for direct distance estimates to achieve an uncertainty of up to 10%, parallax uncertainties of up to ±0.2 milliarcsec are needed, requiring space-based observations. The Hipparcos satellite reported parallaxes for 200 of the nearest Cepheids, but even the best of these had high uncertainties. Recent progress has come with the use of the Fine Guidance Sensor on HST where parallaxes (in many cases) accurate to better than ±10% were obtained for 10 Cepheids, spanning a range of periods from 3.7 to 35.6 days. These nearby Cepheids cover distances from about 300 to 560 pc.

The measured periods, P, and average magnitudes in V, K and I bands are given in **Table 1** as well as the A_V and A_K for extinction in V and K bands, respectively. The measured parallaxes with their uncertainties are also given in milliarcsec (mas). All measured apparent magnitudes have negligible uncertainty.

Table 1: Periods and average apparent magnitudes of 5 Galactic Cepheids with accurate parallax measurements.

	P (day)	**<V> (mag)**	**<K> (mag)**	**A_V (mag)**	**A_K (mag)**	**<I> (mag)**	**parallax (mas)**	**error (mas)**
RT Aur	3.728	5.464	3.925	0.20	0.02	4.778	2.40	0.19
FF Aql	4.471	5.372	3.465	0.64	0.08	4.510	2.81	0.18
X Sgr	7.013	4.556	2.557	0.58	0.07	3.661	3.00	0.18
ζ Gem	10.151	3.911	2.097	0.06	0.01	3.085	2.78	0.18
l Car	35.551	3.732	1.071	0.52	0.06	2.557	2.01	0.20

(D1.1) The observed correlation between the period of a Cepheid and its brightness is usually described by the so-called “Period-Luminosity (PL) relation”, where $L \propto P^{\beta}$. In fact, such a relation is normally expressed in terms of the period and absolute magnitude, instead of luminosity. Hereafter, we shall refer to the Period-Absolute magnitude relation as the conventionally named “PL relation”.

Use the data given in Table 1 to plot a suitable linear graph in order to derive the Cepheid PL relation for the V-band and K-band. You should plot each graph separately on different pieces of graph paper. Determine the slope of the line that best describes the linear relation of the data. (You may find the relation $\Delta(\log_{10} x) \approx \dfrac{\Delta x}{x \log_e 10}$ useful)

[36.5 Marks]

Any apparent differences in PL relations of stars in the different bands can be explained if one also considers differences in colour. Therefore, the PL relation is in fact a PLC (Period-Luminosity-Colour) relation. This is from the reddening effect, due to extinction being a function of wavelength, which can also vary among different Cepheids due to their different metallicities, foreground Interstellar Medium and dust.

A new reddening-free magnitude (or bandpass) called "Wesenheit" has been proposed that does not require the explicit information of the extinction of individual stars but uses colour information from the star itself to get rid of the effect. For example, W_{VI} use V and I band photometry and is defined as

$$W_{VI} = V - \left[\frac{A_V}{E(V-I)}\right](V-I),$$

$$= V - R_v(V-I)$$

where R_v depends on the reddening law. In this case, we shall take the value of R_v to be 2.45.

(D1.2) From the data given in Table 1, plot and derive the reddening-free PL relation using Wesenheit W_{VI} magnitudes. Estimate the linear slope of the relation as well as its uncertainty. [14.5 Marks]

(D1.3) Next, we would like to use the newly-derived PL relations from question (D1.1) & (D1.2) to estimate the distance to the Large Magellenic Cloud (LMC) using periods and magnitudes of classical Cepheids in the LMC. In **Table 2**, the periods, average extinction-corrected apparent magnitudes**,** $<V_{corr}>$, and Wesenheit W_{VI} magnitudes are given.

Estimate the distance modulus, μ, to each star and then use all the information to derive the distance to the LMC (in parsecs) and its standard deviation for each band.

Compare if the derived distances are statistically different for the 2 bands (YES/NO).

Are the standard deviations of the estimated distances for 2 bands different (YES/NO)?

Based on this dataset, which band (V or Wesenheit) is more accurate in estimating the distance to the LMC? [24 Marks]

Table 2: Period, average extinction-corrected apparent magnitude**,** $<V_{corr}>$, and average Wesenheit magnitude measurements of Cepheids in the LMC

	P (day)	**$<V_{corr}>$ mag**	**$<W_{VI}>$ mag**
HV12199	2.63	16.08	14.56
HV12203	2.95	15.93	14.40
HV12816	9.10	14.30	12.80
HV899	30.90	13.07	10.97
HV2257	39.36	12.86	10.54

(D2) **The search for dark matter** **[75 marks]**

A low surface brightness galaxy (LSB) is a diffuse galaxy with a surface brightness that, when viewed from the Earth, is at least one magnitude lower than the ambient night sky.

Some of its matter is in the form of "baryonic" matter such as neutral hydrogen gas and stars. However, most of its matter is in the form of invisible mass – so called "dark matter". In this question, we will investigate the mass of dark matter in a galaxy, the effect of dark matter on the rotation curves of the galaxy, and the distribution of dark matter in the galaxy.

The table below provides the data of a LSB galaxy named UGC4325. The galaxy is assumed to be edge-on. At every distance r from the centre of the galaxy, we measure

1. λ_{obs}, the observed wavelength of the Hα emission line. The Hubble expansion of the Universe has already been excluded from the data.

2. V_{gas}, the contribution of the gas component to the rotation due to M_{gas}, derived from HI surface densities.

3. V_*, the contribution of the stellar component to the rotation due to M_*, derived from R-band photometry.

The rotational velocities of the test particle due to the gas component, V_{gas}, and the star component, V_*, are defined as the velocities in the plane of the galaxy that would result from the corresponding components without any external influences. These velocities are calculated from the observed baryonic mass density distributions.

r (kpc)	λ_{obs} (nm)	V_{gas} (km/s)	V_* (km/s)
0.70	656.371	2.87	20.97
1.40	656.431	6.75	32.22
2.09	656.464	14.14	40.91
2.79	656.475	20.18	46.75
3.49	656.478	24.08	50.10
4.89	656.484	28.08	47.94
6.25	656.481	29.25	45.47
7.10	656.481	27.03	47.78
9.03	656.482	25.90	45.32
12.05	656.482	21.03	42.30

The mass of dark matter $M_{\mathrm{DM}}(r)$ within a volume of radius r can be defined in terms of the rotational velocity due to dark matter V_{DM}, the radius r and gravitational constant G,

$$M_{\mathrm{DM}}(r) = \frac{rV_{\mathrm{DM}}^2}{G}. \tag{1}$$

To a good approximation, the observed rotational velocity V_{obs} can be modelled as

$$V_{\mathrm{obs}}^2 = V_{\mathrm{gas}}^2 + V_*^2 + V_{\mathrm{DM}}^2. \tag{2}$$

The observed rotational velocity V_{obs} depends on the mass of the galaxy $M(r)$ within a volume of radius r measured from the galaxy's centre.

The mass density $\rho_{\mathrm{DM}}(r)$ of dark matter within a volume of radius r can be modelled by a galaxy density profile,

$$\rho_{\mathrm{DM}}(r) = \frac{\rho_0}{1+\left(\frac{r}{r_{\mathrm{C}}}\right)^2} \tag{3}$$

where ρ_0 and r_{C} are the central density and the core radius of the galaxy, respectively. According to the density profile, the mass of dark matter $M_{\mathrm{DM}}(r)$ within a volume of a radius r can be described by

$$M_{\mathrm{DM}}(r) = 4\pi\rho_0 r_{\mathrm{C}}^2 \left[r - r_C \arctan(r/r_C)\right]. \tag{4}$$

Part 1 The mass of dark matter and rotation curves of the galaxy

(D2.1) In laboratories on Earth, Hα has an emitted wavelength λ_{emit} of 656.281 nm. Compute the observed rotational velocities of the galaxy V_{obs} and the rotational velocities due to the dark matter V_{DM} at distance r in units of km s^{-1}.

For the different values of r given in the table, compute the dynamical mass $M(r)$ and the mass of dark matter $M_{\mathrm{DM}}(r)$ in solar masses. [21]

(D2.2) Create rotation curves of the galaxy on graph paper by plotting the points of V_{obs}, V_{DM}, V_{gas}, V_* versus the radius r and draw smooth curves through the points (mark your graph as "D2.2").

Order the contribution of the different components to the observed velocity in descending order. [16]

Part 2 Dark matter distribution

(D2.3) Take a data point at small r and large r to estimate ρ_0 and r_C. Note that for large values of x, $\arctan(x) \approx \pi / 2$ and at small x, $\arctan(x) \approx x - x^3 / 3$. [7]

(D2.4) By comparing Equation (4) to a linear function, the central density ρ_0 could also be found by a linear fit. Plot an appropriate graph so that a linear fit can be used to find another value of ρ_0. Evaluate ρ_0 in units of $M_\odot$ kpc^{-3}. (Mark your graph as "D2.4"). If you cannot find the value of r_C from the previous part, use $r_C = 3.2$ kpc as an estimate for this part. [19]

(D2.5) Compute logarithmic values of the dark matter density, $\ln[\rho_{DM}(r)]$, and plot the distribution of dark matter in the galaxy as a function of radius r on graph paper. (Mark your graph as "D2.5"). [12]

Instructions

1. <u>Do not open the exam envelop yourself.</u>
2. This part of the exam involves observation with real sky. You must complete two tasks using the equipment provided.
3. Hand the exam envelope to the proctor at the exam station.
4. You have 6 minutes to complete the first task, and 4 minutes to complete the second task.
5. Once you complete a task, call out to the proctor to have it graded.
6. Once graded, that answer is considered final and you may not return to it again.
7. Once the timer has expired any ungraded task will be graded as is. (So make sure you complete the task before the timer has elapsed)

N1: Observation with Equatorial Mount Telescope

Instruction: Use the Equatorial Mount Telescope to observe the target given in the star chart. Write down the **Name** and make sure the target is **focused**.

Included:

- Equatorial Mount Telescope
- Mount is already polar aligned
- Eyepiece: 25 mm
- Finder scope already aligned with the main scope
- Telescope starts already pointing at "starting star" (see map)
- Star chart with starting and target position marked.

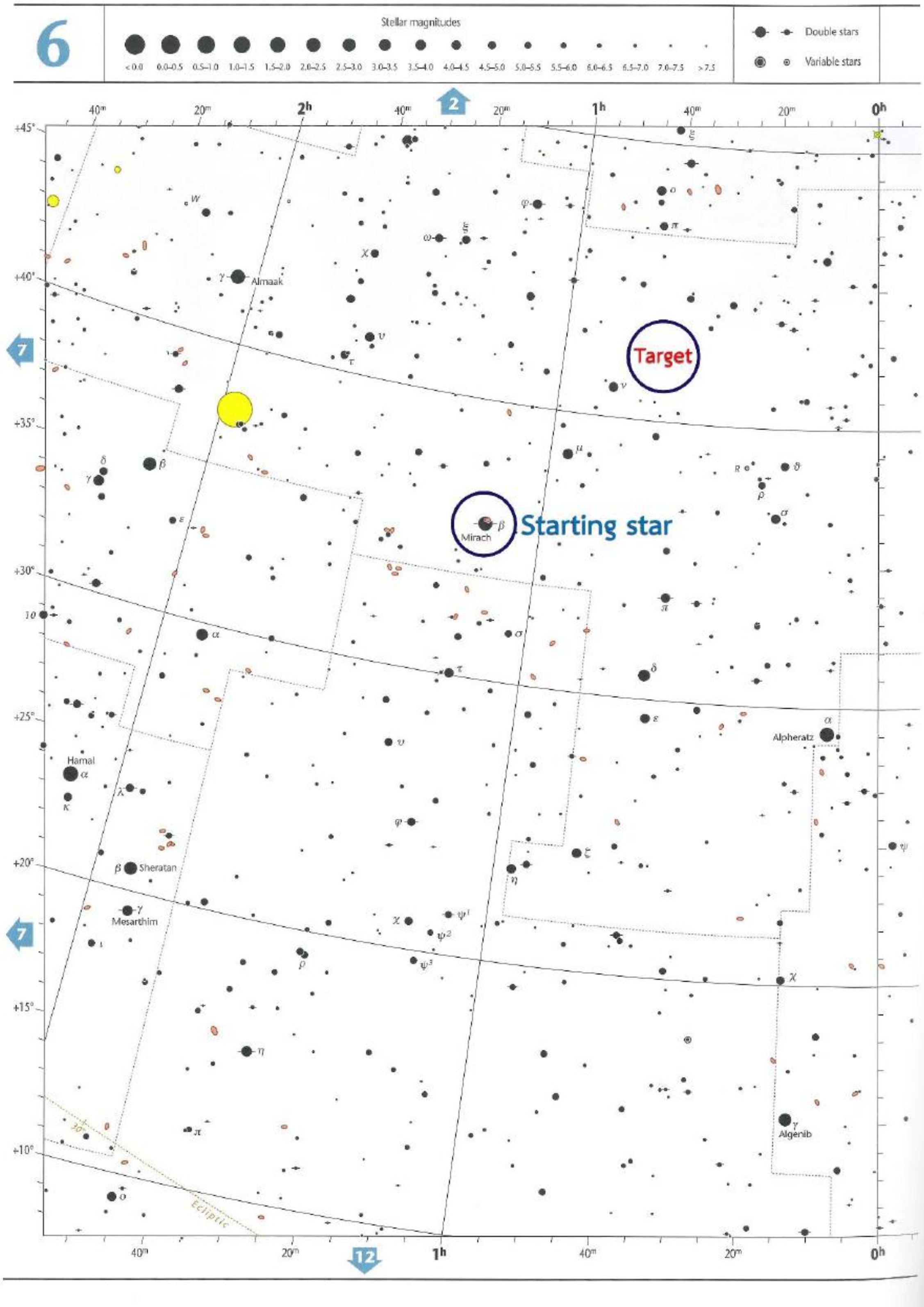
Target name: ____

N2: Observation with Dobsonian Telescope

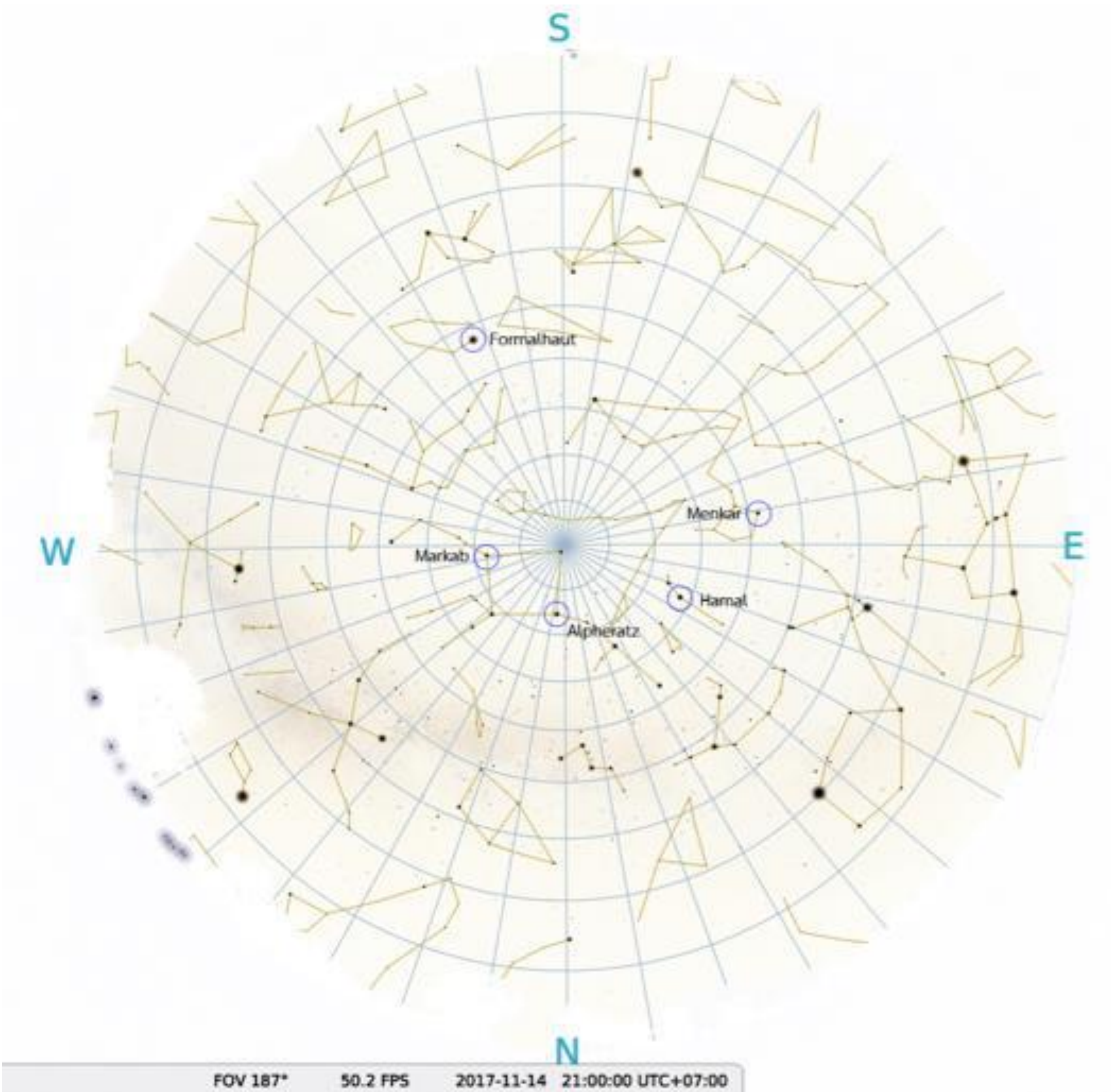
Instruction: Use the Dobsonian Telescope provided to observe **only one** of the following objects and **focus** on selected object.

	Name	Bayer Designation
☐	Menkar	α Cet
☐	Markab	α Peg
☐	Hamal	α Ari
☐	Fomalhaut	α PsA
☐	Alpheratz	α And

Included:
- Dobsonian Telescope
- Eyepiece: 12 and 25 mm
- Finder scope is already aligned with main scope.



Note: To be provided attached to a clipboard at exam station



Note: approximate sky during exam hours (Not provided to the examinees)

Note: provided at exam station

Instructions

1. Do not open the exam envelop yourself.
2. This part of the exam consists of 7* problems located in separate exam stations.
3. You have 5 minutes to complete each problem.
4. After "out of time" is signaled, stop all actions and remain at the same station.
5. After "next station" is signaled, proceed to the next exam station and hand the exam envelope to the station's proctor.
6. Use only blue pen to mark into the answer sheets provided. Any answers written in the answer sheet at the end of timer is considered final and will be graded towards the final score.

* Number of problems will be reduced to 5 if night observation was successful

P1: Naked eye observation from real sky with panoramic 360-degrees image

Instruction: Estimate the LST (Local Sidereal Time) at the time the image was taken, rounded to nearest hour

Included:
- Panoramic 360-degrees image of the sky at night at an unknown location
- Computer screen
- Keypad to pan around the image
- Coordinates of bright stars

Name	Bayer Designation	Declination (Dec)	Right Ascension (RA)
Rigil Kentaurus	**α Cen**	–60° 50′ 02.3737″	14h 39m 36.5s
Arcturus	**α Boo**	+19° 10′ 56″	14h 15m 39.7s
Vega	**α Lyr**	+38° 47′ 01″	18h 36m 56.3s
Capella	**α Aur**	+45° 59′ 53″	05h 16m 41.4s
Altair	**α Aql**	+08° 52′ 06″	19h 50m 47.0s
Aldebaran	**α Tau**	+16° 30′ 33″	04h 35m 55.2s
Antares	**α Sco**	−26° 25′ 55″	16h 29m 24.5s
Spica	**α Vir**	−11° 09′ 41″	13h 25m 11.6s
Deneb	**α Cyg**	+45° 16′ 49″	20h 41m 25.9s
Dubhe	**α UMa**	+61° 45′ 04″	11h 03m 43.7s
Polaris	**α UMi**	+89° 15′ 51″	02h 31m 49.1s
Alpheratz	**α And**	+29° 05′ 26″	00h 08m 23.3s
Schedar	**α Cas**	+56° 32′ 14″	00h 40m 30.4s

LST of the image: ____

Student Code:

P2: Planet observation with real sky in panoramic 360-degrees image

Instruction: Count the number of planets visible in this image above the horizon and name the constellations they're in (with IAU designations).

Included:
- A panoramic 360-degrees image of the sky at night at an unknown location
- Computer screen
- Keypad to pan around the image

Number of Planets visible: ____

List the constellations (with IAU designations, i.e. Ursa Major or UMa):

____

P3: Analemma on another planet

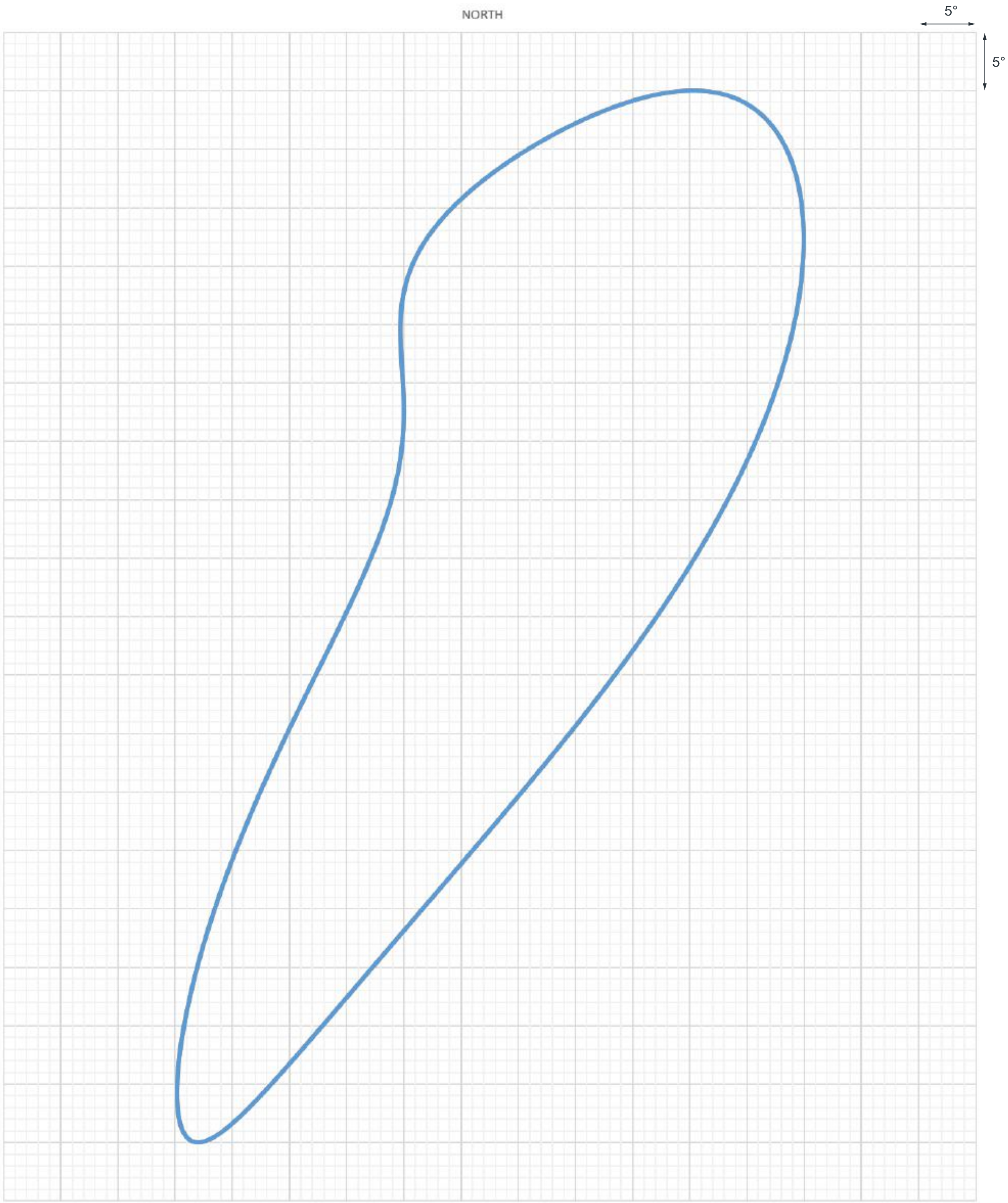
Instruction: Find the Obliquity (Axial Tilt) of the planet

Included:

- A generated analemma (position of a Star taken from the surface of a planet with interval separated by mean solar day of the planet over an orbital period around a Star) of a fictitious planet orbiting around a Star.
- Result is graphed on a paper with each major grid representing 5°

The obliquity of the planet : ______________________________________

Analemma on Planet X



P4: Exposure time from a Photograph

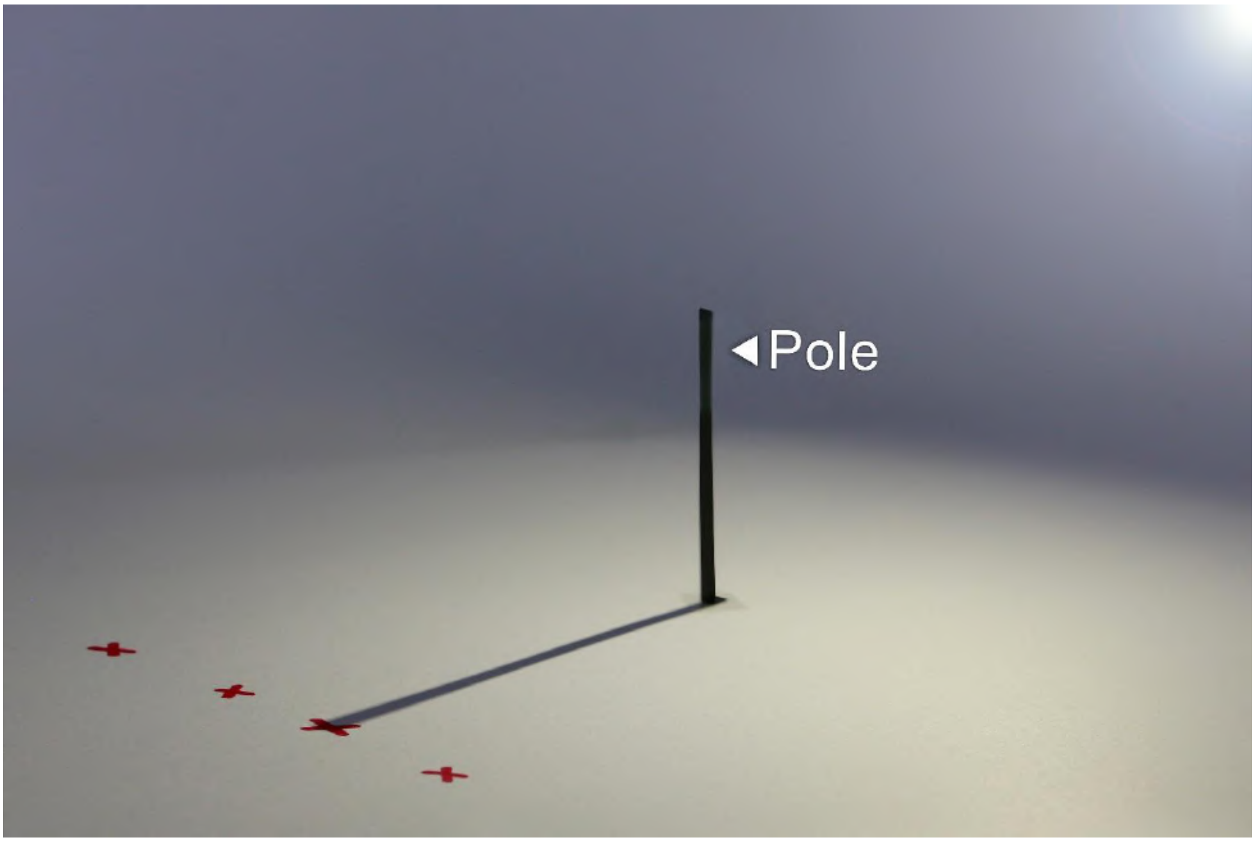
Instruction: Estimate an exposure time of a given "Star Trails" image.

Included:
- A "Star Trails" image that was taken by a still camera capturing image over a period of time.
- Ruler

Exposure time: ____

P5: Find True North from Moon shadow

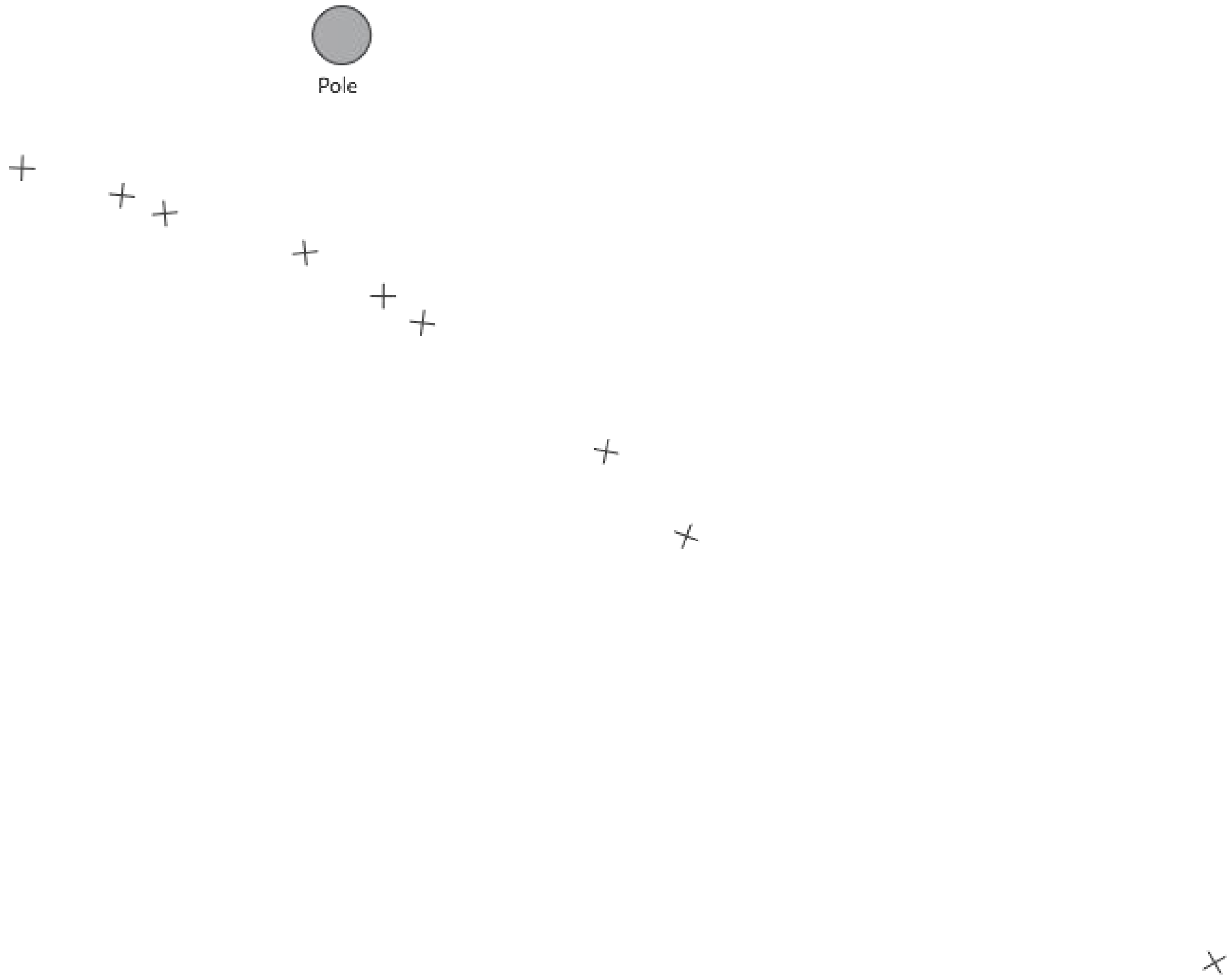
Instruction: Draw an arrow pointing North in the data sheet



Included:

- Simulated position of moon shadows of a pole at certain intervals in the span of a day.
- The observer is located in the Southern hemisphere at latitude 27°S.
- The moon’s declination that night is +15°
- Ruler, Compass (drawing tool), Geometry kit

Data sheet



P6: Find Latitude from Equatorial Mount

Instruction: Without altering the polar alignment, find the observer's latitude based on a previously polar-aligned equatorial mount.

Included:

- An Equatorial Mount Telescope that has already been **properly** aligned to a location in the northern celestial pole.
- Bubble Level.
- Latitude dial on the mount is covered (you may not use it).

Latitude: ____

P7: Precision Polar Alignment with Equatorial Mount

Instruction: Perform a polar alignment on the equatorial mount provided

Included:

- Equatorial Mount with polar scope (has not been polar aligned)
- Date and Time (GMT, UTC+0) of the time performing the polar alignment
- Diagram of the sky's position at the time
- A light source to be substituted with Polaris to be used for proper polar alignment (already visible in the polar scope)
- Longitude of observer

Date and Time : 30 Aug 2017 / 23:30

Longitude : 10 ^{0} E

"Escape"

Instruction

After being shipwrecked (by the theory paper), you are stranded on an unknown island at an unknown location. Luckily, behind the door is a satellite radio that you can use to escape the island. You must use any information available to you to find the correct combination to unlock the door.

Bolted on the door are three combination locks. Enter the correct 4-digit combination and the lock will open.

The information required on the three locks are:

A. The **Latitude** of this island (in "00LL" format, 0000 - 0090)
B. The **Longitude** of this island (in "0LLL" format, 0000 - 0180)
C. The **Month** of the observed sky above you (in "00MM" format, 0001 - 0012)

In order to solve this, the sky above you is simulated from the location of the island, but with time passing faster than real time. The *only* tools you are given are:

- An equatorial sky chart
- A clock showing the current time in GMT (remember, the time is sped up)
- Pencil and paper

The answers, in this case, are literally written in the sky.

You have 2 nights to find an escape. The game will end upon the second dawn.

Your team performance will be rated based upon the time you take to escape the room. **Each wrong combination input will add 1 minute to your total time**. The first two wrong combinations on the latitude and longitude will not be penalized.

Good Luck – you're going to need it!

IOAA 2018 – Beijing, China

12th International Olympiad on
Astronomy and Astrophysics

The 12th IOAA was held from 03rd to 11th November 2018. Total of 46 countries participated in the event.

The 12th IOAA logo is designed in a relax way. Panda is an identifiable icon of China. The head of panda also represents the letter 'O'. Bamboos represent a China's spirit of tenacious. They make up the letters 'I' and 'A's.

Instructions

1. The theoretical competition will be 5 hours in duration and is marked out of a total of 300 points.

2. There are **Answer Sheets** for carrying out detailed work/rough work. On each **Answer Sheet**, please fill in

- Student Code
- Question No.
- Page no. and total number of pages.

3. Start each problem on a separate Answer Sheet. Please write only on the printed side of the sheet. Do not use the reverse side. If you have written something on any sheet which you do not want to be marked, cross it out.

4. Please remember that the graders may not understand your language. As far as possible, write your solutions only using mathematical expressions and numbers. If it is necessary to explain something in words, please use short phrases (if possible in English).

5. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, etc.), please put up your hand to signal the proctor.

6. The beginning and end of the competition will be indicated by a long sound signal. Additionally, there will be a short sound signal fifteen minutes before the end of the competition (before the final long sound signal).

7. At the end of the competition you must stop writing immediately. Sort and put your sheets in

separate stacks,

(a) Stack 1: answer sheets of part 1
(b) Stack 2: answer sheets of part 2
(c) Stack 3: answer sheets of part 3
(d) Stack 4: question papers and paper sheets that you do not want to be graded.

8. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition room.

9. A list of constants for this competition is given on the next page.

Point distribution of this exam

Problem number	**Points**
T1	10
T2	10
T3	10
T4	10
T5	10
T6	25
T7	25
T8	25
T9	25
T10	75
T11	75
Total	300

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity (g)	$9.81\ \mathrm{ms}^{-2}$	
Obliquity of Ecliptic	23°27′	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass ($M_☾$)	7.35×10^{22} kg	**Moon**
Radius ($R_☾$)	1.74×10^{6} m	
Mean Earth-Moon distance	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	−12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	**Sun**
Radius ($R_\odot$)	6.96×10^{8} m	
Luminosity ($L_\odot$)	3.83×10^{26} W	
Absolute Magnitude	4.80 mag	
Surface Temperature	5772 K	
Angular diameter at Earth	30′	
Orbital velocity in Galaxy	$220\ \mathrm{kms}^{-1}$	
Distance from Galactic center	8.5 kpc	
1 *au*	1.50×10^{11} m	**Physical constants**
1 *pc*	206265 au	
Gravitational constant (G)	$6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$	
Planck constant (h)	6.62×10^{-34} Js	
Boltzmann constant (k_B)	$1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$	
Stefan-Boltzmann constant (σ)	$5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	
Hubble constant (H_0)	$67.8\ \mathrm{kms^{-1}Mpc^{-1}}$	
Speed of light in vacuum (c)	$299792458\ \mathrm{ms}^{-1}$	
Magnetic Permeability of free space (μ_0)	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$	
1 Jansky (Jy)	$10^{-26}\ \mathrm{W\ m^{-2}Hz^{-1}}$	

Rayleigh-Jeans law is given by $B_\nu = \frac{2k_BT}{c^2}\nu^2$, which is the power emitted per unit emitting area, per steradian, per unit frequency.

(T1) Super Luminal Galaxies **(10 points)**

Read the statements given below and state if they are true or false:

(a) For some galaxies the apparent recession speed exceeds the speed of light.

(b) The velocity – Distance relation as given by Hubble cannot allow recession velocities to exceed the speed of light.

(c) Hubble-Lemaitre's law (formerly known as Hubble's Law) does not violate special relativity.

(d) If some galaxies would have an apparent recession speed exceeding the speed of light, then the photons from those galaxies can never reach us.

(e) As the expansion of Universe is accelerating, photons emitted right now from galaxies which have apparent recession speed equal to the speed of light will never reach us.

(T2) Distance **(10 points)**

An observer measured trigonometric parallaxes of stars in a star cluster. Due to random errors, the measured parallax values are distributed symmetrically around the expected value with standard deviation equal to 0.05 mas (milliarcsec). Assume there are no systematic errors and assume all stars in the said cluster have the same luminosity. It is known that the distance of this cluster from us is R= 5 kpc.

He gave the data table to 4 of his students (A, B, C and D) and they estimated the distance to the cluster in the following ways:

A. Convert each parallax measurement into distance and then find the average distance (R_A)
B. Take the average of all parallaxes first and then convert the average parallax into distance. (R_B)

C. Convert each parallax measurement into distance and then take the median distance value. (R_C)

D. Find the median value of the parallaxes and then convert the median value into distance. (R_D)

State if the following statements are true or false. **In case a given mathematical relation is false, give the correct relation.**

(l) If the i^{th} star gave the smallest value of parallax and the j^{th} star gave the highest value of parallax, in all likelihood R_i-R > R-R_j

(m) R_A = R (i.e. there is a high chance that the distance estimated by A fairly matches the true distance)

(n) R_B = R (i.e. there is a high chance that the distance estimated by B fairly matches the true distance)

(o) R_C < R (i.e. there is a high chance that the distance estimated by C will be systematically lower than the true distance)

(p) R_D = R (i.e. there is a high chance that the distance estimated by D fairly matches the true distance)

(T3) Atmospheric Refraction **(10 points)**

Consider sunrise at Beijing (φ=40°) on the vernal equinox day.

(a) Let us say r_l, r_d, r_r and r_u are distances from the centre of the undistorted disk of the Sun to the edge of the disk towards the directions left, down, right and up respectively. What will be the hierarchical relation (<, =, >) between the four radii just after the sunrise?

(b) What is the correction in the time of rise of the top edge of the disk as compared to the case without atmosphere? You may assume that typically atmospheric refraction near the horizon is 35'. Please only consider the apparent diurnal motion.

(T4) Height of a Hill **(10 points)**

Two friends wanted to measure the height of the hill next to their village (latitude φ=40°). One of the friends climbed to the top of the hill and she agreed to send a light signal to her friend in the village as soon as she sees the sunset. On March 21, when they did this experiment, the friend in village received the light signal 4.1 minutes after the sunset from the village. Estimate the height of the hill and horizon distance for the person at the hill top. Ignore atmospheric refraction.

(T5) Sidereal Time **(10 points)**

It is very interesting to observe that on one particular calendar day each year, the mean sidereal time will twice be 00:00:00.

(a) What will be the approximate R.A. of the Sun when this event happens?

(b) Estimate the exact date in 2018 for this event.

You may assume that at the Royal Greenwich Observatory, the mean sidereal time ($GMST_0$) was 6.706h at 0h, 1st January, 2018 (JD2458119.5).

(T6) Observe the Sun with FAST **(25 points)**

The Five-hundred-meter Aperture Spherical radio Telescope (FAST) is a single-dish radio telescope located in Guizhou Province, China. The physical diameter of the dish is 500 m, but during observations, the effective diameter of the collecting area is 300 m.

Consider observations of the thermal radio emission from the photosphere of the Sun at 3.0 GHz with this telescope and a receiver with bandwidth 0.3 GHz.

(a) Calculate the total energy ($E_\odot$) that the receiver will collect during 1 hour of observation.
(b) Estimate the energy needed to turn over one page of your answer sheet (E'). Hint: the typical surface density of paper is 80 gm^{-2}.
(c) Which one is larger?

(T7) Sunspot **(25 points)**

Magnetic fields are important in the physics of stars and sunspots. As an approximation, we can model the photosphere of the Sun consisting of a plasma, which can be simply treated as a single component ideal gas, and a magnetic field (**B**), which has an associated magnetic pressure $p_B = \frac{B^2}{2\mu_0}$. It behaves like any other physical pressure except that it is carried by the magnetic field rather than by the kinetic energy of particles.

Assume that the number density of particles in the photosphere is constant everywhere, but the magnetic field inside the sunspot (B_{in}=0.1T) is much stronger than outside ($B_{out}=5\times10^{-3}$T). From the blackbody spectrum, the temperature inside the sunspot is T_{in}~4000K, while the temperature outside is T_{out}~6000K (which is why the sunspot looks darker). For the sunspot to be stable, the inside must be in equilibrium with the outside.

(a) Estimate the number density of plasma particles in the solar photosphere.

(b) Compare your answer with an estimate of the number density of particles in the atmosphere at the surface of the Earth.

(T8) A Possible Dark Matter Deficient Galaxy **(25 points)**

Earlier this year, a team of astronomers reported their discovery of a galaxy with much less dark matter than the galaxy evolution model predicted (van Dokkum et al. 2018, Nature). This galaxy, named NGC 1052-DF2, is located close to the elliptical galaxy NGC 1052 (D=20Mpc from the Sun) in the sky. The shape of NGC 1052-DF2 resembles an ellipse with semi major axis (a) of 22.6" and $\frac{b}{a} = 0.85$. Half of the total light from the galaxy comes from within this ellipse and the mean surface brightness within the ellipse is about 24.7 mag arcsec^{-2}.

(a) Calculate the total apparent magnitude of this galaxy.

(b) The team suggested the galaxy is a companion of NGC 1052. Determine the total mass of stars in NGC 1052-DF2, assuming it has a mass to light ratio $\left(\frac{M/M_\odot}{L/L_\odot}\right)$ of 2.0.

(c) The team identified 10 globular clusters in NGC 1052-DF2 with a mean galactocentric distance of 78.4". They also measured the velocity dispersion of these clusters to be not more than 8.4 km/s. Estimate the dynamical mass of this galaxy. For simplicity, assume the mass distribution in the galaxies is uniform and is spherically symmetric.

(d) This discovery was challenged by other groups (Kroupa et al., Nature, 2018, Truijlo et al., MNRAS, 2018), who claimed that NGC 1052-DF2 is not a satellite of NGC 1052, and it is located at a much smaller distance to us. Show why a smaller distance would weaken the assertion of the dark matter deficiency in NGC 1052-DF2.

(T9) Radio Galaxy **(25 points)**

An observer wants to use the Five-hundred-meter Aperture Spherical radio Telescope (FAST) in China to observe a radio galaxy at redshift of $z = 0.06$. We assume that the radio source is compact compared to the beam size of the telescope at the observing frequencies, i.e., the source is point-like as seen through the telescope. To detect a point source with FAST, it must be sufficiently strong (bright) relative to the noise level (for single polarization observations), σ, which depends on the bandwidth, $\Delta\nu$, and the integration time (the radio astronomy equivalent of exposure time), t_i, as follows:

$$\sigma = \frac{2k_B T_{sys}}{A_e\sqrt{t_i \Delta\nu}}$$

where T_{sys} is the system temperature (about 150 K in the frequency range of 0.28 GHz – 0.56 GHz and 25 K in the frequency range of 1.05 GHz – 1.45 GHz), and $A_e = 4.6\text{x}10^4$ m^2 is the effective area of the telescope taking into account the total efficiency of the instrument.

This radio galaxy has an observed continuum flux density of $f_\nu = 2.5 \times 10^{-3} Jy$ at an observing frequency of 0.4 GHz. The bandwidth $\Delta\nu$ for the continuum observation centered at 0.4 GHz is 2.8×10^8 Hz.

(a) In order to detect the continuum flux density at 0.4 GHz with a signal-to-noise ratio of 30 (a so-called 30σ detection), what is the required integration time, t_i?

(b) We want to search for the neutral Hydrogen (HI) in the galaxy using 21cm absorption line. The HI 21cm line, with rest frame frequency of 1.4204 GHz. Calculate the observed frequency (υ_{obs}) of the HI line for this galaxy.

(c) The radio continuum emission from this galaxy can be described by a power law $f_\nu \sim \nu^\alpha$, with a spectral index of $\alpha = -0.2$. Calculate the continuum flux density at ν_{obs} for this galaxy.

(d) The line width of the HI 21cm absorption line is 90 km/s. Calculate the line width in Hz at the observing frequency of ν_{obs}. According to Figure 1, the HI 21cm line absorbs 4% of the continuum flux density (on average) over the line width of 90 kms^{-1}. In order to detect the absorption line at ≥3σ in three consecutive 30 kms^{-1} channels, what is the required integration time?

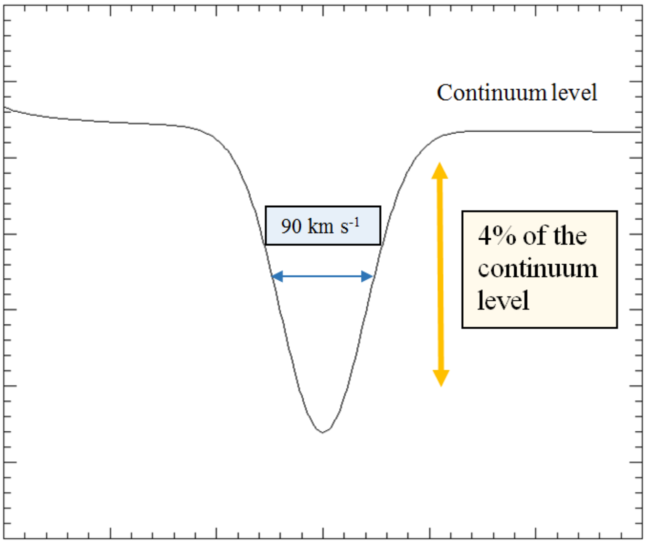


Figure 1: Spectrum of the HI 21cm absorption relative to the continuum emission in the radio galaxy

(T10) Vega and Altair **(75 points)**

As per a very famous Chinese folklore about love, Vega and Altair are two lovers. It is said that they can meet each other once every year on a bridge made up of birds over the Milky Way. The parameters of two stars are given in the table below. For the purpose of this question, assume that the coordinate frame is fixed (i.e. not affected by precession or motion of the Sun).

Star	Right Ascension (J2000.0)	Declination (J2000.0)	Parallax (mas)	Proper Motion		Radial Velocity (km/s)
				$\mu_\alpha \cos\delta$ (mas/year)	μ_δ (mas/year)	
Vega	$18^h36^m56.49^s$	$+38^o\ 47'\ 07.7''$	130.23	+200.94	+286.23	-13.9
Altair	$19^h50^m47.70^s$	$+8^o\ 52'\ 13.3''$	194.95	+536.23	+385.29	-26.1

Based on this data, answer the following questions：
(a) (9 points) What is the angular separation of the two stars?
(b) (6 points) Calculate the distance (in parsecs) between Vega and Altair.
(c) (3 points) Calculate position angles of the proper motion vectors of each of these two stars.

For parts d-g, assume that the angular velocity of the stars on the celestial sphere remains constant. This is not a physical situation but this is an assumption to simplify the problem.
(d) (2 points) How many common points on the celestial sphere are there which can be reached by both these stars?
(e) (20 points) Find the coordinates of the closest such point.
(Note: Drawing the situation on a celestial sphere will help you in visualising the situation)
(f) (8 points) Find when (which year) each of these stars were / will be at that point.
(g) (5 points) When Altair was / will be at that point, what would be its angular separation from Vega?
(h) (22 points) Find coordinates of any point (if it exists) in 3-D space which was /will be visited by both these stars. Do not ignore radial velocities for this part of the question.

(T11) Thermal History of the Universe **(75 points)**

Based on Einstein's general relativity, Russian physicist Alexander Friedmann derived the Friedmann Equation by which the dynamics of a homogeneous and isotropic universe can be well described. The Friedmann Equation is usually written as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}.$$

We define the Hubble parameter as $= \frac{\dot{a}}{a}$, where a is the scale factor and $\dot{a}$ is the rate of change of scale factor with time. Thus, the Hubble parameter is a function of cosmic time. In the Friedmann Equation, ρ_m is the density of matter, including dark matter and baryons, ρ_r is the density of radiation, Λ is the cosmological constant, and k is the curvature of space. Subscript 0 indicates the value of a physical quantity at present day, e.g. H_0 is the present value Hubble parameter. Also, to avoid confusion with the reduced Hubble parameter, we use the reduced Planck Constant $\hbar = h/(2\pi)$ instead of the Planck constant h.

(a) (5 points) What are the dimensions of Hubble parameter? One can define a characteristic timescale for the expansion of the Universe (i.e. Hubble time t_H) using the Hubble parameter. Calculate the present-day Hubble time t_{H0}.

(b) (5 points) Let us define the critical density ρ_c as the matter density required to explain the expansion of a flat universe without any radiation or dark energy. Find an expression of the critical density, in terms *H* and *G*. Calculate the present critical density ρ_{c0}.

(c) (6 points) It is convenient to define all density parameters in a dimensionless manner like $\Omega_i = \frac{\rho_i}{\rho_c}$, i.e. the ratio of density to critical density. The Friedmann Equation can be rewritten using these dimensionless density parameters simply as, $\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k = 1$.

Use this information to find expression for Ω_Λ and Ω_k, in terms *H, c, Λ, k* and *a*.

(d) (7 points) Another equation which is valid for matter, radiation and dark energy is often called the Fluid Equation: $\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$, where p is the pressure of some component, ρ is the density and $\dot{\rho}$ is the rate of change of density over time. Radiation contains photons and massless neutrinos, and they both travel at the speed of light. The pressure exerted by these particles is 1/3 of their energy density. Show that the density of radiation $\rho_r \propto (1+z)^4$, where z is cosmological redshift. You may note that if $\frac{\dot{\rho}}{\rho} = n\,\frac{\dot{a}}{a}$, then $\rho \propto a^n$

(e) (4 points) We know that the value of the cosmological constant Λ doesn't evolve. Its equation of state has a form $p = w\rho_\Lambda c^2$, where *w* is an integer. Find the value of *w*.

(f) (13 points) Planck time, defines a characteristic timescale before which our present physical laws are no longer valid, and where quantum gravity is needed. The expression for Planck time can be written in terms of $\hbar$, G and c and non-dimensional coefficient of this expression in SI units is of the order of unity. Using dimensional analysis, find expression for Planck time and estimate its value.

(g) (7 points) Planck length defines the length scale associated with Planck time is given by $l_P = ct_P$. The minimal mass of a black hole, also called Planck mass, is defined as the mass of a black hole whose Schwarzschild radius is two times the Planck length.

Derive the Planck mass M_P and calculate $M_P c^2$ in GeV. This mass is considered to be an upper threshold for elementary particles, beyond which they will collapse to a black hole.

(h) (4 points) At the very beginning (soon after the Planck time), all the particles were in thermal equilibrium in a primordial soup. As temperature decreased, different particles then decoupled from the primordial soup one by one and could travel freely in the Universe. Photons decoupled at ~300000 years after the Big Bang. These photons emitted at that time are what constitutes the cosmic microwave background (CMB), which follows the Stefan-Boltzmann law for blackbody radiation.

$$\varepsilon_r = \frac{\pi^2}{15\hbar^3 c^3}(k_B T)^4,$$

Show that the temperature of the CMB follows $T/(1+z) = constant$.

(i) (16 points) With the expansion of the Universe, radiation density dropped more quickly than matter density, and at some epoch the matter density was equal to the radiation density. Radiation contains both photons and neutrinos. Apart from photons, neutrinos additionally contribute to the radiation energy density by 68% (i.e. $\Omega_{r0} = 1.68\Omega_{\gamma 0}$, where γ indicates photons). Estimate the redshift of matter-radiation equality z_{eq} in terms of Ω_{m0} and reduced Hubble parameter $h = \frac{H_0}{100km\, s^{-1} Mpc^{-1}}$ You may use the current temperature of the CMB: $T_0 = 2.73K$.

(j) (8 points) The neutrinos decoupled from the primordial soup when the temperature of the universe was around 1 MeV. At this time, the radiation density in the universe was much more than all other components. Estimate the time ($t = \frac{1}{2H}$) when neutrinos decoupled, and express it in seconds since the big bang.

Instructions

1. The data analysis competition lasts for 5 hours and is worth a total of 150 points.

2. Dedicated IOAA **Summary Answer Sheets** are provided for writing your answers. Enter the final answers into the appropriate boxes in the corresponding **Summary Answer Sheet**. On each Answer Sheet, please fill in

- Student's Code

3. **Graph Paper** is required for your solutions. On each Graph Paper sheet, please fill in

- Student's Code
- Question no.
- Graph no. and total number of graph paper sheets used.

4. There are **Answer Sheets** for carrying out detailed work/rough work. On each Answer Sheet, please fill in

- Student's Code
- Question no.
- Page no. and total number of pages.

5. Start each problem on a separate Answer Sheet. **Please write only on the printed side of the sheet. Do not use the reverse side.** If you have written something on any sheet which you do not want to be marked, cross it out.

6. Use as many mathematical expressions as you think may help the graders to better understand your solutions. The graders may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible in English).

7. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, need more Answer Sheets, Graph Paper etc.), please put up your hand to signal the invigilator.

8. The beginning and end of the competition will be indicated by a long sound signal. Additionally, there will be a short sound signal fifteen minutes before the end of the competition (before the final long sound signal).

9. At the end of the competition you must stop writing immediately. Sort and put your **Summary Answer Sheets**, **Graph Papers**, and **Answer Sheets in one stack**. Put all other papers in another stack. You are not allowed to take any sheet of paper out of the examination area.

10. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition room.

11. A list of constants is given on the next page.

Table of constants

Mass ($M_\oplus$)	5.98×10^{24} kg	**Earth**
Radius ($R_\oplus$)	6.38×10^{6} m	
Acceleration of gravity (g)	$9.81\ \mathrm{ms^{-2}}$	
Obliquity of Ecliptic	23°27’	
Length of Tropical Year	365.2422 mean solar days	
Length of Sidereal Year	365.2564 mean solar days	
Albedo	0.39	
Mass ($M_☾$)	7.35×10^{22} kg	**Moon**
Radius ($R_☾$)	1.74×10^{6} m	
Mean Earth-Moon distance	3.84×10^{8} m	
Orbital inclination with the Ecliptic	5.14°	
Albedo	0.14	
Apparent magnitude (mean full moon)	−12.74	
Mass ($M_\odot$)	1.99×10^{30} kg	**Sun**
Radius ($R_\odot$)	6.96×10^{8} m	
Luminosity ($L_\odot$)	3.83×10^{26} W	
Absolute Magnitude	4.80 mag	
Surface Temperature	5772 K	
Angular diameter at Earth	30′	
Orbital velocity in Galaxy	$220\ \mathrm{kms^{-1}}$	
Distance from Galactic center	8.5 kpc	
1 *au*	1.50×10^{11} m	**Physical constants**
1 *pc*	206265 au	
Gravitational constant (G)	$6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$	
Planck constant (h)	6.62×10^{-34} Js	
Boltzmann constant (k_B)	$1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$	
Stefan-Boltzmann constant (σ)	$5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	
Hubble constant (H_0)	$67.8\ \mathrm{kms^{-1}Mpc^{-1}}$	
Speed of light in vacuum (c)	$299792458\ \mathrm{ms^{-1}}$	
Magnetic Permeability of free space (μ_0)	$4\pi\times10^{-7}\ \mathrm{Hm^{-1}}$	
1 Jansky (Jy)	$10^{-26}\ \mathrm{W\ m^{-2}Hz^{-1}}$	

$$\Delta \log_{10}(x) = \frac{\Delta x}{x\ ln10}$$

(D1) Dust and Young Stars in Star-forming Galaxies **[75 points]**

As a by-product of the star-forming process in a galaxy, interstellar dust can significantly absorb stellar light in ultraviolet (UV) and optical bands, and then re-emit in far-infrared (FIR), which corresponds to a wavelength range of 10-300 μm.

1.1. In the UV spectrum of a galaxy, the major contribution is from the light of the young stellar population generated in recent star-formation processes, thus the UV luminosity can act as a reliable tracer of the star-formation rate (SFR) of a galaxy. Since the observed UV luminosity is strongly affected by dust attenuation, extragalactic astronomers define an index called the *UV continuum slope* (β) to quantify the shape of the UV continuum:

$$f_\lambda = Q \cdot \lambda^\beta$$

where f_λ is the monochromatic flux of the galaxy at a given wavelength λ (in the unit of W m^{-3}) and Q is a scaling constant.

(D1.1.1) (6 points) AB magnitude is a specific magnitude system. The AB magnitude is defined as:

$$\mathrm{m_{AB}} = -2.5\log\frac{f_\nu}{3631\ \mathrm{Jy}}$$

The AB magnitude of a typical galaxy is roughly constant in the UV band. What is the **UV continuum slope** of this kind of galaxy? (Hint: $f_\nu \Delta\nu = f_\lambda \Delta\lambda$)

(D1.1.2) (12 points) Table 1 presents the observed IR photometry results for a $z = 6.60$ galaxy called *CR7*. **Plot** the AB magnitude of *CR7* **versus the logarithm of the rest-frame** wavelength on graph paper and labelled as **Figure 1**.

(D1.1.3) (5 points) Calculate *CR7*'s UV slope, **plot** the best-fit UV continuum on *Figure 1* and make a comparison with the results you obtained in (D1.1.1). Is it dustier than the typical galaxy in (D1.1.1)? **Please answer with [YES] or [NO]**. (Hint: Express $\mathrm{m_{AB}}$ as a function of λ and m_{1600} , where m_{1600} is the AB magnitude at $\lambda_0 = 160$ nm (1600 Å))

Table 1. (Observed Frame) IR Photometry of CR7 at z = 6.60

Band	*Y*	*J*	*H*	*K*
Central Wavelength (μm)	1.05	1.25	1.65	2.15
AB Magnitude	24.71 ± 0.11	24.63 ± 0.13	25.08 ± 0.14	25.15 ±0.15

1.2. Under the assumption that dust grains in the galaxy absorb the energy of UV photons and re-emit it by blackbody radiation, the relation between the UV continuum slope (β), UV brightness (at 1600 Å) and FIR brightness could be established:

$$\mathrm{IRX} \equiv \log\left(\frac{F_{FIR}}{F_{1600}}\right) = S(\beta)$$

where F_{FIR} is the observed far-infrared flux and F_{1600} is the observed flux at rest-frame wavelength 160 nm (1600 Å) (The "flux" F_λ is **defined** as $F_\lambda = \lambda \cdot f_\lambda$). Table 2 presents 20 measurements of β, F_{FIR} and F_{1600} in nearby galaxies (Meurer et al. 1999).

Table 2. UV slope, flux and FIR flux of 20 nearby galaxies

Galaxy Name	UV Slope β	$\log(F_{1600}/10^{-3}\mathrm{Wm^{-2}})$	$\log(F_{FIR}/10^{-3}\mathrm{Wm^{-2}})$
NGC4861	-2.46	-9.89	-9.97
Mrk 153	-2.41	-10.37	-10.92
Tol 1924-416	-2.12	-10.05	-10.17
UGC 9560	-2.02	-10.38	-10.41
NGC 3991	-1.91	-10.14	-9.80
Mrk 357	-1.80	-10.58	-10.37
Mrk 36	-1.72	-10.68	-10.94
NGC 4670	-1.65	-10.02	-9.85
NGC 3125	-1.49	-10.19	-9.64
UGC 3838	-1.41	-10.81	-10.55
NGC 7250	-1.33	-10.23	-9.77
NGC 7714	-1.23	-10.16	-9.32
NGC 3049	-1.14	-10.69	-9.84
NGC 3310	-1.05	-9.84	-8.83
NGC 2782	-0.90	-10.50	-9.33
NGC 1614	-0.76	-10.91	-8.84
NGC 6052	-0.72	-10.62	-9.48
NGC 3504	-0.56	-10.41	-8.96
NGC 4194	-0.26	-10.62	-8.99
NGC 3256	0.16	-10.32	-8.44

(D1.2.1) (14 points) Based on the data given in Table 2, **plot** the IRX $-\ \beta$ diagram on graph paper and labelled as **Figure 2** and find a linear fit to the data. **Write down** your best-fit equation (i.e. $\mathrm{IRX} = a \cdot \beta + b$) by the side of your diagram.

(D1.2.2) (6 points) Quantify the **dispersion** (in 'units' of dex, where **for example,** $\mathbf{\log(10^9) - \log(10^4) = 5\ dex}$) between the observed $\mathrm{IRX_{obs}}$ and predicted $\mathrm{IRX_{pred}}$ using the following equation:

$$\sigma = \sqrt{\frac{\sum(\Delta \mathrm{IRX}_i)^2}{N-1}}\ (\mathrm{unit: dex})\ where\ \Delta\mathrm{IRX}_i = \mathrm{IRX}_{i,\mathrm{obs}} - \mathrm{IRX}_{i,\mathrm{pred}}$$

1.3. Under the previous assumption of the energy transfer process, the ratio of F_{FIR} to F_{1600} can be expressed as:

$$\frac{F_{FIR}}{F_{1600}} \approx 10^{0.4A_{1600}} - 1$$

Where F_{1600} is the unattenuated flux and A_λ is the dust absorption (in magnitudes) as a function of wavelength λ.

(D1.3.1) (6 points) Express A_{1600} as a function of IRX.

(D1.3.2) (12 points) Based on Table 2 data and the function of $A_{1600}(IRX)$ you derived above, **plot** the $\mathrm{A}_{1600}-\beta$ diagram on graph paper and label it as **Figure 3** and find a linear fit to the data. **Write down** your best-fit equation (i.e.$\mathrm{A}_{1600}=a'\cdot\beta+b'$) by the side of your diagram.

(D1.3.3) (2 points) If your linear model in (D1.3.2) is correct, what is the expected **UV continuum slope** β_0 of a dust-free galaxy?

1.4. After establishing the local relation between UV continuum slope and IRX, we could probably test this empirical law in the high-redshift universe. In 2016, researchers obtained an Atacama Large Millimeter / submillimeter Array (ALMA) observation of CR7, and the FIR continuum corresponded to a 3σ upper limit of an FIR flux of 1.5×10^{-19} W m^{-2}.

(D1.4.1) (6 points) Calculate the **IRX of *CR7***. Is it an upper limit or lower limit?

Hint: here F_{1600} should be written in the form of:

$$F_{1600}=\lambda_0\cdot f_{1600}$$

where $\lambda_0=160$ nm (1600 Å) *and* f_{1600} *is the observed flux in the rest-frame*

(D1.4.2) (6 points) Is the current observation long enough to show any deviation of *CR7* from the IRX$-\beta$ relationship you just derived in the local universe? **Please answer with [YES] or [NO] on the summary answer sheet, give the IRX difference and show the working used to calculate it on the answer sheet**.

(D2) Compact Object in a Binary System **[75 points]**

Astronomers discovered an extraordinary binary system in the constellation of Auriga during the course of the Apache Point Observatory Galactic Evolution Experiment (APOGEE). In these questions, you will try to analyse the data and recreate their discovery for yourself.

The research team is aiming to find compact stars in binary systems using the radial velocity (RV) technique. They examined archival APOGEE spectra of "single" stars and measured the apparent variation of their RV within this data. Among ~200 stars with the highest accelerations, researchers searched for periodic photometric variations in data from the All-Sky Automated Survey for Supernovae (ASAS-SN) that might be indicative of transits, ellipsoidal variations or starspots. After this process, they spotted a star named 2M05215658+4359220, with a large variation in RV and photometric variability.

2.1. The following table presents the radial velocity measurements of 2M05215658+4359220 during three epochs of APOGEE spectroscopic observation. Here we assume the variation of its RV is due to the existence of an unseen companion. The proper motion of the stars can be ignored.

Table 3. APOGEE Radial Velocity Measurements of 2M05215658+4359220

Observation	MJD	RV (km s^{-1})	Uncertainty (km s^{-1})
1	56204.9537	-37.417	0.011
2	56229.9213	34.846	0.010
3	56233.8732	42.567	0.010

(D2.1.1) (6 points) Use the data and a simple linear model to obtain an initial estimate of the apparent **maximum acceleration** of the star**:**

$$a_{max} = \left.\frac{\Delta RV}{\Delta t}\right|_{max}, \text{ unit: km s}^{-1}\text{ day}^{-1}$$

(D2.1.2) (9 points) Now use the data to obtain an initial estimate of the **mass** of its unseen companion.

2.2. After discovering this peculiar star, astronomers conducted follow-up observations using the 1.5-m Tillinghast Reflector Echelle Spectrograph (TRES) at the Fred Lawrence Whipple Observatory (FLWO) located on Mt. Hopkins in Arizona, USA. The following table presents the RV measurements using this instrument:

Table 4. TRES Radial Velocity Measurements of 2M05215658+4359220

MJD	RV (km/s)	Uncertainty (km/s)
58006.9760	0	0.075
58023.9823	-43.313	0.075
58039.9004	-27.963	0.045
58051.9851	10.928	0.118
58070.9964	43.782	0.075
58099.8073	-30.033	0.054
58106.9178	-42.872	0.135
58112.8188	-44.863	0.088
58123.7971	-25.810	0.115
58136.6004	15.691	0.146
58143.7844	34.281	0.087

(D2.2.1) (14 points) **Plot** the diagram of RV variation (measured with TRES) versus time on your graph paper and label it as Figure 4. Draw a suitable **sinusoidal function** to fit the given data. **Estimate** the orbital period (P_{orb}) and radial velocity semi-amplitude (K) from your plot.

(D2.2.2) (4 points) If the star is moving in a circular orbit, calculate the **minimum value** of the orbital radius (r_{orb}) of the star in units of both $R_\odot$ and au.

(D2.2.3) (7 points) The mass function of a binary system is defined as:

$$f(M_1, M_2) = \frac{(M_2 \sin i_{orb})^3}{(M_1 + M_2)^2}$$

where the subscript "1" represents the primary star and "2" represents its companion. The parameter i_{orb} is the orbital inclination of the binary system. This mass function can also be expressed in terms of observable parameters. Calculate the **mass function of this system** in units of $M_\odot$.

2.3. Based on a detailed analysis on APOGEE, TRES spectra and GAIA parallax measurements, astronomers derived the following stellar parameters:

Table 5. Selected Physical Properties of 2M05215658+4359220

Effective Temperature T_{eff} (K)	Surface Gravity $\log g$ (cm s^{-2})	Parallax π (mas)	Measured Rotation Velocity $v_{rot} \sin i$ (km s^{-1})	Bolometric Flux F (J s^{-1} m^{-2})
4890 ± 130	2.2 ± 0.1	0.272±0.049	14.1±0.6	(1.1±0.1)×10^{-12}

Photometric observations indicate that the period of its light curve is identical to its orbital period, thus we may assume that the rotation period satisfies $P_{rot} = P_{orb} \equiv P$, and the inclination satisfies $i_{orb} = i_{rot} \equiv i$.

(D2.3.1) (16 points) **Calculate** the luminosity (L_1, in unit of $L_\odot$), radius (R_1, in unit of $R_\odot$), sine of the inclination angle ($\sin i$), as well as mass (M_1, in unit of $M_\odot$) of the visible star. Please **include** the uncertainty in your results.

(D2.3.2) (4 points) **Choose** the correct type of this star from the following options: (1) Blue Giant (2) Yellow main sequence star (3) Red Giant (4) Red main sequence star (5) White Dwarf.

(D2.3.3) (10 points) Based on the mass function $f(M_1, M_2)$ of the binary system, **plot** the rough relationship of M_2 (as vertical axis) and M_1 (as horizontal axis) on your graph paper and label it as Figure 5. Plot the most probable relation (by using $\sin i$), upper limit (with $\sin i + \Delta \sin i$) and lower limit (with $\sin i - \Delta \sin i$) derived in (D2.3.1).

(D2.3.4) (5 points) Draw a vertical **shadowed region** of $[M_1 - \Delta M_1, M_1 + \Delta M_1]$, as well as two horizontal **dashed lines** showing the maximum mass of the white dwarf and neutron star, on your *Figure 5*. What is the possible mass of the invisible companion, and what kind of celestial object could it be?

Instructions

1. This part of the contest consists of 4 problems, is 1 hour long and is worth a total of 100 points.
2. Only blue pen should be used to fill in the answer boxes, draw and mark on the star chart.
3. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, need to visit a restroom, etc.), please put up your hand to signal the supervisor.
4. The beginning and end of the competition will be indicated by a long sound signal.
5. Wait at your table until your envelope is collected. Once all envelopes are collected, your student guide will escort you out of the competition area.

O1 **(40 points)**

Figure 1 is a whole sky star chart of Yanqing, Beijing at 20:30 tonight (UTC+8) with the limit magnitude = 5^m (m = magnitude). Four stars (about 1^m - 3^m) and one planet (brighter than 2^m) are missing in this chart. In the chart, the distance from the centre is in proportion to zenith distance

(1) (20 points) Draw a cross (X) on the location of each missing star and mark "T" on the chart, and draw a cross (X) on the location of the missing planet and mark "P" on the chart.

(2) (5 points) Please mark the orientation of the star chart with "N" "E" "S" "W" at the edge of the star chart.

(3) (10 points) On the chart, the celestial equator passes through many constellations. Please write down the name of any five of these constellations (IAU codes).

Answer：

(4) (5 points) Using the star chart, estimate the altitude of Aldebaran (α Tau), to the nearest degree.

Answer：

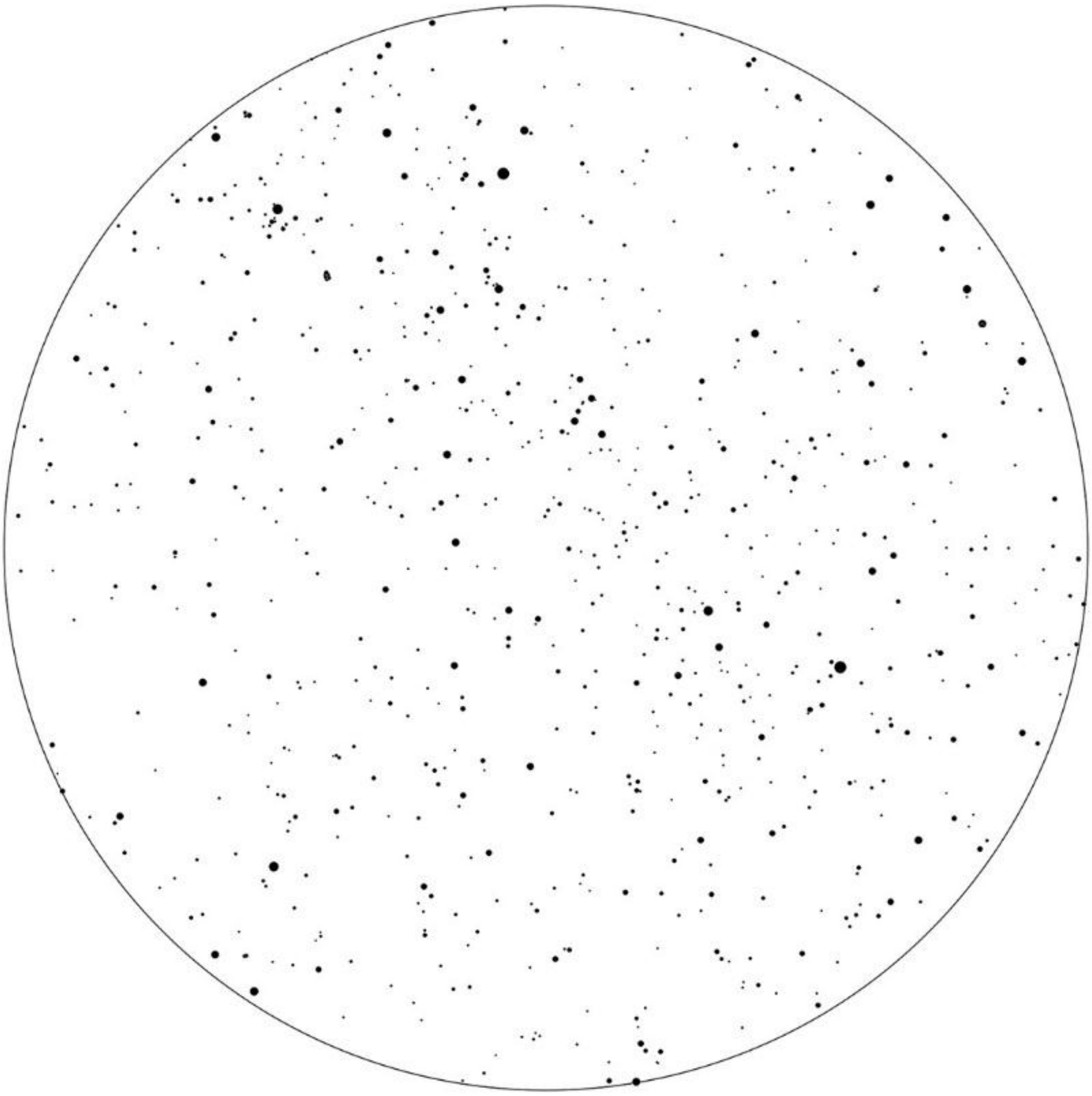


Figure 1

O2 **(20 points)**

Figure 2 is a star chart of a recent opposition of Jupiter. The grid in the figure is the ecliptic coordinates. Please estimate the date of this opposition, to the nearest day.

Answer：

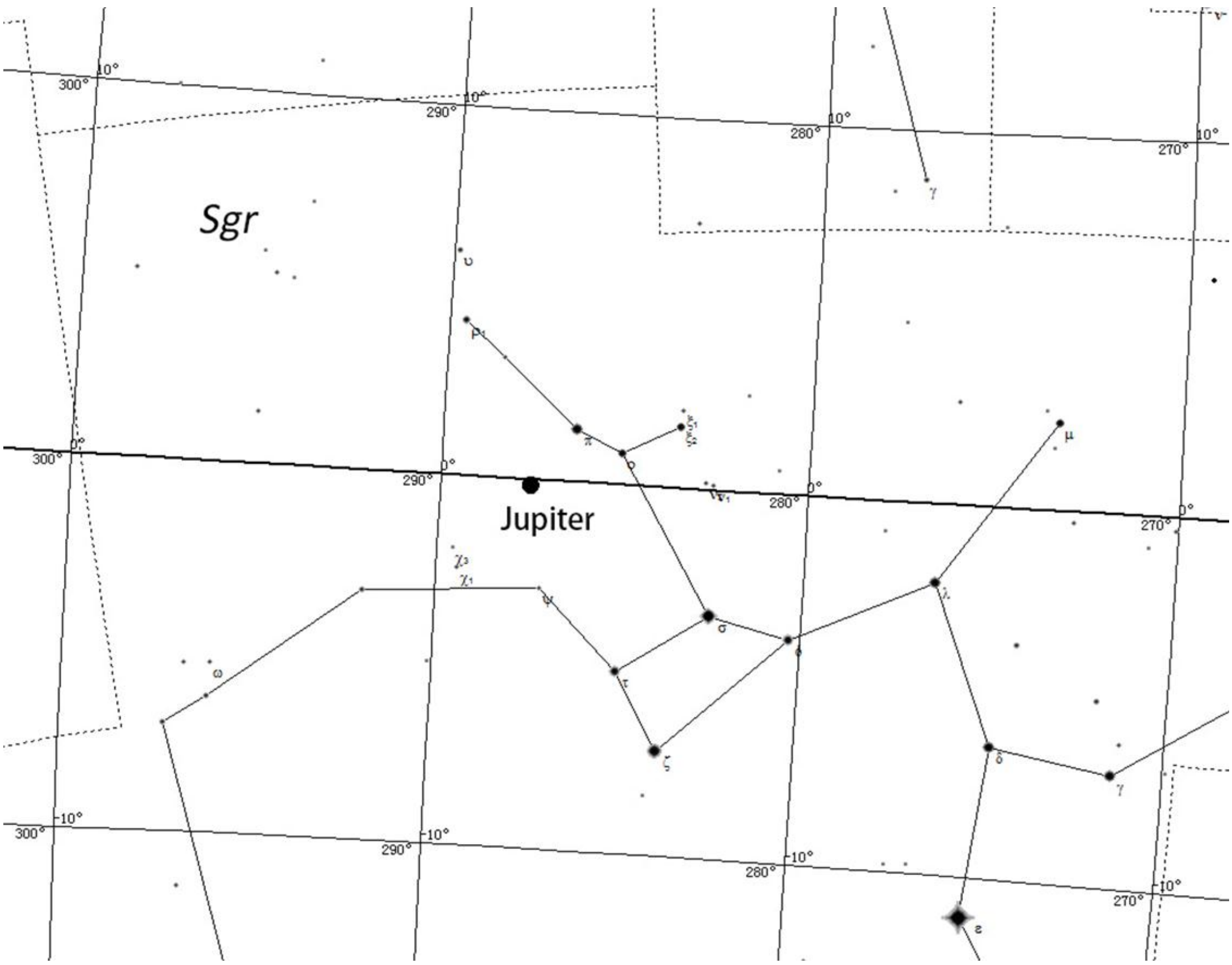


Figure 2

O3 **(20 points)**

Figure 3 is a star chart of a part of the sky on March 21, 2018. The longitude and latitude of the observation site is 120 °E, 40 °N (UTC+8). The grid in the figure is an equatorial grid. The thicker vertical line in the centre is the meridian. Estimate the mean solar time to an accuracy of better than 0.5h.

Answer:

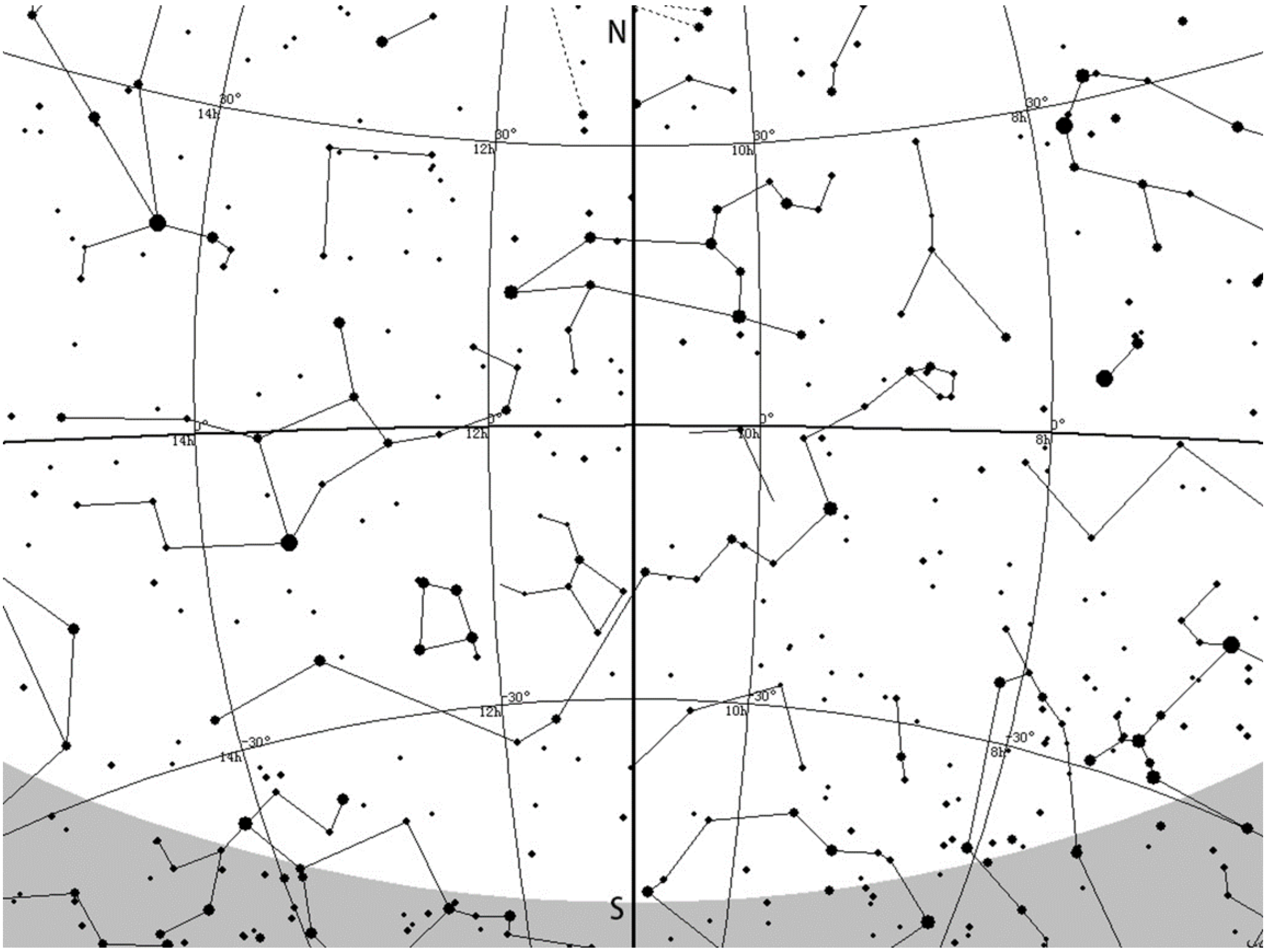


Figure 3

O4 **(20 points)**

Figures 4.1 – 4.4 are four photos of Messier objects. For each of them, please write down the Messier catalogue number and name the constellation where it is located.

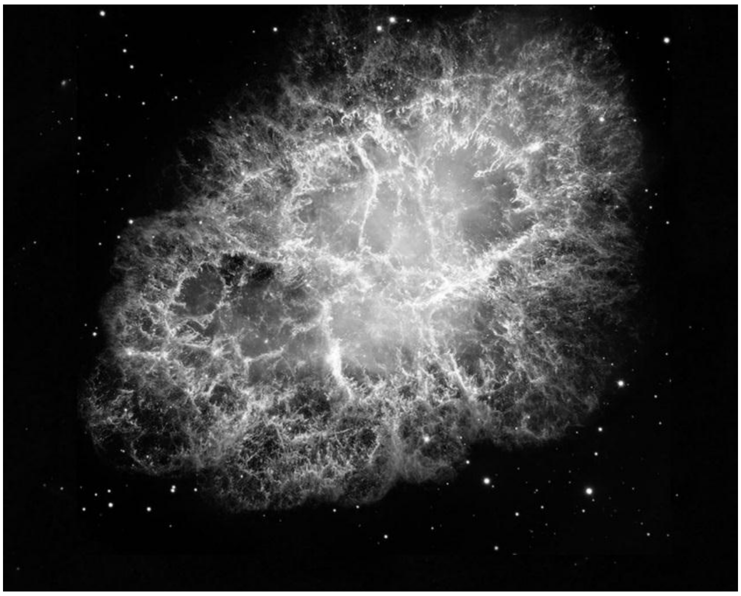


Figure 4.1

Answer：

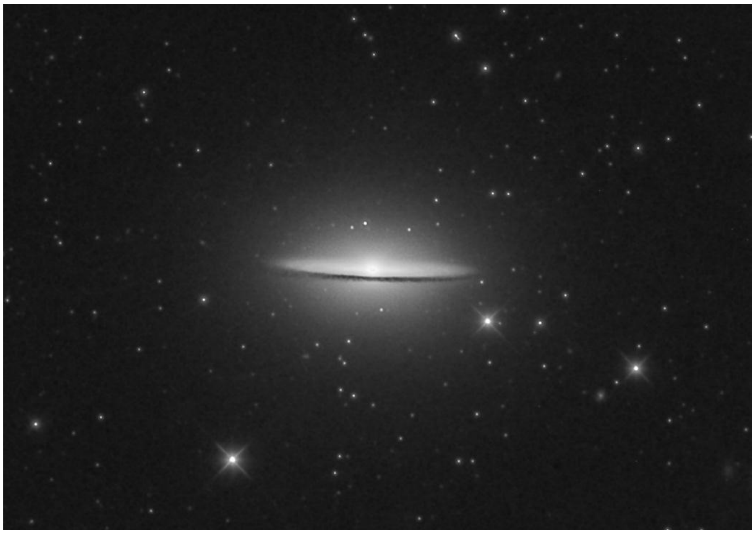


Figure 4.2

Answer：

Figure 4.3

Answer:

Figure 4.4

Answer:

Instructions

1. **This part of the competition consists of 2 problems (O5 and O6), and is worth a total of 50 points.**
2. **You have 10 minutes to complete these problems.**
3. **Stop working once the timer expires.**
4. **At the end of the examination, hand over the test envelope to the supervisor at the station.**
5. **You will use a telescope to observe five red LED screens at a distance. The telescope is equipped with one eyepiece, but no finderscope. Some of these screens display the names of planets, some show Messier numbers, and some show equatorial coordinates.**

O5. (10 points) The eyepiece's field of view is 45°. Please estimate the field of view (with an accuracy of 0.1 °) of the telescope when observing.

Answer:

O6. (40 points) Write down the text you observed on the screen.

Answers:

Mountain

Instruction

First and foremost, safety should be your main concern！

You can see mountains around the hotel, and in the picture below, the highest peak is marked with an arrow. Estimate the height of this peak relative to the ground of the hotel. You can use everything you can get or make, except climbing up! The team which gives a number closest to the actual answer will win the ‘game’.

Give this page to the organizers or volunteers before 23:59, November 8th.

The height: ______________ m; margin of error: _________ m.

Brief description of your process:

No. of Team:

Time of completion (for jury):

IOAA 2019 – Keszthely, Hungary

The 13$^{\text{th}}$ IOAA was held from 02$^{\text{nd}}$ to 10$^{\text{th}}$ August 2019. Total of 47* countries participated in the event.

1. Famous astronomical events **(10 p)**

Arrange the following astronomy-related events in chronological order from the oldest to the most recent. **Write the correct serial number (between 1 and 11) in the appropriate box on the answer sheet.**

1. Launch of the Hubble Space Telescope
2. Viking probes arrived at planet Mars
3. Discovery of Phobos and Deimos
4. Latest perihelion of comet 1/P Halley
5. Discovery of Ceres (asteroid / dwarf planet)
6. Discovery of Uranus (planet)
7. First successful measurement of a stellar parallax
8. Discovery of the first planetary nebula
9. Discovery of stellar populations (I and II)
10. First identification of a quasar with an optical source
11. Discovery of the expansion of the Universe

2. Deflection of radio photons in the gravitational field of solar system bodies **(10 p)**

A. Eddington and F. Dyson from Principe, and C. Davidson and A. Crommelin from Sobral, Brazil measured the deflection of light coming from stars apparently very close to the Sun during the total solar eclipse in 1919. The deflection was found to agree with the theoretically predicted value of 1.75".

A light ray (or photon) which passes the Sun at a distance d is deflected by an angle

$$\Delta\theta \propto \frac{4GM_{\odot}}{dc^2}$$

The present-day accuracy of the VLBI (Very Long Baseline Interferometry) technique in the radio wavelength range is 0.1 mas (milliarcsecond). Is it possible to detect a deflection of radio photons from a quasar by (a) Jupiter, (b) the Moon? Estimate the angle of deflection in both cases and **mark "YES" or "NO" on the answer sheet**.

3. The supermassive black hole in the centre of Milky Way Galaxy and M87 **(10 p)**

The first image of a black hole was constructed recently by the international team of the Event Horizon Telescope (EHT). The imaged area surrounds the supermassive black hole in the centre of the galaxy M87. The observations resulting in the final image were carried out at a wavelength $\lambda = 1.3$ mm where the interstellar extinction is not prohibitively large.

a) How large an instrument would be needed to resolve the shadow (in effect the photon capture radius, which is three times the size of event horizon) of a supermassive black hole in the centre of a galaxy? Express the result as a function of the distance d and the mass M of the black hole. (6 p)

b) Give the size of the instrument in units of Earth radius for

i. the supermassive black hole in the centre of M87, (1 p)
($d_{\text{BH-M87}} = 5.5 \times 10^7$ ly, $M_{\text{BH-M87}} = 6.5 \times 10^9\ \mathrm{M}_{\odot}$)

ii. and Sgr A*, the supermassive black hole of our own galaxy, Milky Way. (1 p)
($d_{\text{Sgr A*}} = 8.3$ kpc, $M_{\text{Sgr A*}} = 3.6 \times 10^6\ \mathrm{M}_{\odot}$)

c) What type of technology is needed for the development of such an instrument? **Mark the letter of your answer with × on the answer sheet.** (2 p)

(A) Gravitational lensing by dark matter
(B) Interferometry with an array of radio telescopes
(C) Photon deceleration in a dense environment
(D) Reducing the effect of incoming wavefront distortions
(E) Neutrino focusing with strong electromagnetic fields

4. Improving a common reflecting telescope **(10 p)**

A student has an average quality Cassegrain telescope, with primary and secondary mirrors having $\varepsilon_1 = 91\ \%$ reflectivity aluminium layers.

a) What will be the change in the limiting magnitude of this telescope by replacing the mirror coatings with "premium" quality $\varepsilon_2 = 98\ \%$ reflectivity ones? (5 p)

b) Assuming the student also uses a star diagonal mirror, also with reflectivity ε_1 with the original telescope - what will be the improvement if he/she also replaces this piece with an $\varepsilon_3 = 99\ \%$ reflectivity ("dielectric" mirror) model, combined with the new ε_2 mirrors? (3 p)
(A star diagonal mirror is a flat mirror, inclined to the optical axis by 45°.)

c) Is this difference obviously detectable by the human eye? **Mark "YES" or "NO" on the answer sheet.** (2 p)

Consider the whole visual band and disregard any wavelength dependence and geometric effects.

5. Cosmic Microwave Background Oven **(10 p)**

Since the human body is made mostly of water, it is very efficient at absorbing microwave photons. Assume that an astronaut's body is a perfect spherical absorber with mass of $m = 60$ kg, and its average density and heat capacity are the same as for pure water, i.e. $\rho = 1000\ \mathrm{kg\ m^{-3}}$ and $C = 4200\ \mathrm{J\ kg^{-1}\ K^{-1}}$.

a) What is the approximate rate, in watts, at which an astronaut in intergalactic space would absorb radiative energy from the Cosmic Microwave Background (CMB)? The spectral energy distribution of CMB can be approximated by blackbody radiation of temperature $T_{\mathrm{CMB}} = 2.728\ \mathrm{K}$. (5 p)
b) Approximately how many CMB photons per second would the astronaut absorb? (3 p)
c) Ignoring other energy inputs and outputs, how long would it take for the CMB to raise the astronaut's temperature by $\Delta T = 1\ \mathrm{K}$? (2 p)

6. The height of the chimney of Tiszaújváros power plant **(20 p)**

The European Copernicus Earth-observation programme operates two Sentinel-2 remote sensing satellites. These satellites orbit the Earth on Sun-synchronous polar orbits at about 800 km altitude. They pass over a given area once every few days, always taking images at the same local time (accurate to within a few minutes). The cameras are sensitive to 13 different optical and near-infrared spectral bands. The resolution of the images is 10 meters.

The 3rd tallest building in Hungary is the chimney of a power plant near the town Tiszaújváros. You can see two Sentinel-2 satellite images. One is from June 29, another one is from December 16, close to the summer and winter solstices, respectively. The orientation of the images is as normal, i.e. north is up and east is to the right.

The estimated shadow lengths, based on the images above and the scales given in the lower-left corners, of them are $x_1 = 125$ m and $x_2 = 780$ m. Answer the following questions:

a) On which date do we expect the shadow to be longer? **Mark the letter of your answer with × on the answer sheet.** (1 p)

(A) on June 29
(B) on December 16

b) At which time of the day did the Sentinel-2 satellites fly over this area? **Mark the letter of your answer with × on the answer sheet.** (1 p)

(A) early morning
(B) late morning
(C) early afternoon
(D) late afternoon

c) Based on the given shadow lengths, estimate the height of this chimney. For this calculation only, assume that the satellite images were taken at local noon. (16 p)

d) What could affect the accuracy of the chimney's derived height (more than one choice is possible)? **Mark the letter of your answer with × on the answer sheet.** (2 p)

(A) The oblate spheroid shape of the Earth.
(B) Limited resolution of the satellite images and the ill-defined edge of the shadow.
(C) The elevation of the base of the tower above sea level.
(D) Seasonal variation in the tilt of the Earth's rotational axis.
(E) Taking into account the effect of atmospheric refraction.

7. Effect of sunspots on solar irradiance (20 p)

Since 1978, the solar constant has been almost continuously measured by detectors on-board artificial satellites. These accurate measurements revealed that there are seasonal, monthly, yearly, and longer timescale variations in the solar constant. While the seasonal variations have their origins in the periodically varying Earth–Sun distance, the decade-long quasi-cyclic variations mainly depend on the activity cycle(s) of the Sun.

a) Calculate the value of the solar constant at the top of the Earth's atmosphere, when the Earth is 1 au from a perfectly quiet Sun, assuming that Sun emits as a perfect black-body. (4 p)

b) Calculate the solar constant of this perfectly quiet Sun in early January and early July, and find their ratio. (4 p)

c) Calculate the solar constant again at 1 au in the presence of a near equatorial sunspot with mean temperature of $T_{\mathrm{sp}} = 3300$ K and diameter of sunspot, $D_{\mathrm{sp}} = 90\,000$ km. Calculate the ratio - blank Sun to Sun with sunspot. (7 p)

Assume the sunspot is circular and ignore the effects of its spherical projection. Neglect any other activity features. Also assume that the Sun is rotating fast enough, hence solar irradiance is still isotropic.

d) In reality solar irradiance is no longer isotropic. Calculate the ratio of solar irradiance for the cases when the sunspot is not visible from the Earth to the case when it is fully visible. (5 p)

8. Amplitude variation of RR Lyrae stars (20 p)

Hungarian astronomers significantly contributed to the study of pulsating variable stars of RR Lyrae type, which show cyclic amplitude modulation in their light variation (Blazhko effect)

The light variation of an RR Lyrae star is observed at two different wavelengths: $\lambda_1 = 500$ nm and $\lambda_2 = 2000$ nm. At each wavelength, we see into the star at a different depth. We refer to the depths as layer 1 and layer 2. One can approximate the light intensity of a star at a given wavelength by the radius and the black-body radiation intensity at the appropriate depth. Moreover, the Wien approximation can be used for the black-body radiation to calculate the emitted power from unit surface area:

$$F(\lambda, T) \propto \frac{1}{\lambda^5} \exp\left(-\frac{hc}{k\lambda T}\right),$$

where h is the Planck constant, k is the Boltzmann constant, and c is the speed of light. To simplify the calculations we introduce a new constant $C_b = hc/k \approx 0.0144$ m K.

a) Assume that the temperature varies between $T_1 = 6000$ K and $T_2 = 7400$ K in each of the layers and ignoring radius variation, what is the ratio of the amplitudes of variation in magnitudes at the two wavelengths? (5 p)

b) What is the peak to peak amplitude of the light curve at λ_1? Use the magnitude scale. (3 p)

c) Ignoring temperature variation, what is the contribution of the radius variation to the light-curve amplitude for a given wavelength, if $R_{\text{min}} = 0.9\,\langle R\rangle$ and $R_{\text{max}} = 1.05\,\langle R\rangle$? $\langle R\rangle$ is the mean radius of the given layer. (3 p)

d) Recent observations and models show that the radius of the photosphere is only minimally modulated during the Blazhko cycle; however, the temperature variation is significant. As a result, the amplitude of light curve itself keeps varying. Let us assume that during the minimum amplitude of pulsation the temperature variation is reduced to $T_{\text{min}} = 6100$ K and $T_{\text{max}} = 6900$ K. What is the modulation amplitude at the two wavelengths? For the maximum amplitude, use the temperature values given in part a). (5 p)

e) Which statement is correct? (There can be more than one correct selection.) **Mark the letter of your answer with × on the answer sheet.** (4 p)

(A) It is easier to observe the Blazhko effect in the infrared band.
(B) The temperature variation dominates the visual light curve.
(C) If we neglect the radius variation, the amplitude is inversely proportional to the wavelength.
(D) Multi-colour observations are not useful in understanding the Blazhko effect.

9. Distance of the Lagrangian point L_2 of the Earth–Moon system (20 p)

On 3 January 2019, the Chinese spacecraft Chang'e-4 landed on the far side of the Moon in the area of the von Kármán crater, which was named after the world famous Hungarian-born physicist Theodore von Kármán.

As the Earth remains below the horizon of the spacecraft all the time, a relay station is also necessary for the communication with mission control on the Earth. For this purpose, The Chinese Space Agency launched a spacecraft, Queqiao, which was placed into a halo orbit around the outer Lagrangian point of the Earth–Moon system, L_2, the far side of the Moon.

Calculate the distance (h) of this satellite above the surface of the Moon. The Moon's orbit should be considered as a perfect circle with a radius of $R = 384\,400$ km. Neglect perturbations from the Sun and other planets.

Hint: You can use the following approximation: $1/(1+x)^2 \approx 1 - 2x$, if $|x| \ll 1$.

10. South → East → North (20 p)

Consider the Earth as a perfect, rigid, sphere with radius $R = 6378$ km. There exist some points on the surface of the Earth from which we can travel first 6378 km South, then 6378 km East, and after that 6378 km North, and as a result return to the original point of departure. Find such points and paths. Calculate the geographic coordinates of the turning points of your solutions and draw the paths.

For the sake of simplicity measure the geographic longitude from 0° to +360° eastward from Greenwich, the geographic latitude from 0° to +90° north of the Equator and from 0° to −90° south of the Equator. Solutions resulting from rotational symmetry should not be considered to be different.

11. Identification of light curves of types of selected variable stars **(25 p)**

TESS (Transiting Exoplanet Survey Satellite) is NASA's most recent exoplanet hunter mission. It has been surveying the southern sky to locate exoplanets around the brightest and closest stars, along with a large number of time-variable phenomena (pulsating and eclipsing variable stars, supernovae, stellar flares, and asteroids among others).

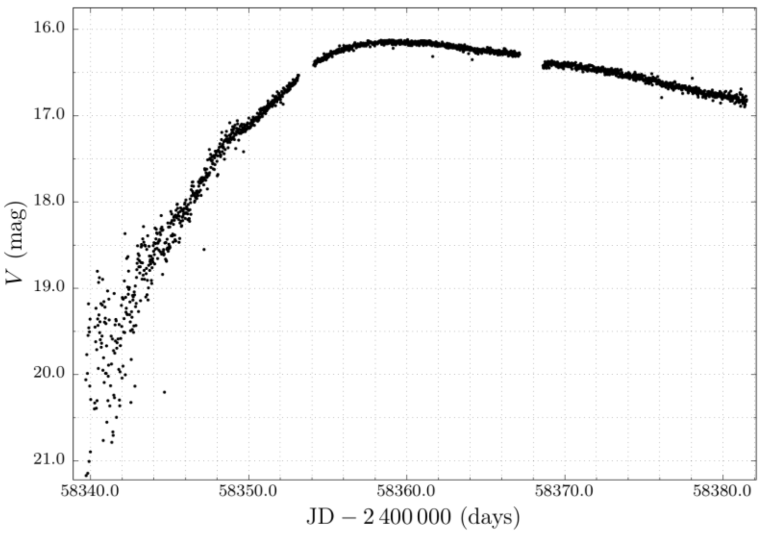
On the separate sheet you will find graphs with light curve plots of 8 periodic variable stars (intrinsic, eclipsing and rotational; namely FO Eri, RW Dor, 24 Eri, TIC 147272181, ST Pic, UY Eri, VV Ori, AH Col) from the TESS target list, numbered from 1 to 8. The horizontal axis is BJD − 2 400 000 in days, and the vertical axis is V in magnitudes, where BJD is the Barycentric Julian Date, and V is the visual brightness.

a) Below is a list of variable star types. Match each light curve of the star with its variability type **by writing its number into the rectangle on the answer sheet**. (8 p)

- Heart-beat star
- RR Lyrae type (RRab subclass) pulsating variable star
- Eclipsing binary of Algol type (semi-detached) with a pulsating component
- α^2 CVn pulsating variable star
- W Vir type (Population II) Cepheid pulsating variable star
- Detached eclipsing binary with strong reflection effect
- Contact eclipsing binary of W UMa type
- Rotationally variable (spotted) star

b) Based on the light curve plots, estimate the periods of each variable star in days. **Give your answers on the answer sheet up to 2 decimal places.** Periods in the range ±5% of the true periods are acceptable. (16 p)

c) What type of astronomical object produced the TESS light curve shown in the figure below? **Mark the letter of your answer with × on the answer sheet.** (1 p)



(A) Microlensing event
(B) Saturated galaxy due to the proximity of Mars
(C) Comet's coma close to the edge of the camera field
(D) Supernova in a distant galaxy
(E) Superflare on a supergiant star

12. Distance to a Near-Earth Asteroid (25 p)

Assume that a Near-Earth Asteroid is observed by two astronomers, one from Nagykanizsa, Hungary and one from Windhoek, Namibia. The longitudes of the two cities are exactly 17° east of Greenwich. They observe the asteroid when it crosses their respective meridians. The Nagykanizsa-observer sees the asteroid 25° south of his zenith at this instant, while the Windhoek-observer sees it 45° north of his zenith at the same instant. The latitudes of the two cities are 46°27' N and 22°34' S, respectively. The sites of both astronomers are at sea level.

a) Draw a diagram of the geometric configuration. (5 p)

b) What is the distance of this asteroid from the centre of the Earth, expressed in units of Earth-radii and the average Earth-Moon distance? Provide a solution which makes use of all available information. Neglect the effect of the atmospheric refraction. (20 p)

13. Distance to the Coma galaxy cluster (40 p)

The Coma galaxy cluster (Abell 1656) has an angular diameter on the sky of about 100 arcminutes, and contains more than 1000 individual galaxies, most of which are dwarf and giant ellipticals orbiting the common center of mass of the cluster in approximately circular orbits. The table below lists the measured radial velocities of a few individual cluster member galaxies.

No.	v_r (km/s)	No.	v_r (km/s)	No.	v_r (km/s)	No.	v_r (km/s)
1	6001	6	7116	11	7156	16	7111
2	7666	7	7004	12	7522	17	8292
3	6624	8	4476	13	7948	18	5358
4	5952	9	6954	14	4951	19	4957
5	5596	10	8953	15	7797	20	7183

a) Derive the distance of the cluster from the mean radial velocity of the galaxies listed in the table. (8 p)

b) Estimate the physical diameter of the cluster (in Mpc). (4 p)

c) The virial theorem states that if the galaxy cluster is in dynamic equilibrium, then the mean kinetic energy, $\langle K \rangle$, and the mean gravitational potential energy, $\langle U \rangle$, are related by

$$-2\langle K \rangle = \langle U \rangle$$

assuming the Coma cluster is spherical.

For simplicity, assume that each galaxy has approximately the same mass, m.

Use the virial theorem to prove that, in this case, the cluster mass M (also called as the *virial mass*) can be expressed as

$$M = \frac{5R}{G}\sigma_r^2$$

where σ_r^2 is the velocity dispersion of the cluster. (10 p)

The formula for standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

d) Using the data in the table, estimate the virial mass of the Coma cluster in solar masses. (12 p)

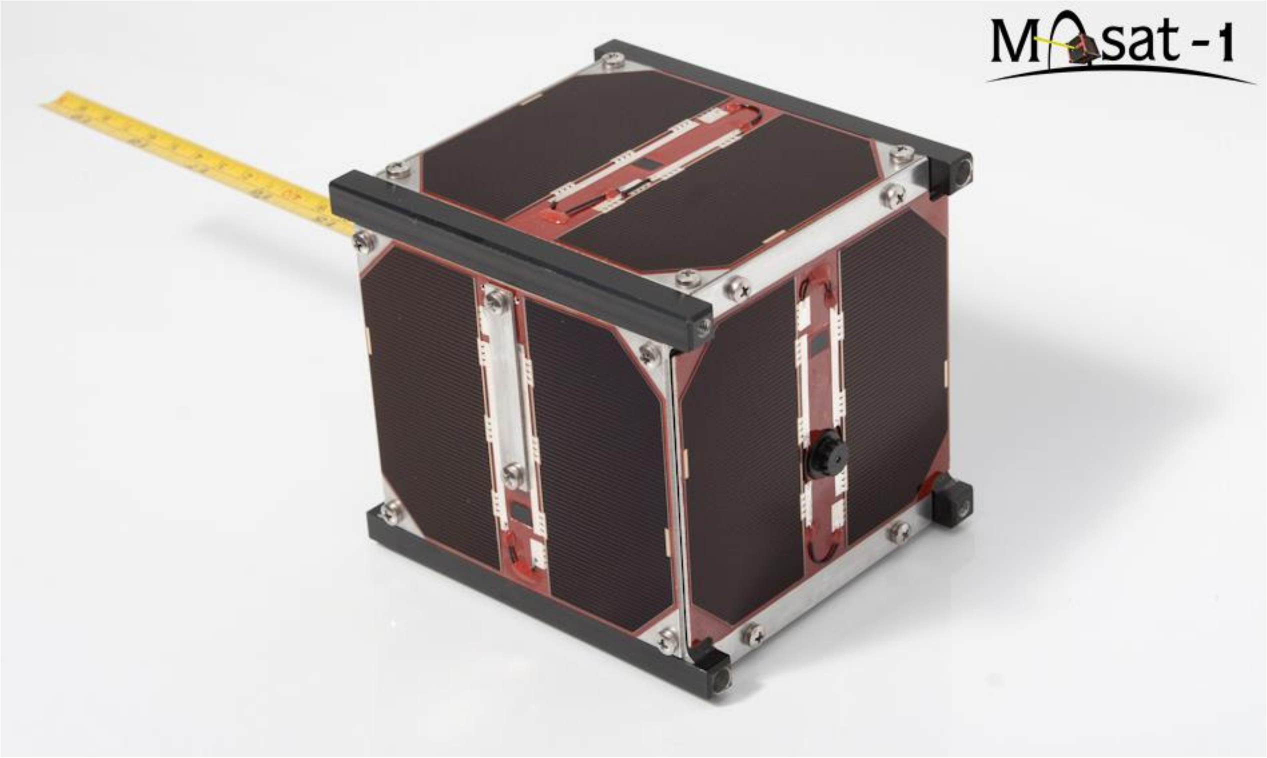
e) The total luminosity of the Coma cluster (in solar luminosity, $L_{\odot}$) is $L \approx 5 \times 10^{12} \mathrm{L}_{\odot}$. Calculate the mass-luminosity ratio of the cluster in solar mass per solar luminosity units. (2 p)

f) Which of the following statements is true (more than one answer is possible)? **Mark the letter of your answer with × on the answer sheet.** (4 p)

(A) The mass-luminosity ratio of the Coma cluster is much higher than that of a typical spiral galaxy, like the Milky Way.
(B) The mass-luminosity ratio of the Coma cluster is similar to that of a typical spiral galaxy.
(C) The mass-luminosity ratio of the Coma cluster is much less than that of a typical spiral galaxy.
(D) The Coma cluster contains much more dark matter than a typical spiral galaxy.
(E) The Coma cluster contains much less dark matter than a typical spiral galaxy.

14. Photographing a nanosatellite (60 p)

The very first, entirely Hungarian-made, satellite was "MASAT–1", a nanosatellite "cubesat". It was made mostly of aluminium with total mass 1 kg, sides length l = 10 cm and a longer communication aerial. It was designed and prepared at the Technical University of Budapest (BME) in 2009 by students. The launch occurred on February 13, 2012, using a Vega rocket from the Kourou launch site – together with several other cubesats from other countries. It operated successfully right up to the last minutes of its lifetime (the transmitted last data packages were captured by radio receivers on the evening of January 9, 2015, a few hours later the nanosatellite re-entered the atmosphere and disintegrated).



The orbital altitude of MASAT–1 changed between $h_{\mathrm{min}} = 350$ km and $h_{\mathrm{max}} = 1450$ km (due to its highly eccentric orbit), but in this question assume it to be in a circular orbit 900 km above the sea level.

The MASAT team wished to photograph their nanosatellite from the ground. Thus, they called the staff of Baja Astronomical Observatory (South Hungary, $\lambda_{\mathrm{B}} = 19.010843°$, $\varphi_{\mathrm{B}} = 46.180329°$, $h_{\mathrm{B}} = 100$ m) to photograph the orbiting MASAT–1 with their telescope.

The observatory has a Ritchey-Chrétien type reflecting telescope with a diameter of 50 cm, and focal ratio of $f/8.4$. The CCD camera installed on the telescope had a 4096×4096 chip with 9 µm sized square pixels. The quantum efficiency of the CCD was about 70 %. The practical detection limit was about 19.5^{m} visual band applying a $\tau_{\mathrm{exp}} = 2$ min exposure time. Disregard any effects of seeing. The orbital inclination of MASAT–1 was about i=70° and the direction of the orbital motion was the same as the Earth rotation. In addition, let us assume that the reflection area always is 100 cm^2. Throughout this event, the telescope was pointed to the local zenith, and its RA motor was tracking the stars. Consider only light from the Sun, you can ignore light reflected from Earth and Moon. Ignore the effects of atmospheric extinction.

a) Calculate the apparent visual magnitude of this cubesat under ideal observational circumstances, i.e. when it was at the local zenith of the observing site (Baja, Hungary) at midnight. Omit all atmospheric effects, and consider the Earth to be a sphere. The albedo of a specular aluminium plate is $a \approx 0.70$. (10 p)

   Hint: Use a comparison of MASAT–1 with the Full Moon.

b) What was the Observatory's answer to the MASAT team; would it ever be possible to photograph their nanosatellite with the existing equipment at the observatory? **Mark "YES" or "NO" on the answer sheet.** Support your answer with detailed calculations. (42 p)

c) What would have been the answer if they had taken into account the blurring effect of the atmosphere, the so-called seeing? In Hungary the typical FWHM (Full Width at Half Maximum) value of the blurred stellar images – which can be approximated by a symmetric 2D Gaussian profile close to the zenith – is about 3.5″. **Mark "YES" or "NO" or "Close to the limit" on the answer sheet.** Support your answer with a short calculation. (8 p)

   Hint: Although the illumination of the seeing spot in the focal plane can be approximated by a symmetric 2D Gaussian profile, you can take it to be homogeneous in your calculation.

1. Photometry and spectroscopy of Nova Del 2013 **(60 p)**

Classical nova V339 Del (Nova Delphini 2013) was discovered by Koichi Itagaki at 6.8 magnitude on 14 August 2013 at 14:01 UT (MJD = 56518.584). Both professional and amateur astronomers analysed the photometry and spectroscopy of the nova. Less than 10 hours after the alert, when the night falls at the Piszkéstető Mountain Station of the Konkoly Observatory of the Hungarian Academy of Sciences, Hungarian astronomers took the first spectrum data of the nova using the eShel echelle spectrograph in the Gothard Astrophysical Observatory of Loránd Eötvös University attached to the 1 meter telescope of the Konkoly Observatory.

Refer to Fig 1.1 and Fig 1.2 to complete the questions. The larger versions of Fig 1.1 and Fig 1.2 are found on separate A3 papers.

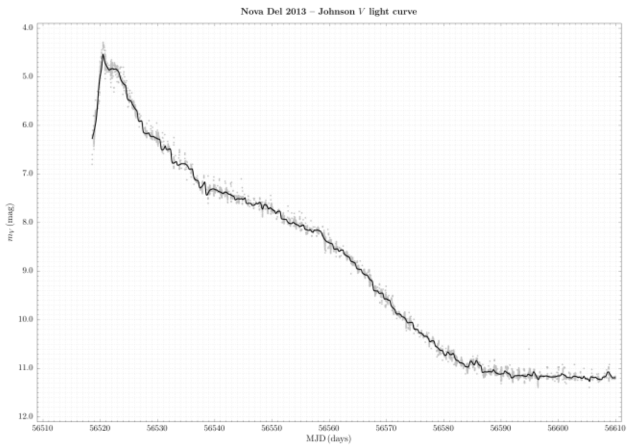


Fig. 1.1: Nova Del 2013 – Johnson V light curve

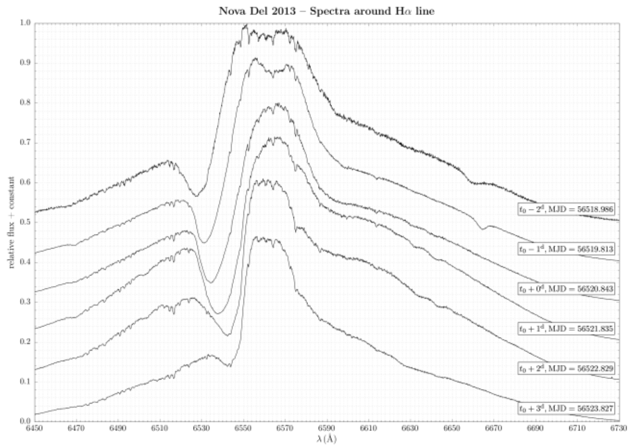


Fig. 1.2: Nova Del 2013 – Spectra around Hα line

Fig. 1.1 shows the visual light curve of the nova based on the data downloaded from the website of AAVSO (American Association of Variable Star Observers). On the horizontal and vertical axes, the Modified Julian Date (MJD = JD−2 400 000.5) of the observations and the Johnson V magnitudes are plotted, respectively. The grey circles (about 38000 data points) represent the measured values, while the continuous black line is the result of smoothing the data with a Gaussian filter (Full Width at Half Maximum = 0.5 day) to define an "average" light curve from the data points.

The rate of decline can be characterized by the values t_2 and t_3, which show the time interval in days in which a nova fades from its maximum brightness by 2 and 3 magnitudes.

A few empirical formulae between the peak of the absolute magnitude in the V band (M_0) and t_2, t_3 values can be found in the following literature:

(a) $M_0 = -7.92 - 0.81 \arctan \frac{1.32 - \log t_2}{0.23}$ (Della Valle, M. & Livio, M.: 1995, *ApJ* **452**, 704)

(b) $M_0 = -11.32 + 2.55 \log t_2$ (Downes, R.A. & Durbeck, H.W.: 2000, *AJ* **120**, 2007)

(c) $M_0 = -11.99 + 2.54 \log t_3$ (Downes, R.A. & Durbeck, H.W.: 2000, *AJ* **120**, 2007)

The $E(B-V)$ color excess of Nova Del 2013 (Chochol, D. et al.: 2014, *Contrib. Astron. Obs. Skalnaté Pleso* **43**, 330) is:

$$E(B-V) = 0.184 \pm 0.035$$

Fig 1.2 shows the nova spectra taken in the wavelength region around the Hα line on six consecutive nights before and after the time of the maximum brightness (t_0). The individual spectra have been shifted vertically for clarity. The Modified Julian Dates (MJD) of the observations are listed on the right hand side of each spectrum slice.

The Hα line shows the so-called P Cygni profile with very broad wings, which is typical not of novae only, but is present in almost all spectral types and is a reliable sign of a massive radial motion of matter ejected from the star. The P Cygni profile is composed of a strong, broad emission peak which is considered to be centered at the rest wavelength in air λ_0 of the line – for Hα $\lambda_0 = 6562.82$ Å – and a usually weaker, blueshifted absorption component. The expansion (radial) velocity of the shell can be approximated from the measured wavelength λ of the absorption peak using the well known Doppler formula connecting the displacement $\Delta\lambda = \lambda - \lambda_0$, the radial velocity v_r, and c, the speed of light.

Assume that the Hα line showing P Cygni profile is excited in the outermost part of the spherically expanding shell, and its extent at the moment of taking the first spectrum was still negligible.

a) From Fig. 1.1, estimate the Modified Julian Date of the peak magnitude (MJD_0) and the value of the peak magnitude itself. Consider the error of this brightness value to be 0.05^{m}. (3 p)

b) Estimate the Modified Julian Dates based on the time interval (days) in which the nova has faded by 2 and 3 magnitudes, then calculate t_2 and t_3 values. (6 p)

c) With reference to t_2 and t_3 from (b), determine the peak absolute magnitude of the nova using all three empirical formulae listed earlier, calculate their mean (M_0) and their standard deviation, and consider this latter as the uncertainty of M_0. (5 p)

The formula for standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

d) Using the value of the color excess $E(B-V)$, determine the interstellar extinction A_V and its uncertainty in the direction of the nova. Use $R_V = 3.1$, without error. (4 p)

e) Estimate the distance to the nova and its uncertainty. Give the result in kpc. (11 p)

f) Measure the central wavelengths of the P Cygni absorption features plotted in Fig. 1.2 (refer to magnified version), and calculate the corresponding radial velocities. No error estimation is needed. (14 p)

g) Plot these radial velocities against the Modified Julian Dates of the observations. (6 p)

h) From the graph in (g), estimate the physical radius of the envelope at the end of the time interval. Give the answer in astronomical units (au). (7 p)

i) Knowing the distance to the nova and the physical radius of the spherical envelope 5 days after the discovery, estimate the apparent angular diameter of the envelope then. (4 p)

2. Triply eclipsing hierarchical triple stellar system (90 p)

HD 181068 was one of the brightest targets which was continuously observed during the almost 4-year-long primary mission of NASA's exoplanet-hunter *Kepler* space telescope. The spacecraft observed $\approx 3-4\times10^{-3}$ magnitude dimmings every 0.453 days. (Note: The even dimmings were slightly smaller amplitude than the odd ones.) Furthermore, additional 0.007 magnitude, 2.3-day-long dimmings were detected every 22.7 days.

The correct explanation of this very unusual photometric behaviour was given by Hungarian astronomers. They found that HD 181068 is a compact hierarchical triple stellar system seen almost edge-on.

Hierarchical triple star systems consist of three stars; A, B, and C. Two of these stars (B and C) form an inner or close stellar binary system, whilst the outer component (star A) orbits at a distance from the inner system significantly larger (usually orders of magnitude) than the semi-major axis of the inner system. The schematic view of an example of a triple star system is illustrated in Fig. 2.1.

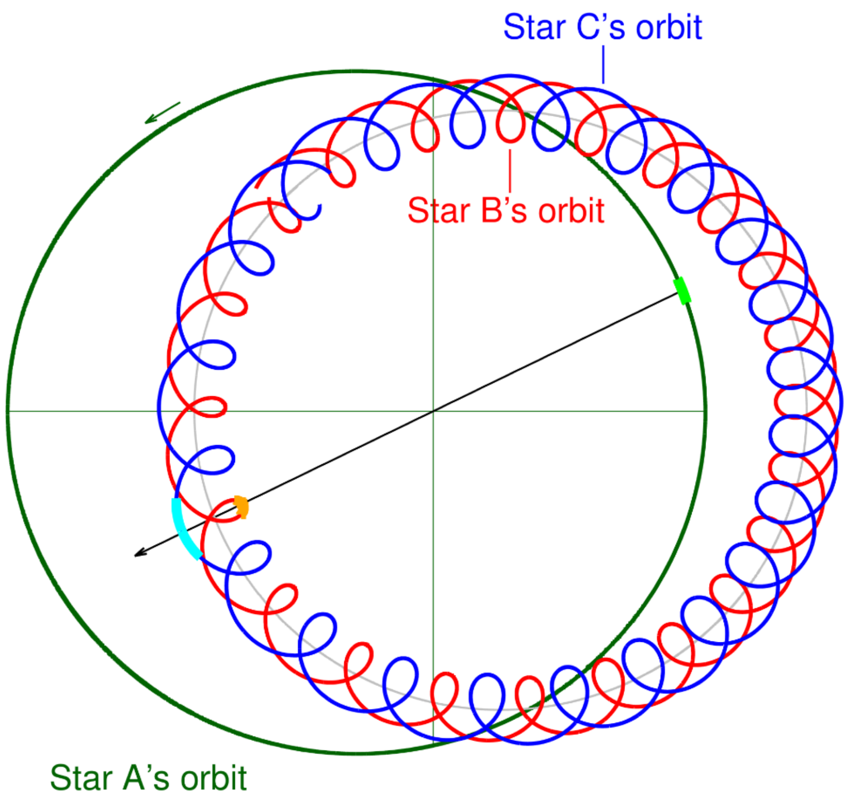


Fig. 2.1: The schematic pole-on view of a hypothetical , hierarchical triple stellar system. The black arrow is directed towards the Earth. The thick segments of the three orbits represent the stars' orbital arcs during an outer eclipse.

Mathematically, the motion of a hierarchical triple system can be well approximated with two unperturbed Keplerian two-body motions; (1) Keplerian motion of the inner binary. (2) The centre of mass of this close binary and the third star revolves on a second Keplerian orbit, "outer binary".

In this problem, stars B and C form a $P_1 = 0.9056768$-day-period eclipsing binary, while the centre of mass of these stars with star A forms the $P_2 = 45.4711$-day-period outer binary. As the orbital plane of this outer orbit is seen almost edge-on from the Kepler spacecraft (and from the Earth), during their revolution on the outer orbit, stars B and C eclipse not only each other, but also star A or, a half outer revolution later are eclipsed by it, causing the extra dimmings.

i. *Determination of the physical stellar sizes (and other quantities) from the geometry of the eclipses*

These assumptions are used throughout this section: (1) both the inner and outer orbits are exactly circular, (2) the orbital planes of the inner and outer orbits are identical, and (3) this plane is seen exactly edge-on (i.e. $i_1 = i_2 = 90°$ and $i_{\mathrm{rel}} = 0°$). Let's consider the extra dimmings, which are central eclipses (i.e. either occultations or transits – annular eclipses), therefore, these events have four contacts. In the case of an ordinary eclipsing binary (or of a transiting exoplanet) at the times of the outer contacts the sky-projected disks of the two objects connect with each other at one point from outside, while at the inner contacts they approach each other from inside. While this last statement is also valid for outer eclipses, the situation becomes more complex, because instead of two, three stars are involved into the eclipses. However, despite this fact, we can certainly define the times of each contact from the light curve, and furthermore, we can also decide explicitly which member of the inner binary is involved in a given contact. (The other member, of course, is always star A.)

In the table below, the accurate times of some contacts of different eclipses observed by the Kepler spacecraft, the contact types and the stars are documented. Time is expressed in barycentric Julian Days (BJD).

event no.	contact	stars	BJD	φ_1	φ_2
1	I	A, B	2455476.1096		
	II	A, C	2455476.4245		
	III	A, B	2455477.9677		
	IV	A, B	2455478.4722		
2	I	A, B	2455521.5217		
3	III	A, C	2455568.9434		
4	I	A, C	2455612.4733		
	III	A, C	2455614.3571		
5	III	A, B	2455659.9241		
	IV	A, C	2455660.2422		

a) Given that $T_{01} = 2455051.2361$ and $T_{02} = 2455522.7318$ denote the time of an inferior conjunction of the inner and outer binaries respectively (i.e. that time, when, from the perspective of the observer, star C eclipses star B, and when star A eclipses the centre of mass of stars B and C.)

Define

$\varphi_1(t) = \{(t - T_{01})/P_1\}$, and $\varphi_2(t) = \{(t - T_{02})/P_2\}$

as the photometric phases of the inner and outer binaries respectively. $\{x\}$ denotes the decimal part of the real number x. If $\{x\} < 0$, use $\{x\} + 1$ instead. Calculate the phases for the times of the tabulated contact events and **write the answers in the appropriate columns of the table on the answer sheet**. Round your answers to four decimal places. (10 p)

b) Determine, whether star A, or the close binary (i.e. stars B and C) were closer to the observer during each eclipsing event. **Write your answer in the table on the answer sheet.** (5 p)

c) Using the table above, calculate (1) the dimensionless radius of each star relative to the semi-major axis of the outer orbit ($R_{\mathrm{A,B,C}}/a_2$), (2) the ratio of the semi-major axes of the two orbits (a_1/a_2) and (3) the mass ratio of stars B and C ($q_1 = m_\mathrm{C}/m_\mathrm{B}$). *Hint:* Use at least four decimal place accuracy in your calculations. Be cautious, it may not be possible to use all theoretically possible contact combinations with a given limited accuracy of time data. (30 p)

d) Based on the results obtained above, calculate the outer mass ratio ($q_2 = m_\mathrm{BC}/m_\mathrm{A}$). (8 p)

ii. *Dynamical determination of the stellar masses using radial velocity (RV) and eclipse timing variations (ETV) measurements*

To obtain RV data, ground-based spectroscopic follow up observations were carried out with four different instruments. Only the lines of star A were detectable in all spectra. Plotting all the measurements against time, the RV curve was nicely fitted in the following form:

$$V_\mathrm{rad,A} = V_\gamma + K_\mathrm{A} \sin \phi_\mathrm{RV},$$

where V_γ is the systemic velocity and K_A is the velocity amplitude:

$$V_\gamma = 6.993 \pm 0.011 \text{ km s}^{-1},\ K_\mathrm{A} = 37.195 \pm 0.053 \text{ km s}^{-1},$$

$$P_2 = 45.4711 \pm 0.0002 \text{ d},\ \phi_\mathrm{RV} = \frac{2\pi}{P_2}[t - (2455522.7318 \pm 0.0095)].$$

Furthermore, the researchers determined the mid-times of the regular eclipses of the close binary (formed by stars B and C), and found that the occurrence of for example the eclipsing minima belonging to the N^th orbital revolution can be described by the simple expression:

$$T_N = T_0 + P_1 N + A_\mathrm{ETV} \sin\left(\frac{2\pi}{P_2} P_1 N + \phi_0\right),$$

where

$$T_0 = \text{BJD } 2455051.23607 \pm 5 \times 10^{-5},\ P_1 = 0.9056768 \pm 3 \times 10^{-7} \text{ d},$$

$$A_\mathrm{ETV} = 0.001446 \pm 0.000110 \text{ d},\ \phi_0 = -0.76779 \pm 0.01937 \text{ rad}.$$

In this expression A_ETV is the amplitude of the eclipse timing variation, T_0 denotes the mid-eclipse time of the reference (zeroth) primary eclipse, and N is the cycle number, which is an integer for primary eclipses (i.e. when the slightly fainter star C eclipses star B), and half-integer for secondary ones (i.e. when star B eclipses star C).

Determine (1) again the mass ratio ($q_2 = m_\mathrm{BC}/m_\mathrm{A}$) of the centre of mass of the inner binary and star A using only the results obtained in point ii., (2) the mass of component A (m_A) and (3) the total mass of the inner, close binary (m_BC). Calculate the errors for (1), (2), and (3) in masses. *Hint:* You can save much time by expressing the masses in solar mass and the orbital separations either in solar radius or au. (22 p)

iii. Using results obtained in questions 1 and 2, determine the masses of stars B and C respectively and calculate the physical dimensions of all three stars (i.e. stellar radii in physical units). (15 p)

Student ID code: ☐☐☐–☐☐

PLANETARIUM ROUND – ANSWER SHEET

3 projected images with questions. Each part is 15 minutes long. Total time: 45 minutes.

PROBLEM 1

This is the sky above Keszthely at midnight. The projected sky does not show any Solar System objects.

QUESTIONS / TASKS:

1.1. There are 3 novae on the projected sky at 2nd magnitude. Mark their positions by circles on the star chart. (Please circle only 3 stars. If there are more than 3 circles, then each one in a wrong location will result in 1 point deduction.) [ ]

1.2. The Messier objects have been removed from the star chart given you. Mark all the Messier list globular clusters present in the projected sky on the star chart using crosses (X) and write the Messier number of each object near the cross marks. [ ]

1.3. The projected sky corresponds to the second half of which month (at midnight CEST) in Keszthely? Circle the correct month.

JAN / FEB / MAR / APR / MAY / JUN / JUL / AUG / SEP / OCT / NOV / DEC [ ]

1.4. What is the local sidereal time? (To an accuracy of 15 minutes.)

………………………………… [ ]

1.5. List six zodiacal constellations, which are partly or entirely visible. (Use the official IAU abbreviations or IAU designation. Every constellation named which is not visible in the projected sky will result in 1 point deduction.)

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[ ]

PROBLEM 2

We are standing somewhere on the Earth. The projected sky does not show any Solar System objects.

QUESTIONS / TASKS:

2.1. Determine the geographical latitude of this observing site: …………….° [ ]

In which hemisphere is the site situated? N / S (Circle the right one.) [ ]

2.2. Determine the azimuth of the 3 brightest stars on the projected sky. Azimuth is measured from North towards the East. Write the name of these stars in English or using their Bayer designation and their azimuths in the list below.

Bright star / name: ………………………………… Az: ……………….° [ ]

Bright star / name: ………………………………… Az: ……………….° [ ]

Bright star / name: ………………………………… Az: ……………….° [ ]

2.3. Yellow × signs show the position of 3 comets. Which comet is closest to the ecliptic? (Circle the number below.)

1 / 2 / 3 [ ]

2.4. List nine constellations that contain circumpolar stars seen from the given observing site. (Use the official IAU abbreviations or IAU designation.)

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[ ]

2.5. Mintaka (δ Orionis) is setting at this moment. How many hours earlier did it rise? (To an accuracy of 15 minutes.)

……………… [ ]

IOAA 2019 – Keszthely, Hungary

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PROBLEM 3

For this view, we are now standing on the Moon. At this instant viewed, the Earth is centrally eclipsing the Sun (see the red circle on the sky). Consequently the Moon is at one of its nodes now. Assume the longitudinal and latitudinal librations are exactly 0° at this moment.

QUESTIONS / TASKS:

3.1. At the time of this observation, which season is it in Hungary? (Circle the correct answer.)

Spring / Summer / Autumn / Winter [ ]

3.2. There is a yellow circle on the projected sky (next to the red circle), which denotes minor planet Juno, which is at a distance of exactly 3 au from the Sun at this moment. Estimate its distance to the Moon at this instant. (Rounded to the nearest integer in units of million km.) Assume all orbits to be circular.

……………………………million km [ ]

3.3. Approximately how much time (in Earth days) after the projected event will …

…the Sun set at your observing site? ……………………………… [ ]

…the Earth set at your observing site? ……………………………… [ ]

3.4. Determine the Lunar (Selenographic) coordinates of this observing site (as defined in the lunar map on the next page):

……………………………………. [ ]

What is the name of the large surface lunar area, where your observing site is situated? Do not use your national language, please use the official IAU nomenclature.
(See lunar map on next page.)

……………………………… [ ]

3.5. Estimate the distance from the observing site to the Apollo-11 landing site (0.6875 N, 23.4333 E):

……………………………… km [ ]

OBSERVATIONAL ROUND – QUESTION SHEET

Telescope: 150/750 Newton
Eyepieces: 25 mm, 10 mm, Barlow lens: 2x

Note:

- Telescope is already polar aligned.
- In case of bad sky conditions at low altitudes, task 1 and 2 will be replaced by alternative tasks 1 and 2 (see page 3). In this situation, the telescope assistant will cross out Tasks 1 and 2.
- You have to use 25 mm eyepiece for tasks 1, 3 and 4.
- For these tasks, if you finish before the allotted time, you must keep tracking the object with the telescope till the end of allotted time. The telescope assistant will check the object only at the end of the allotted time.
- For task 2, we recommend using 10 mm eyepiece and Barlow 2x.
- For task 5, you are not allowed to use the telescope.

TASK 1: FINDERSCOPE ALIGNMENT

available time: 5 minutes 5 points

- The finderscope is NOT aligned at the beginning. **Point the telescope to Saturn and align the finderscope parallel to the main tube.**

If the alignment of Saturn is not within the crosshair of the finderscope, the telescope assistant will correct it – and you receive only partial or no points.

TASK 2: OBSERVATION OF SATURN

available time: 10 minutes 15 points

- In the upper box, the circle represents the disk of Saturn and the horizontal line is the E-W direction on the sky. Pay attention to direction of North (see top right corner).
  Mark position of Titan by a cross.

- The smaller box on the bottom right corner of first box is for drawing the rings of Saturn. Again the circle represents the disk of Saturn.
  Draw the rings of Saturn in this box with the correct size and orientation.
  Both the outer and inner edges of the ring are necessary, no faint ring details or gaps are needed.
  Keep orientation of the image the same as the orientation in the upper box.
- **Estimate the angular distance (in arcsec) and position angle (in degrees) of Titan relative to the center of Saturn.** You may do your calculations besides the answer.

Apparent major axis of the ring: 43"

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TASK 3: M57 – FIELD OF VIEW

available time: 10 minutes total: 10 points

- **Find the planetary nebula M57 (in constellation Lyra), and put it in the centre of the field of view in the main scope.**
- The star chart in the answer sheet shows a part of constellation Lyra. In this chart, **draw the FOV circle around M57 as accurately as possible.**

If you cannot find M57, the assistant will help you, but only after 5 minutes. In this case you will lose the marks for pointing to the object.

TASK 4: VARIABLE STAR – AF CYGNI

available time: 15 minutes total: 15 points

- **Use the given charts of the constellation Cygnus to find the variable star AF Cyg.**
  The large scale finder chart has normal orientation (N is up E is to the left)
  The smaller scale chart has 'telescope' orientation (S is up W is to the left)
  Brightness of reference stars are given without decimal points. e.g. '97' means 9.7 magnitude
  If you do not find AF Cyg, the telescope assistant cannot help you to point to it in this task.

- **Estimate the magnitude of AF Cyg by comparing it with the reference stars** and write it down, with decimal point, at one decimal accuracy (i.e. 9.7).
  Write the time of your observation in UTC. You may ask telescope assistant for the time in the local time zone (CEST).

TASK 5: NAKED EYE BRIGHTNESS ESTIMATION

available time: 5 minutes 5 points

- **Estimate the visual magnitude of the two naked-eye stars marked on the stellar chart of constellation Ursa Minor:**
  a) ζ UMi (zeta UMi = Alifa) – STAR 2
  b) γ UMi (gamma UMi = Pherkad) – STAR 1
  c) Write your estimate with one decimal accuracy (e.g. 8.6).
- **Estimate the angular distance between γ UMi (STAR 1) and Polaris in degrees.**

<u>TASK 1/ ALTERNATIVE</u>: FINDERSCOPE ALIGNMENT

available time: 5 minutes 5 points

- The finderscope is NOT aligned at the beginning. **Point the telescope to Altair (α Aql) and align the finderscope parallel to the main tube.**

If the alignment is not satisfactory, the telescope assistant will correct it – and you receive only partial or no points.

<u>TASK 2 / ALTERNATIVE</u>: EPSILON LYRAE

available time: 10 minutes 15 points

- **Find ε Lyr, and make a drawing of the field of view (with the object and other stars) with 10mm eyepiece.**
  Label the directions North and East by two arrows and mark them as 'N' and 'E'.
- **Estimate the angular distance between the wide pair (ε1-ε2), and estimate the position angle of the same pair.**
- **Increase the magnification with 2x Barlow lens to be able to resolve and separate the two close pairs. Estimate the angle** (in degrees to the nearest integer) **subtended by the two close pairs relative to each other.** (The enclosed angle of the two lines going through the two narrow pairs).
  Do not give any PA, only the relative angle of the two close pairs. No drawing is needed.

If you cannot find ε Lyr, the assistant can point to it for you, but only after 5 minutes. In this case you will lose the marks for pointing the telescope to the object.
The telescope assistant will check the object at the end of the 10 min limit. If you are ready sooner, keep the star in the FOV, and wait for the check.

GeCAA 2020 – Estonia (host)

Due to CoViD-19 pandemic situation, IOAA 2020 was cancelled. For the benefit of participants who were selected in different national competitions to participate in IOAA 2020, the IOAA international board had organised Global e-Competition on Astronomy and Astrophysics from 25th September to 23rd October 2020.

Theory Round

1 Astrophotography

An astrophotographer, based on the Equator, uses a good digital camera on a tripod, but with no tracking. The camera has a telescopic lens with focal length of 150 mm and aperture (objective diameter) of 84 mm. The sensor has effective light collecting diameter of 22.5 mm. The photographic target is a star field at the observer's Zenith.

(a) **(2 points)** Calculate the field of view (the angular width of the image) which can be captured on the sensor using this equipment.

(b) **(5 points)** The pixels in the camera's sensor are separated by a distance of 3.23 µm. What is the maximum possible exposure time for a single frame, so that no star trails appear on the exposed image?

(c) **(3 points)** For a better-quality image of the star field, the photographer decides to take multiple shots at the exposure time calculated in b) and then to stack them together. The total time for all these shots is 30 s (ignore any time taken to write data to the memory card) What proportion of the total field of view is possible at the higher signal to noise ratio?

2 Flat Earth

(10 points) A new model of the world is gaining in popularity among some people. These people believe in the "Flat Earth" view of the world, where the Earth is not a spheroid, but rather a circle with radius $R_\oplus$. The central axis of the Earth (normal to the circle passing through its centre C) is passes through the observer's zenith.

This model must at least remain consistent with the observed phenomena, as listed below:

- The value of the solar constant is $S_\odot = 1366\ \mathrm{W/m^2}$.
- The Earth's central axis precesses in a circle with a period 25800 years.
- The radius of the precession circle is 23.5°.

We assume that the Earth is a perfect blackbody radiator and the Sun is sufficiently far away that all sun rays are parallel. Let us also assume that the Sun's current (initial) location is at the zenith.

Determine how many years it will take for the Earth's equilibrium temperature to decrease by 1 °C.

3 Mirror

A bored cosmologist comes up with a thought experiment to determine the Hubble constant (H_0) for his model of a Steady-State-Universe. In this experiment, a large, fully reflecting flat mirror – carrying several gyroscopes that would maintain its spatial orientation in the same plane – would be placed at a distance D from the Solar System in a region without gravitational influences. From the Earth, a laser beam would be directed towards that region for a long period of time. After a time T, the radiation would return and be detected, allowing the determination of the fixed constant H_0.

(a) **(7 points)** Find an expression for H_0 as a function of D, c (speed of light) and T. Consider that the separation S between the Solar System and the mirror increases only due to the expansion of the universe according to the law $S = se^{H_0 t}$, where s is the initial separation. You may use $e^x \approx 1 + x$ for $x \ll 1$, if necessary.

(b) **(3 points)** Imagine that such a mirror is located in the vicinity of the star Vega (which also features on the logo of the 1st GeCAA). Vega was the first star outside the Solar System to be photographed and one of the first stars whose parallax ($p = 0.125''$) was accurately measured in 1840 by G. W. von Struve.
Estimate the total duration of this H_0 measurement experiment.

4 Light Curves

The light curve A shown below, shows a fictional edge-on eclipsing binary system containing stars X (radius r_X, luminosity L_X) and Y (radius r_Y, Luminosity L_Y) . Assume that star X is brighter, but star Y is hotter.

(a) **(1 point)** Which of the two stars is likely to be on the main sequence? (Write "X" or "Y")

(b) Based on light curve A, estimate:

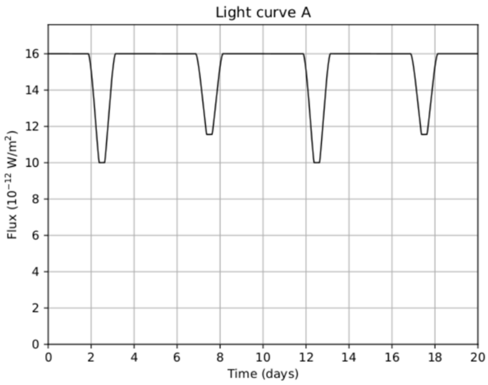
(I) **(2 points)** $\frac{r_X}{r_Y}$, the ratio of the radii of the two stars.

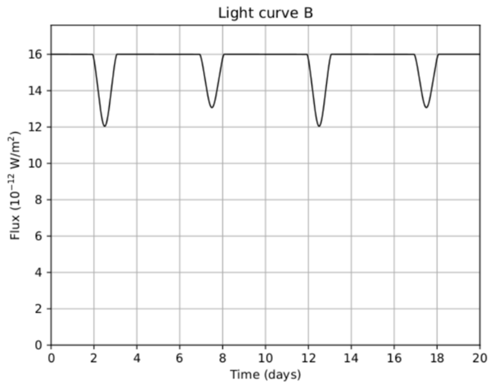
(II) **(2 points)** $\frac{L_X}{L_Y}$, the ratio of the Luminosity of the two stars.

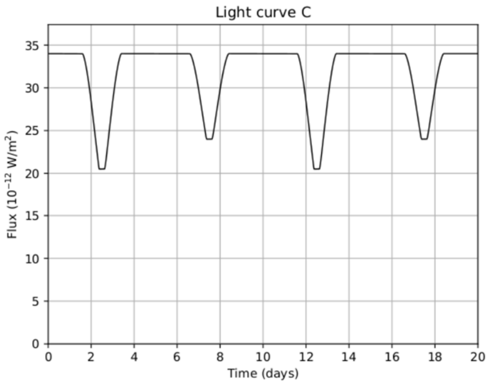
(c) **(15 points)** For light curves B to F, in each case only one parameter of the binary system has been changed from the case in light curve A. For each case, choose the description from the following list that best corresponds to the change (Write the appropriate roman numeral in the answer sheet).

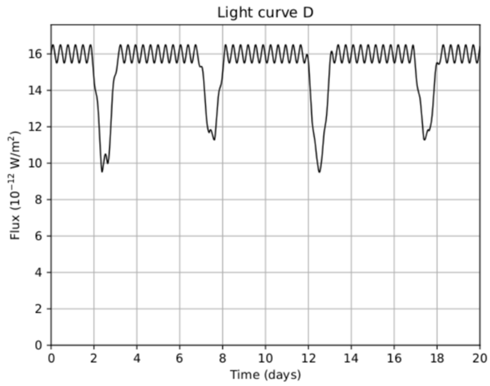
(i) Star X increased in size.
(ii) Star X increased in luminosity.
(iii) Star X decreased in size.
(iv) Star X decreased in luminosity.
(v) Star Y increased in size.
(vi) Star Y increased in luminosity.
(vii) Star Y decreased in size.
(viii) Star Y decreased in luminosity.
(ix) Star X is a variable star.
(x) Star Y is a variable star.
(xi) The inclination of the system relative to the Earth has changed.
(xii) The distance of the system from the Earth has decreased.
(xiii) The distance of the system from the Earth has increased.
(xiv) The orbital period of the system increased.

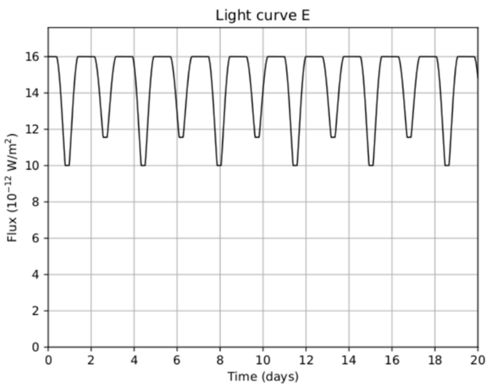
(xv) The orbital period of the system decreased.

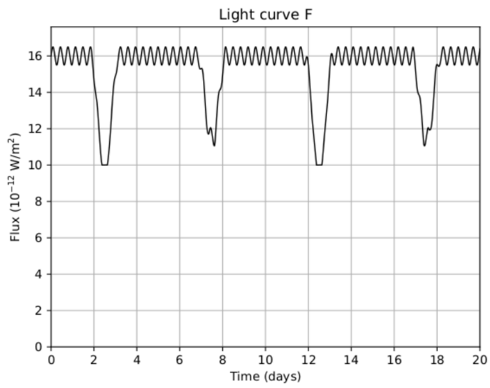












5 HII region

Luminous Blue Variable (LBV) are massive, unstable, supergiant stars that can undergo episodes of very strong mass loss, due to an instability in their atmospheres. After such an event, a dense nebula is formed around the star. LBV are also very hot stars and produce a large amount of high-energy photons that are able to ionise hydrogen atoms ($E_{\mathrm{ph}} > h\nu_0 = 13.6$ eV) creating a roughly spherical region of ionized hydrogen (HII region).

In this problem, we consider a static, homogeneous, pure hydrogen nebula with a concentration of $n_{\mathrm{H}} = 10^8$ m^{-3} and temperature $T_{\mathrm{HII}} = 104$ K, ionized by photons from a single LBV star with a stable rate of ionizing photons $Q = 1049$ ph/s. Assume that each photon can ionise only one hydrogen atom. At a particular location within an HII region, the rate of photoionization is balanced by the rate of recombination per unit volume. This sets the radius of the fully ionized region and this region is called the Stromgren sphere with the radius R_{S}.

The total number of recombinations per volume is proportional to the concentration of protons $n_{\rm p}$, the concentration of electrons $n_{\rm e}$ and the recombination coefficient for hydrogen $\alpha(T_{\rm HII}) = 10^{-19}\ {\rm m^3\ s^{-1}}$. For simplification, ignore the fact that the process of recombination can also release ionising photons.

(a) **(5 points)** Derive an algebraic expression for the radius of the Stromgren sphere and calculate its value for the given parameters. Express your answer in units of parsecs (pc).

(b) **(3 points)** The photoionization cross-section of H-atoms in the ground state encountering photons with frequency ν_0 is equal to

$$\sigma \approx 10^{-21}\ {\rm m^2}$$

Calculate the mean-free path l_{ν_0} of an ionising photon. Compare l_{ν_0} to $R_{\rm S}$ to determine if this ionized nebulae is sharp-edged or not? (answer "YES" or "NO")

(c) **(4 points)** On what timescale (in years) do you expect the Stromgren sphere to form?

(d) **(4 points)** Radiation from an ionized hydrogen cloud (HII region) is often called free-free emission because it is produced by free electrons scattering off the ions without being captured: the electrons are free before the interaction and remain free afterwards. In this process, the electron retains most of its pre-scattering energy. An electron, while passing by a much more massive singly ionized hydrogen atom, produces a radio photon of $\nu = 10$ GHz. Calculate the mean electron thermal energy in the HII region, for the given temperature of the Stromgren sphere. Is this an example of free-free emission? (answer "YES" or "NO")

(e) **(4 points)** Since the HII region is in local thermodynamic equilibrium, one can calculate the absorption coefficient that is proportional to the optical depth $\tau_\nu \propto \nu^{-2.1}$ and it turns out that at the sufficiently high radio frequencies, the hot plasma is nearly transparent and hence, $\tau_\nu \ll 1$.

The flux density of photons has power-law spectra of the form $S_\nu \propto \nu^\beta$.

Find β for the radio frequencies.

6 Occultation of a X-ray Source

Consider a satellite observing x-ray sources, while orbiting the Earth in the equatorial plane with orbital radius r, and orbital time period P. Let us assume that this satellite is pointed to one fixed direction in space for a given length of time. Take the radius of the earth as R.

When the satellite moves 'behind' the earth, naturally, the x-ray source is 'occulted' and the measured x-ray flux from the source drops to zero. However, due to Earth's atmosphere, this drop is gradual. If the line of sight of the source passes through the atmosphere, the attenuation depends on the air-mass (i.e. length of air column) along the line of sight.

(a) **(1 point)** Let us assume that pointing towards a fixed source at 0° declination. We consider that the source is occulted when 50% of the light coming from the source gets attenuated due to the atmosphere. Let us say that this happens when the minimum height of the line of sight from the surface of the Earth is h.

If θ_0 is the angle between the direction to the source and the direction to the Earth, as measured from the spacecraft, find an expression for θ_0.

(b) **(4 points)** The time duration Δt between the source getting attenuated from 90% of pre-occultation flux to 10% is defined as the 'occultation time' for the source. Assume the flux attenuates to 90% when the minimum height of the line of sight $(h + 0.5\Delta h)$ and similarly the flux attenuates to 10% at $(h - 0.5\Delta h)$, where $\Delta h \ll R$.
Find the expression for Δt in terms of r, P, Δh and θ_0.

(c) **(15 points)** If the satellite was pointing towards a source at declination β instead (β not too large), what will be the expression for Δt?

Note:
If the satellite was not in the equatorial plane, then the problem could have been simply rephrased by assuming the satellite's orbital plane to be the equatorial plane. In that case, β becomes 'relative declination'.

7 Radiant of a Meteor Shower

A stargazer in Chiayi, Chinese Taipei (23.5°N, 120.4°E, GMT+8) saw two meteors streaking through the sky at 21:00 (Chinese Taipei time) on 25$^{\text{th}}$ September 2020. One of the meteors appeared at horizon exactly due west and streaked to a point at 15° altitude directly above the northern horizon. The second meteor originated at an altitude of 23.5° and an azimuth of 210° and ended at an altitude of 75° and an azimuth of 255°.

(a) **(6 points)** What is the Local Sidereal Time (LST) at the time of observation?

(b) **(16 points)** Find the alt-az coordinates of the apparent radiant of the two meteors.

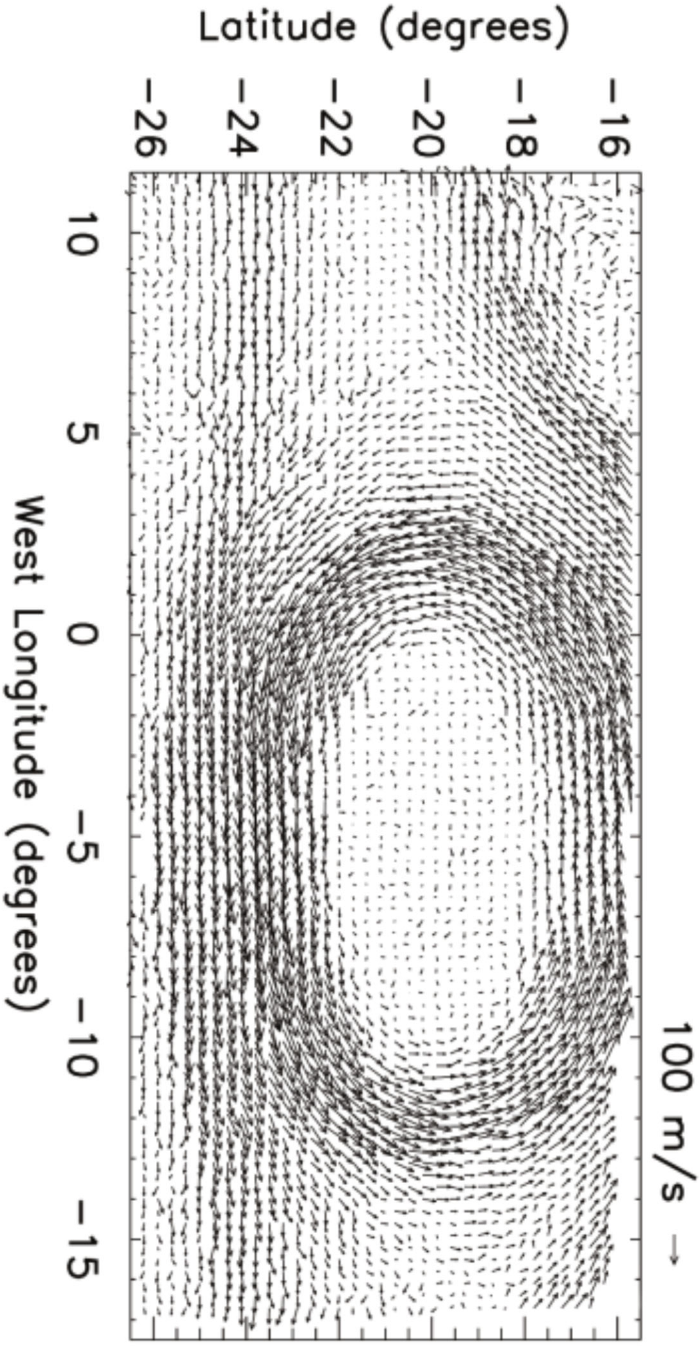
(c) **(6 points)** Find the equatorial coordinates of the apparent radiant.

(d) **(2 points)** Which of the following constellations is closest to the radiant?
Crux / Dorado / Pavo / Tucana / Triangulum Australes
(choose one and write the same name in the answer box)

Notes:

- Azimuths are measured from the North (0°) towards the East.
- The Greenwich Sidereal Time (GST) at 00:00 UT on 1$^{\text{st}}$ January 2020 is $6^{\text{h}}\,40^{\text{m}}\,30^{\text{s}}$.

8 Jupiter's Great Red Spot



In the following problem the fluid mechanics of Jupiter's Great Red Spot (GRS) is studied based on the velocity field data. The diagram on the next page shows a map of relative velocity for GRS and the surrounding region. The arrows are oriented and scaled as per the directions and magnitudes of winds at different points.

Due to the combined effects of gravity and rotation, Jupiter is slightly flattened at its poles. The equation of a spheroid approximating for the shape of Jupiter can be stated as:

$$\frac{x^2 + y^2}{R_e^2} + \frac{z^2}{R_p^2} = 1,$$

where $R_e = 7.15 \times 10^7$ m is the equatorial radius of Jupiter, and $R_p = 6.69 \times 10^7$ m the polar radius. The radii of curvature of this spheroid in any direction can be calculated by the following equations

($\epsilon = \frac{R_e}{R_p}$):

$$r(\phi) = R_e(1+\epsilon^{-2}\tan^2\phi)^{-1/2}$$

$$R(\phi) = R_e\epsilon^{-2}\left(\frac{r(\phi)}{R_e\cos\phi}\right)^3$$

where r and R are the zonal (aka in the zone of a particular latitude) and meridional (aka longitudinal) radii of curvature, respectively, as a function of planetographic latitude ϕ. The sidereal rotation period of Jupiter is $P = 3.57 \times 10^4$ s.

(a) **(4 points)** Calculate the zonal and meridional radii values ($\overline{r}$ and $\overline{R}$ respectively) at the location of the centre of the GRS.

(b) **(5 points)** Estimate the eccentricity of the GRS.

(c) **(6 points)** 'Vorticity' at any point is a measure of local spinning of the fluid as measured by an observer situated in the reference frame of the fluid. Mathematically, it is calculated as 'curl' (vector derivative product) of the velocity field. In this case, the average relative vorticity may be estimated by the equation:

$$\xi = \frac{V_w L_{\mathrm{GRS}}}{A_{\mathrm{GRS}}}$$

where V_w is the maximum value of winds as per the velocity field, L_{GRS} is the length of the circumference of the GRS and A_{GRS} is the area of the GRS.
Estimate average relative vorticity of the GRS.
Hint: The circumference of an ellipse is well approximated by an average of circumferences of the corresponding auxiliary and minor circles.

(d) **(2 points)** Find the absolute vorticity $\xi_a = (\xi + f)$ by adding the Coriolis parameter

$$f = 2\Omega\sin\phi$$

where Ω is the angular velocity of the Jupiter (due to axial rotation) and is the appropriate latitude.

(e) **(1 point)** If the absolute vorticity has the same sign as the latitude, we call the storm a 'cyclonic storm'. If they have opposite signs, the system is 'anticyclonic'. Is the GRS cyclonic or anticyclonic?

(f) **(12 points)** Imagine that the GRS moves to another latitude ϕ_1, where the absolute vorticity changes the sign (changes from anti-cyclonic to cyclonic or vice versa). Assuming minimum possible displacement of the GRS, at what value of ϕ_1 do we expect this change?
In your analysis, assume that the GRS at the new location would occupy the same angular span in latitude, as well as have the same wind velocities and eccentricity as the original.

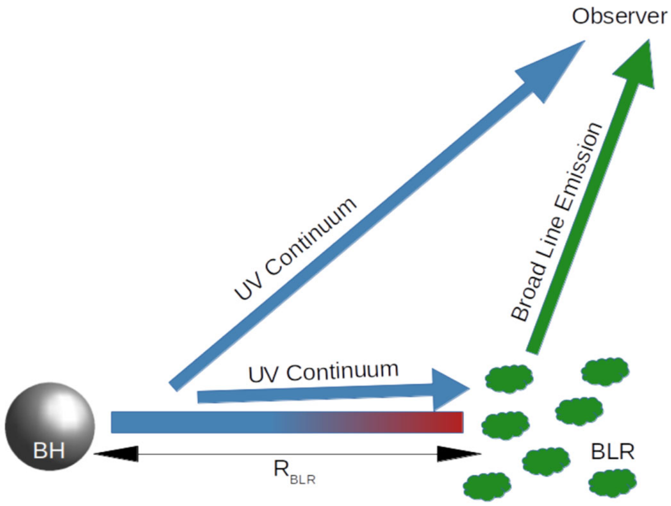
Data Analysis Round

1 AGN

It is believed that the accretion disk around supermassive black holes (BH) at galactic centres gives rise to UV thermal emission. This emission is associated with Active Galactic Nuclei (AGNs).

The optical spectra of bright AGNs show additional bright broad emission lines. Those emission lines arise from the dense gas in the Broad Line Region (BLR), which is ionized by the UV photons from the accretion disk. See the sketch to visualise this model.

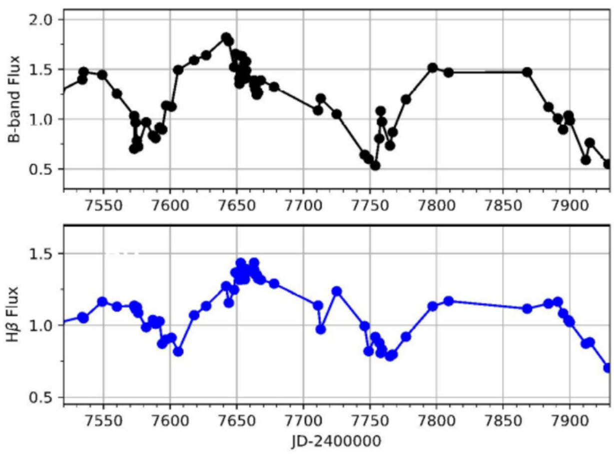
We can assume that the flux of broad emission lines varies in response to the variation of the



UV continuum with a time delay. This time delay should be proportional to the separation R_{BLR} between the BH and the BLR.

Assume that the size of the accretion disk is negligible as compared to R_{BLR}.

(a) **(1 point)** Estimate the time lag (days) between the B-band continuum and broad emission line ($\mathrm{H}_{-\beta}$) using the light curves shown below. The x-axis is in reduced Julian Dates (JD).



(b) **(3 points)** Estimate R_{BLR} in parsecs (pc).

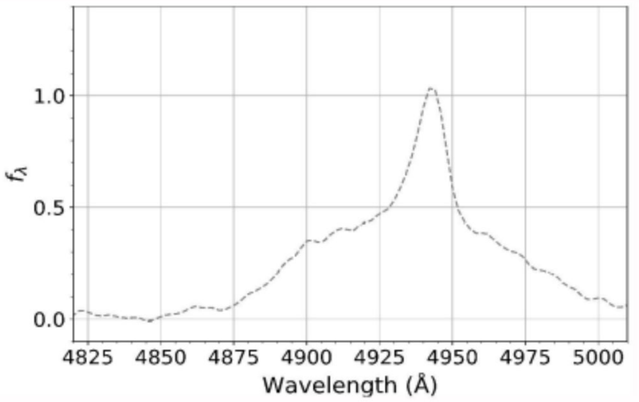
(c) **(2 points)** Estimate the angular separation of this region $\theta_{\rm BLR}$ (in arcsec) from the blackhole, if this AGN is 100 Mpc away from us.

It is possible to estimate the mass of the system using the Virial theorem, if the velocity dispersion of the gasses in the BLR and the size of the system are known. Assume that the masses of the accretion disk and broad line region are negligible, as compared to the black hole. The velocity dispersion v_σ may be estimated from the broadening of the given emission line. We will take the corresponding wavelength dispersion to be

$$\sigma = \frac{\text{FWHM}}{2.35}$$

where FWHM is the full width at half maximum of the broad emission line.

(d) **(5 points)** Calculate the velocity dispersion v_σ in units of km s^{-1}, from the spectral line shown below.



(e) **(4 points)** Calculate the mass of the central BH ($M_{\rm vir,BH}$) in a unit of $M_\odot$.

2 Minor Planet

Table 1 gives ecliptic longitude (λ) and parallax (p) at different times (t), for a certain hypothetical minor planet. The baseline for the parallax is the diameter of the earth. The time is expressed in years and for your reference ecliptic longitudes of the Sun($\lambda_\odot$) for the same dates are also given the table. Let us assume that the orbital inclination of this minor planet, with respect to the ecliptic, is negligible and the eccentricity of the Earth's orbit is negligible.

(a) **(38 points)** Calculate the coordinates of the minor planet in the heliocentric polar coordinate system and put them in a polar plot. The x-axis in the plot should be directed towards the initial position of the minor planet. Draw the major axis of the orbit of the minor planet.

Identify erroneous observation(s), if any.

(b) **(6 points)** Assuming the heliocentric orbit of the minor planet to be elliptical, determine

(i) the semimajor axis length $a_{\rm p}$.

(ii) eccentricity e.

(iii) the period P.

(c) **(6 points)** Estimate the errors in the values of P, a_p and e.

Table 1: Minor planet data

t [year]	λ [°]	$\lambda_\odot$ [°]	p [″]
2012.3	336.73	40.95	3.82
2012.6	3.44	134.83	7.24
2012.9	50.71	242.08	7.09
2013.4	94.52	64.84	2.40
2013.6	121.40	134.59	2.16
2013.9	154.31	241.82	2.75
2014.2	25.33	353.29	3.16
2014.5	148.51	99.04	1.99
2014.8	176.26	205.45	1.83
2015.0	216.33	280.19	2.03
2015.3	187.5	28.55	2.897

3 Hypervelocity stars

In recent years, a new field of research has emerged, that of Hypervelocity Stars (HVS for short). These are stars in our Galaxy (mostly at its outskirts), which are moving with excessive velocities and may be escaping from the Milky Way.

In this question, you will use spectroscopic and astrometric measurements in order to calculate the velocity of one such star, called "HVS1", consider its origin and whether it may escape the Galaxy.

Figure 1 shows a spectrum of HVS1 in the blue to UV part of the spectrum:

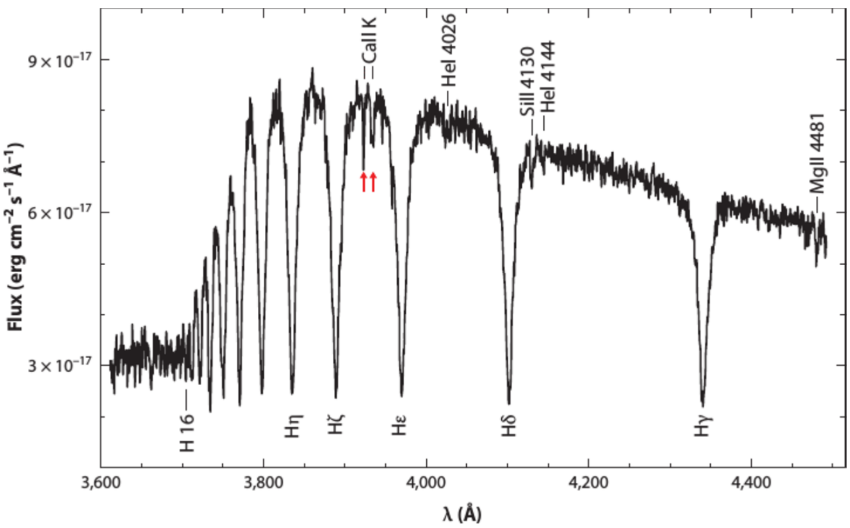


Figure 1: *The spectrum of HVS1 shifted to the rest frame of the star (i.e., there is no Doppler shift due to the motion of the star along the line of sight).*

(a) **(7 points)** Determine the spectral type of the star using the standard spectra in Appendix 3 and the absorption lines identified on the spectrum of HVS1. (Note that the spectrum above contains both stellar and interstellar absorption lines.)

(b) **(18 points)** Detailed modeling of the spectral lines places the star between luminosity classes V (Main Sequence) and IV (subgiant).

i. The apparent magnitude of the star in the visual band is $m_V = 19.84$. Find the absolute magnitude M_V of the star using Appendix 1 for the two possible luminosity classes.

You may ignore the uncertainty in m_V since the uncertainty in your calculation will be dominated by the uncertainty in M_V .

ii. For both these possible luminosity classes, calculate the star's distance from the Sun, ignoring interstellar absorption.

iii. The galactic coordinates of HVS1 are $l = 227.335\,372\,67°$, $b = 31.331\,993\,86°$. Is the assumption of ignoring the interstellar absorption justified? Write "YES" or "NO".

iv. The Gaia mission of the European Space Agency has been mapping the Milky Way since 2014, measuring the parallax and proper motion of 1.5 billion stars to an accuracy between 0.04 and 0.1 milliarcseconds (mas). Could Gaia have measured the parallax of HVS1,

(A) if it is a MS star? Write "YES" or "NO".

(B) if it is a subgiant? Write "YES" or "NO".

For the rest of this question, adopt the larger of the two distances you have calculated above.

v. Assume that the distance of the Sun from the Galactic center is 8.0 kpc. Make a rough sketch of the relative positions of HVS1, the Sun and the Galactic center. Use it to calculate the distance (r) of HVS1 from the Galactic center.

(c) **(17 points)** Here, you will calculate the actual velocity of HVS1.

i. The spectrum in Fig1 shows two absorption lines due to Ca II. One is caused by the atmosphere of the star and the other is due to the interstellar medium. The shift of this line is due to the motion of the star with respect to the interstellar medium. Measure this Doppler shift and calculate the radial velocity of HVS1 with respect to the Sun.

ii. We are interested in the velocity with respect to the Galactic center. For this we first need to take into account the velocity of the Sun due to the rotation of the Galaxy. The following equation transforms the velocity of a star of heliocentric radial velocity v_{rHC} to one in the Galactic rest frame (rf), v_{rf} :

$$v_{\mathrm{rf}} = v_{\mathrm{rHC}} + 11.1 \cos l \cos b + 247.24 \sin l \cos b + 7.25 \sin b$$

where the speeds are measured in km s^{-1}.

Find v_{rf} for HVS1.

iii. HVS1's proper motion has been measured as:

$$(\mu_\alpha, \mu_\delta) = (+0.08 \pm 0.26, -0.12 \pm 0.22) \text{ mas/yr}.$$

Calculate the tangential velocity component (in km s^{-1}) of HVS1. (You may ignore the correction for declination as the star is near the celestial equator).

iv. Calculate the velocity v_T of the star with respect to the Galactic center (magnitude in km s^{-1} and angle with respect to direction of the Galactic center).

v. Assuming this star was born within the Galactic disc, use your calculation of the velocity to estimate where in the Galactic disc it is more likely to have come from:
(A) near to the Galactic center

(B) further out in the Galactic disc

(d) **(6 points)** From the energy considerations,

i. Write expression for the escape speed v_{esc} as a function of the distance from the Galactic center and the enclosed mass.

ii. Calculate the mass of the galaxy (in solar masses) within the radius of the distance of HVS1.

$$M_r = 4\pi\rho_0 r_c^2 \left[r - r_c \tan^{-1}\left(\frac{r}{r_c}\right)\right]$$

where $r_c \approx 8$ pc is an constant of the equation and $\rho_0 = 1.396 \times 10^4\ M_\odot\ \mathrm{pc}^{-3}$.

iii. Calculate the magnitude of the escape velocity at the distance of HVS1.

iv. Is this a runaway star? Write "YES" or "NO".

(e) **(2 points)** How long has it taken for HVS1 to reach this position?

(f) **(3 points)** On the basis of the spectral type and the luminosity class of this star, estimate the age of HVS1 and compare this with your result in the previous part. Which one of the following statements about the origin of the star is true:
(A) The star was ejected when or shortly after it was formed.
(B) The star was ejected mid-way through its time on the Main Sequence.
(C) The star was ejected towards the end of its time on the Main Sequence.

(g) **(2 points)** Astronomers looking for HVS-s start by finding a sample of stars in the Galactic halo which are of a spectral type similar to that of HVS1. Explain why by choosing which one of the following statements is true:
(A) Stars of this spectral type are young and so belong to the native population of the halo.
(B) Stars of this spectral type are old and so belong to the native population of the halo.
(C) Stars of this spectral type are young and so do not belong to the native population of the halo.
(D) Stars of this spectral type are old and so do not belong to the native population of the halo.

GeCAA - Observation

Constants and materials

Here we provide all links that are inserted for the student:

In theory round there are 4 questions with a sum of 80 points, point value of question should be in accordance with question difficulty level. Solving time is 1:30h
Points for questions are:

1. 15 points
2. 10 points
3. 40 points
4. 15 points

Correct answers in bold

Comets in the "air"

(5 points) The figure below shows a star chart of the night sky. The location of comet C/2020 F3 Neowise on July 31st, 2020 is marked by a red dot.

Name the five brightest stars in the field shown. Please use IAU star names in your answer (i.e. like Sirius or Rigel).

The brightest stars[5]

Sort the brightest stars visible on the figure in descending order of brightness.

A. **1st brightest - Arcturus**
B. **2nd brightest - Regulus**
C. **3rd brightest - Pollux**
D. **4th brightest - Spica**
E. **5th brightest - Capella**

[CATEGORIES]

1. Achernar
2. Acrux
3. Aldebaran
4. Altair
5. Antares
6. Arcturus
7. Canopus
8. Capella
9. Fomalhaut
10. Hadar
11. Pollux
12. Procyon
13. Regulus
14. Shaula
15. Spica

Where is the Sun? [2]

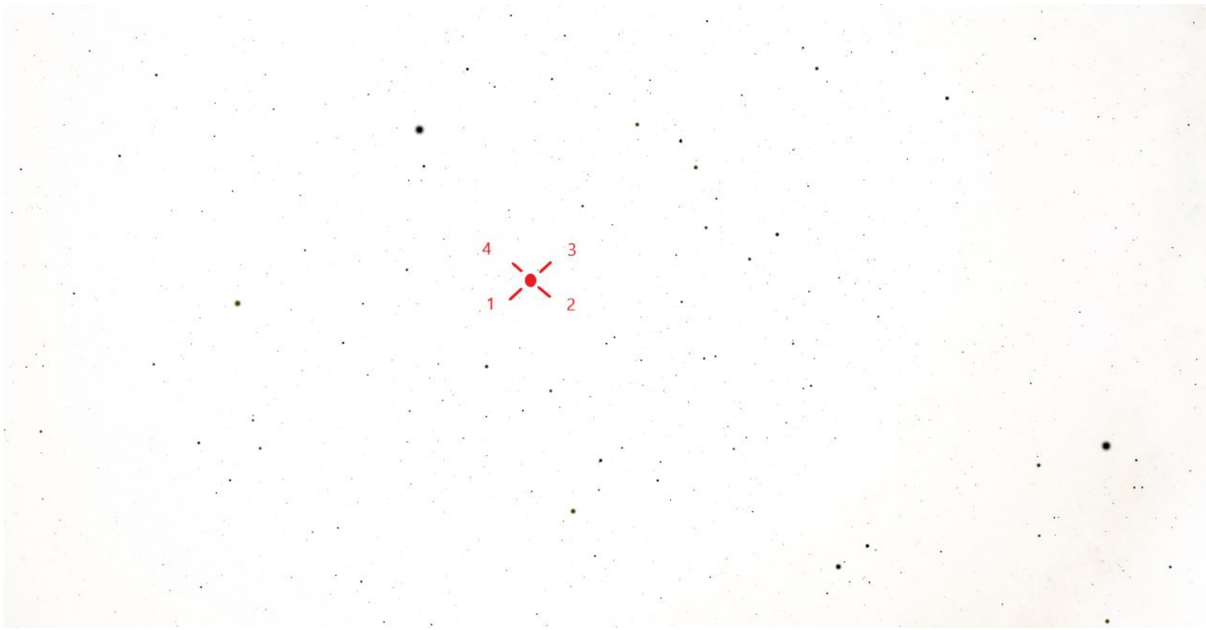
(2 points) Write the latin name abbreviation (you can find accepted abbreviated names here: https://en.wikipedia.org/wiki/IAU_designated_constellations) of the constellation in which the Sun is present on 31st July 2020.

(Cnc)

Point on chart[3]

(3 points) Mark the position of the Sun on the chart, in case it is not present on the chart, mark the direction to the Sun at the edge of the image.

[CANVAS] - not graded



1. {"name":"Sun","x":0.57899,"y":0.91937”}

Tail of the comet Neowise [2]

(2 points) Mark which line on the chart corresponds most accurately to the position of the comet’s gas tail (1, 2, 3 or 4 as indicated on Figure). Write the correct number as your answer.

[4]

In which constellation is comet Neowise [3]

(3 points) Name the constellation in which the comet is seen in Figure 1. Write the answer using the IAU abbreviation

Com

Neowise with MAGIC

The Figure shows the Astronomy Picture of the Day on July 24, 2020 (image credit & copyright: Urs Leutenegger, https://apod.nasa.gov/apod/ap200724.html) taken near the MAGIC telescopes at European Northern Observatory. Comet C/2020 F3 Neowise is visible in the image.

Photographer’s location [5]

Estimate the latitude of the telescope’s location. (5 pts)
[NUMBER: 30 (10%)]

When was this picture taken? [3]

(**3 points**) Is this picture taken in the morning, evening or midnight sky

A. Morning sky
B. Evening sky
C. Midnight

Comet tails[2]

(**2 points**) Two tails of the comet are visible in Figure 2. Which tail is the gas and which one is the dust tail?

A. The Dust tail is on the left and the gas tail is one the right of the image.
B. The Gas tail is on the left and the dust tail is on the right of the image.

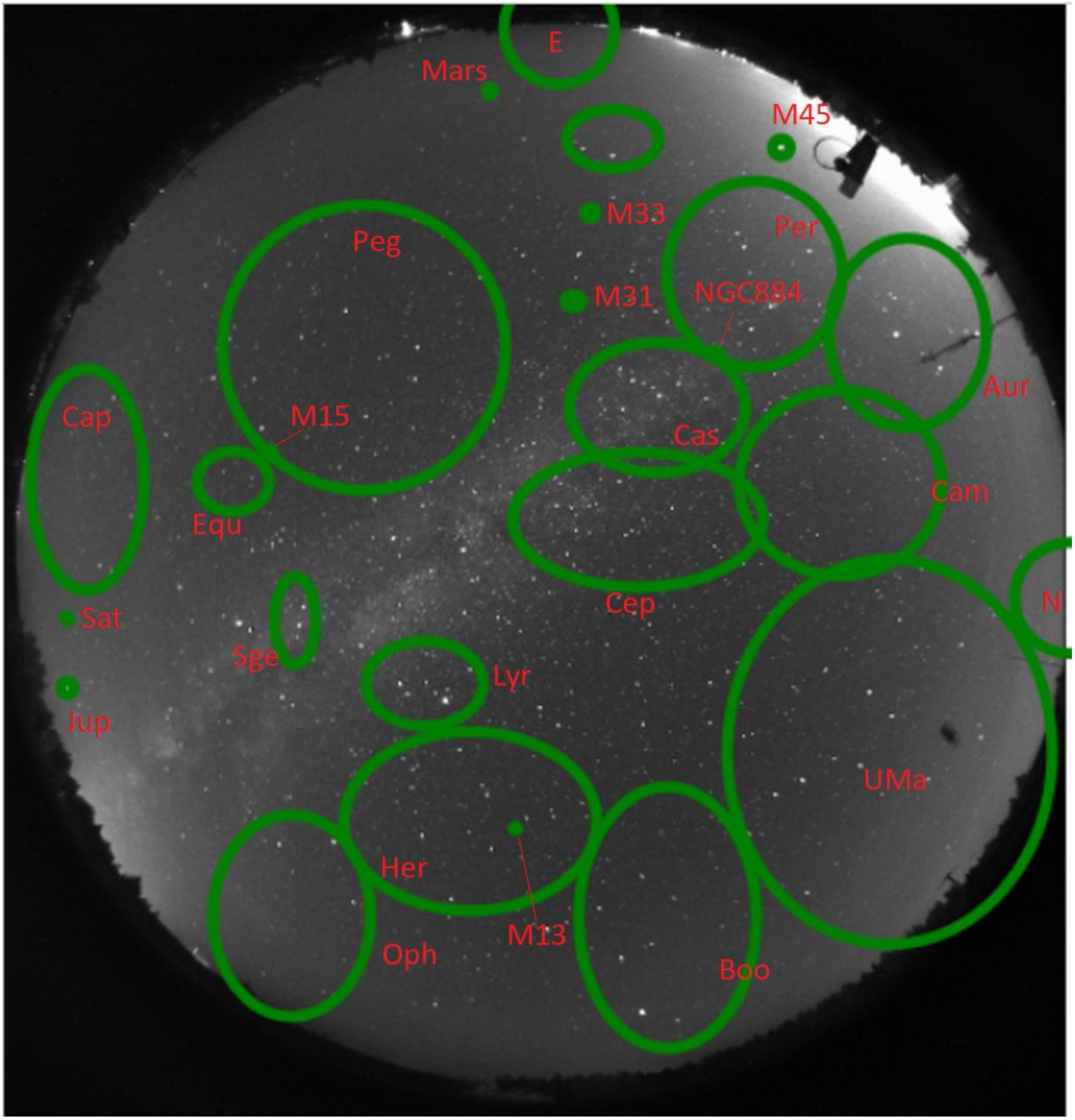
All sky

The Figure shows an all-sky image taken somewhere at local midnight.

Directions in the sky

(**26 points**) In this question you are asked to mark the following visible objects or directions

- Mark north and east directions on the horizon by clicking with the mouse. (4 pts)
- Mark all Solar System objects that are visible (3 points)
- Identify and mark on the all-sky image the following deep-sky objects: M31, M13, NGC 884, M45, M33, M15 (6 pts)
- Identify and mark the following constellations by clicking on their approximate center: Pegasus, Ophiucus, Ursa Major, Cepheus, Capricornus, Camelopardalis, Cassiopeia, Lyra, Sagitta, Perseus, Equuleus, Aries, Hercules, Bootes, Auriga (15 pts)



1.

Geographic latitude of an observatory [4]

(**4 points**) Estimate the geographic latitude of the site where the image was taken. The right ascension of Altair and Capella are $19^h\ 51^m$ and $5^h\ 17^m$, respectively. Write the latitude in integer format.

[**58 (accepted error 5%)]**

Sidereal time [4]

(4 points) Estimate the approximate sidereal time when the image was taken. The right ascension of Altair and Capella are $19^h\ 51^m$ and $5^h\ 17^m$, respectively. Please give your answer using the format HH:MM !

Answer: **20:21 - 20:51 (20:36 +- 15 min)**

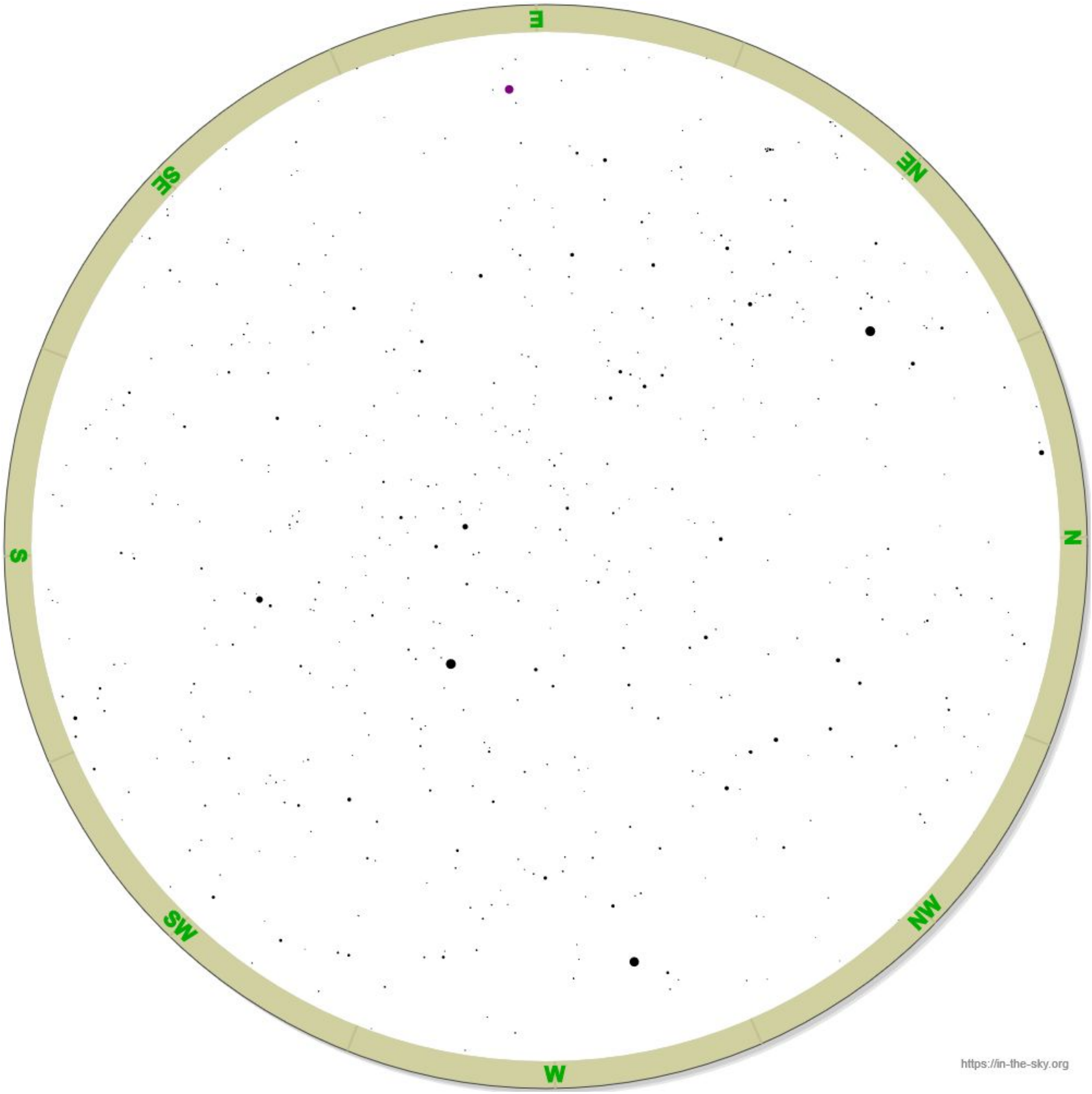
Galactic equator [6]

(**6 points**) On the Figure, the Galactic Equator passes through 12 constellations. Mark all the constellations that are on the Galactic Equator in this Figure.

Galactic equator: **Aquila, Auriga, Camelopardalis, Cassiopeia, Cepheus, Cygnus, Perseus, Sagitta, Sagittarius, Scutum, Taurus, Vulpecula**

Skymap

On the Figure, a star chart is shown for Tallinn, Estonia (Lat 59.43 N, Long: 24.75 E) on 14th September 2020 at 22:00 (UTC+3). The chart is not distorted and shows all altitudes from 0° to $+90^{\circ}$. Stars to magnitude $+4.7^m$ and one planet are shown.



Planets [4]

(**4 points**) Four relatively bright (about 1.5^m - 3.5^m) stars in well-known constellations or asterisms are missing. Identify them (in any order) using the Bayer classification.

Mark all the planets that should be visible at this time on this chart. Mars is marked as a red dot:

A. Mercury
B. Venus
C. Earth
D. Jupiter
E. Saturn
F. Uranus
G. Neptune

NB! Marking Earth was penalized -2 p and Venus and Mercury -1point, minum score was 0

Missing stars[8]

(8 points) Mark the missing stars indicating their rank order as greek letter with IAU Designation

epsilon Cassiopeiae, gamma Pegasi, beta Tauri, delta Ursa Majoris

Mars [3]

(**3 points**) What is the RA of Mars (to nearest 10 minutes, **write in format HH:MMm, where H and M-s are replaced with correct numbers, round answer to nearest 10 minutes, so it must end with “0” for example 12:10**)?

01:40 - 02:00 (01:50+- 10m).

Measuring the distance to the Moon[1]

The ancient Greek astronomer Hipparchus (c. 190–c. 120 BCE) noticed that the Moon shows *parallax*, that is, it appears in a slightly different place in the sky relative to background objects when seen at the same time from different locations on Earth. Knowing the distance between the two locations and having descriptions of the position of the Moon against the Sun during an eclipse (and so, at the same time), he was able to determine the distance to the Moon using trigonometry.

This year, the members of each team are spread out across the globe, which makes for a unique opportunity to perform an analogous experiment and use simultaneous observations from different places on Earth to determine the distance to the Moon in a similar way.

However, the distance of the Moon from the Earth varies over time due to the slight eccentricity of its orbit. The Moon is closest at perigee with a distance of about 360 000 km (called a "supermoon" if the Moon is full) and furthest at apogee, with a distance of about 405 000 km (called a "micromoon"). You are therefore asked to find the distance to the Moon at a specific time.

Your task, as a team, is to plan and carry out a series of observations and calculations to determine the distance of the centre of the Moon from the centre of the Earth at 12:00 UT on 6 October 2020 as well as you can.

1. You may use any observational methods (visual, photographs, video ...), and different members of the team can use different methods, but your final result must be based *only* on observations made by members of the team.
2. You can perform your observations at any time during the Team Competition period (28 September to 12 October) and as many times as you need to. (Note that you do not actually have to observe the Moon exactly at 12:00 UT on 6 October.)
3. If simultaneous observations are impossible you can interpolate the position of the Moon from your own observations, but you must use *only* your own observations with an explanation. Do not use table values or planetarium programs to "guess" the position of the Moon.
4. The more observations you make from different places and the more simultaneous they are, the more accurate your final answer will be.
5. You can look up the diameters of the Earth and Moon (if you need to) and the geographical coordinates (latitude and longitude) of your locations, but any distances (on Earth or to the Moon) must be calculated by you, showing your method.
6. You should prepare a short "paper" which includes at least:
   (a) a brief description of your observations (a short paragraph for each observing "session") including the weather conditions and a discussion of sources of error;
   (b) a table of all of your measurements with estimates of uncertainty;
   (c) discussion of any systematic effects;
   (d) a description of your calculations, including any graphs and diagrams;
   (e) the final result with an uncertainty derived from the measurement errors;
   (f) a short discussion of the result;
   (g) references/bibliography.

[1]In memory of Prof. John H. Seiradakis (1948–2020), team leader from Greece and one of the founders of the IOAA.

7. If planned observations failed for any reason, mention this.

8. Any numerical values taken from external sources (e.g. your location) should be marked and you should provide a reference to the source.

9. You should include an attachment with the raw data (images etc.) or a link to download them (Google, Dropbox etc), so that it is possible to reproduce your work.

10. You should also prepare a short presentation (either as a Powerpoint or PDF of around 10 slides, or as a video no longer than 10 minutes) presenting your team: who you are, where you are from, how the observations were made, and a summary of the results and conclusions. Think of what you would say to present yourselves and your experiment to the other participants at a conference.
    The presentations may be put on the IOAA website, so please do not include detailed personal information (first name/country/nearest large city is enough).

The Academic Committee will be looking for careful and creative planning, observation, calculation and assessment of uncertainties rather than necessarily obtaining a result closest to the "textbook" value or with the smallest uncertainty "on paper". Good scientific methodology counts for more than a lucky result or expensive equipment!

Spectral Distortions of the CMB

In this problem, you will investigate whether a proposed satellite mission, the Primordial Inflation Experiment (PIXIE, Kogut et al., 2011, 2016), will be able to detect spectral distortions of the Cosmic Microwave Background (CMB).

Over the past 30 years, the CMB has provided research opportunities for *precision cosmology*[1], the results of which have solidified our understanding of the history of our universe. The spatial anisotropies of the CMB have been at the forefront of both experiment and theory, and have driven the field forward. As per our understanding, these spatial anisotropies can reveal details about the size and physical properties of the primordial fluctuations in matter distribution leading to the rise of the structure that we see today. The interaction of matter with photons, on the other hand should lead to spectral anisotropies. Such anisotropies have been theoretically predicted, but have never been detected so far.

In 1992, the FIRAS instrument on NASA's Cosmic Background Explorer (COBE) (Fixsen et al., 1996; Mather et al., 1994; Fixsen, 2009) mission showed that the CMB spectrum is a near-perfect blackbody, isotropic to the level of one part in 10^5, with temperature $T_{\mathrm{CMB}} = (2.725 \pm 0.001)\,\mathrm{K}$.

Radiation is thermalized at very high redshifts (well before and up until the decoupling of radiation and matter) due to a number of radiative processes, such as Compton, inverse Compton, double Compton scattering and bremsstrahlung, each one dominating at different energy regimes (and thus redshifts).[2]

Physical processes such as reionization and structure formation, decaying or annihilating particles, dissipation of primordial density fluctuations, adiabatic cooling of matter and recombination, as well as cosmic strings, primordial black holes, small-scale magnetic fields, can lead to distortions to the blackbody shape of the spectrum of CMB (Chluba, 2014; Chluba and Jeong, 2014; Hill et al., 2015; Tashiro, 2014).

Spectral distortions of the CMB, if present, were not detectable at the precision level of the FIRAS instrument. PIXIE will provide a sensitivity improved by 76 times compared to its similar predecessor.

As the Universe expands, processes responsible for maintaining thermalization become less effective. This then allows spectral distortions to be developed. The distortions created between redshift $z \approx 10^6$ (when double Compton scattering decouples) and $z \approx 10^5$ (when Compton scattering no longer contributes to thermalization) are characterized by a chemical potential[3] and are called "μ distortions" (Zeldovich and Sunyaev, 1969; Sunyaev and Zeldovich, 1970; Illarionov and Siuniaev, 1975; Sunyaev and Khatri, 2013). At redshifts below 1.5×10^4, after thermalization decoupling, Compton and inverse Compton scattering produce the so-called "y distortions"[4] (Zeldovich and Sunyaev, 1969; Sunyaev and Zeldovich, 1972). For $1.5 \times 10^4 < z < 2 \times 10^5$, we get a distortion spectrum that is intermediate between those of y and μ-type distortions (Khatri and Sunyaev, 2012).

The two different types of distortions have different frequency dependence and this can suggest the type (as well as redshift) of interactions that give rise to spectral distortions.

Assume PIXIE will have the sensitivity as described in Table 1

[1] The branch of cosmology that makes detailed quantitative predictions and measurements of properties of the Universe

[2] You can read more at `https://ned.ipac.caltech.edu/level5/Sept05/Gawiser2/Gawiser1.html`

[3] The chemical potential μ is a quantity that expresses energy that is absorbed or released when the number of particles -in this case, photons- in a system changes. Here, it is measured in units of kT and is thus dimensionless.

[4] y is a parameter that expresses by how much a photon will change its energy due to repeated scatterings in a medium of finite extend.

Frequency [GHz]	14.4	374.6	734.9	1095.1
PIXIE Sensitivity [Jy/sr]	3.6035	3.7700	4.2859	5.1598

Table 1: PIXIE Sensitivity (Jy refers to the unit Jansky)

Spectral Distortions Modeling

y distortion PIXIE can shed light on the history of star formation by looking at the spectral distortions produced during the epoch of reionization. The CMB photons undergo inverse Compton scattering off the gas that has been ionized by early stars. These distortions are parametrized by y which expresses the mean number of scatterings times the average energy change that a photon suffers in a scattering. The respective intensity contribution is given by:

$$\Delta I_{\nu,y} = I_0 \frac{x^4 e^x}{(e^x - 1)^2} \left[x \coth\left(\frac{x}{2}\right) - 4 \right] y, \tag{1}$$

where $I_0 = \frac{2h}{c^2}(\frac{kT_0}{h})^3 = 270\,\mathrm{MJy/sr}$ for $T_0 = 2.726\,\mathrm{K}$, and $x = \frac{h\nu}{kT_0}$. We take $y = 1.7 \times 10^{-6}$ for our fiducial value (assumed value for comparison).

μ distortion The amplitude of density fluctuations during inflation translates into energy injection in the CMB for redshifts $10^5 < z < 10^6$ which leads to μ distortions characterized by the chemical potential, with intensity given by

$$\Delta I_{\nu,\mu} = I_0 \frac{x^4 e^x}{(e^x - 1)^2} \left[\frac{1}{\beta} - \frac{1}{x} \right] \mu, \tag{2}$$

where $I_0 = \frac{2h}{c^2}(\frac{kT_0}{h})^3 = 270\,\mathrm{MJy/sr}$ for $T_0 = 2.726\,\mathrm{K}$, $x = \frac{h\nu}{kT_0}$ and $\beta = 2.1923$.

A suitable fiducial value for μ is $\mu = 2 \times 10^{-8}$ (Abitbol et al., 2017).

Your task, as a team, is to calculate the expected standard deviations of the y and μ parameters, if PIXIE were to try to measure them in the bands given in Table 1. This will tell us if the y and μ distortions will be detectable, assuming the fiducial values given above.

In order to calculate that, you can use the matrix[5]:

$$F_{ij} = \sum_{a,b} \left(\frac{\partial(\Delta I_\nu)}{\partial p_i} \right)_a C_{ab}^{-1} \left(\frac{\partial(\Delta I_\nu)}{\partial p_j} \right)_b \tag{3}$$

called the Fisher Information Matrix (see for example Verde, 2010). a, b are indexing frequency, and p_i, p_j are parameters of the model (in our case, the two free parameters for the spectral distortions, y and μ, all other parameters assumed fixed). C_{ab}^{-1} is the inverse of the covariance matrix of the experiment. Inverting the F_{ij} matrix and taking the square root of the diagonal gives us the standard deviations expected for each parameter.

(a) **(18 points)** Make a plot of $\Delta I_{\nu,y}$ and $\Delta I_{\nu,\mu}$ over the range of frequencies 14.4GHz to 1100.1 GHz, assuming the fiducial values for y and μ.

[5] For students not familiar with matrix notation and/or partial derivatives, a basic introduction for these can be found in any good introductory textbook on calculus / analysis

(b) **(10 points)** Calculate the analytical forms of $\frac{\partial \Delta I_{\nu,\mu}}{\partial \mu}$ and $\frac{\partial \Delta I_{\nu,y}}{\partial y}$.
Then, evaluate them at the 4 frequencies given in Table 1.

(c) **(8 points)** Calculate the covariance matrix C_{ab} for PIXIE sensitivity in frequency bands using Table 1. The covariance matrix has the variance (square of the sensitivity) of the frequency bands on the diagonal, and the covariance of the bands on the other entries. In this problem, assume the frequency bands are uncorrelated and their covariance is thus 0.

(d) **(4 points)** Calculate the inverse C^{-1}_{ab} of the PIXIE covariance matrix.

(e) **(20 points)** Assuming PIXIE is trying to model only the y and μ distortions, and ignoring other foregrounds, **calculate** the Fisher Information Matrix, and the standard deviations for the parameters y and μ.

(i) Would the y distortion be detectable with PIXIE?

(ii) Would the μ distortion be detectable with PIXIE?

For the distortion to be detectable, the standard deviation on the parameter should be less than the value of the parameter.

Notes:

- Useful Constants and Units
  - $h = 6.626\,070\,04 \times 10^{-34}\,\mathrm{J\,s}$
  - $k = 1.380\,648\,52 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
  - $1\,\mathrm{Jy} = 10^{-26}\,\mathrm{W/m^2/Hz}$
- In this analysis, for each given frequency, the only free parameters of the functions $\Delta I_{\nu,y}$ and $\Delta I_{\nu,\mu}$ are y and μ respectively.
- A partial derivative $\frac{\partial f}{\partial x}$ of a multi-variable function $f(x, y, z, ...)$ with respect to a single variable x is the process of calculating derivative with respect to the said variable x, while all the other variables are kept constant.
- In reality, PIXIE will have to model other foregrounds as well, such as the foregrounds coming from the synchrotron emission in the galaxy, the thermal radiation from the dust present in the interstellar medium, the free-free emission, cosmic infrared background etc. But we will ignore these contributions for the purpose of the problem.

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IOAA 2021 – Bogota, Columbia

The 14th IOAA was held from 14th to 21st November 2021 in hybrid mode, with academic committee and IOAA executive committee in Bogota, and team leaders and students joining remotely from around the world.

The logo is conformed by the acronym IOAA, where the first letter is represented by the silhouette of the building of the National Astronomical Observatory of Colombia, the oldest observatory in America. It is located in Bogotá where it was founded in 1803. The capital city of Colombia is bordered by two famous hills, Monserrate and its neighbor Guadalupe, which are icons of Bogota's cityscape that decorate the logo's background. The map of Colombia is placed inside the letter O. A star is displayed at the base of the same letter

LIGO (5 points).

The first detection of gravitational waves GW150414 was announced in 2016 by the collaboration LIGO (Laser Interferometer Gravitational-Wave Observatory). The detected signal corresponds to the merger of two black holes with masses of $35M_{\odot}$ and $30M_{\odot}$, which when joined formed a black hole of $62M_{\odot}$. Ignoring the rotational energies of the black holes, you may assume that the energy released by this process (E_{GW}) is emitted solely in the form of gravitational waves, that were observed by the interferometer in 2015. You are given that the explosion of a supernova (SN) releases $E_{SN} = 2 \times 10^{44}\ J$.

1.1	To find out which of these two events (SN, GW) releases more energy, estimate the energy ratio $\frac{E_{SN}}{E_{GW}}$.	5.0pt

Temperature of the Earth (10 points).

For at least the last few million years, the Earth has been in roughly thermal equilibrium with the radiation from the Sun at the Earth's orbital distance.

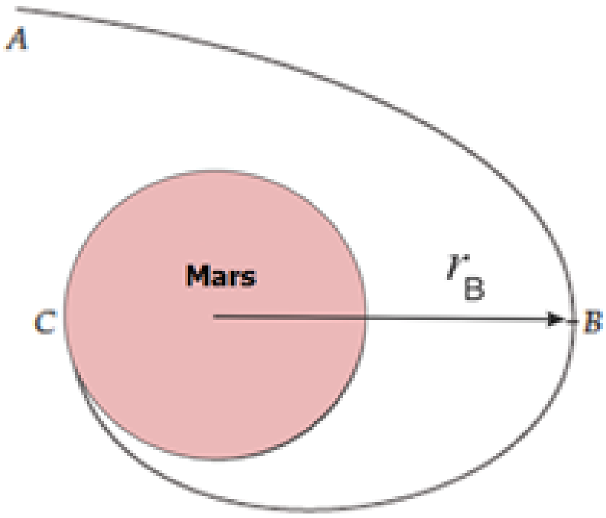
2.1	Assuming our planet to be an ideal black body, calculate what the Earth's equilibrium temperature (in Celsius) would be.	4.0pt

2.2	The Earth's albedo is approximately 30%. Calculate the Earth's surface temperature (in Celsius) considering its albedo.	2.0pt

2.3	The Earth's absorbed radiation is reemitted as black body radiation from its surface, but its atmosphere re-absorbs 58% of that energy, causing a greenhouse effect. Considering this effect, calculate the Earth's surface temperature (which will be the same as the temperature of the lower atmosphere). Give your answer in Celsius. For simplicity, consider the reabsorption effect as happening only once, and do not consider the atmosphere as a separate black body.	4.0pt

Mars (10 points).

A spacecraft of mass m=5.0×10^4 kg approaches in a parabolic orbit AB, with respect to Mars. When the spacecraft reaches point B of least distance to the center of Mars, $r_B = 6.8 \times 10^6$ m, it undergoes an instantaneous deceleration using its rockets and goes into a perfectly calculated orbit so that it will touch the Martian surface exactly at point C, diametrically opposite B, as shown in the figure.



3.1	Determine the speed ($km\,s^{-1}$) of the spacecraft at point B just before the deceleration.	3.0pt
3.2	Calculate the total energy (J) of the spacecraft as it is moving between points B and C.	4.0pt
3.3	Calculate the speed ($km\,s^{-1}$) of the spacecraft at point C.	3.0pt

ALMA - Calculating photons (10 points).

ALMA is a radio observatory with a revolutionary design. It consists of 66 high-precision antennas, operating in the wavelength range from $0.32\,mm\;to\;8.60\,mm$. The principal array has fifty antennas of $12\,m$ diameter each that can work together as a single telescope in the so-called interferometric mode. There is also another array of four $12\,m$ antennas, and twelve smaller antennas of $7\,m$ diameter each.

Imagine that a single $12\,m$ antenna is being calibrated, pointing to a source with a known incident flux of $1\times10^{-20}\;W/m^2$

4.1	Assuming that all the flux arrives at the shortest wavelength of ALMA sensitivity, determine the average number of photons that would reach the detector every second.	2.0pt

4.2	Compare it to the average number of photons that would have reached the detector, if all the flux arrived at the longest wavelength of operation.	2.0pt

4.3	What is the angular resolution (in arcsec) of a single $12\,m$ antenna, operating at $74.9\,GHz$?	2.0pt

4.4	Imagine the principal array operating at $74.9\,GHz$ in the interferometric mode. Assuming for simplicity that the spatial resolution is solely given by the longest baseline (largest distance between any pair of antennas), which turns to be $D_{\text{max}}=16\,km$, what would be the angular resolution (in arcsec) in this case? Treat this case as a single slit aperture instead of a circular one.	2.0pt

4.5	For a radio antenna, the term SEFD refers to 'System Equivalent Flux Density', which is a characteristic energy flux density of the antenna, depending on its temperature and size. We also note that for energy estimation of radio photons, Rayleigh-Jeans approximation is valid. Assuming a system temperature of $691\;K$, what would be the SEFD of the full ALMA observatory in Jansky if all the 66 antennas could work together?	2.0pt

Under pressure (10 points).

Magnetic fields in the Sun are constantly shaping the structure of various different features in the Solar atmosphere. Inside any feature, the magnetic field (*B*) adds to the total pressure exerted by the gas. This so-called magnetic pressure is a function of the height *z* and can be expressed as:

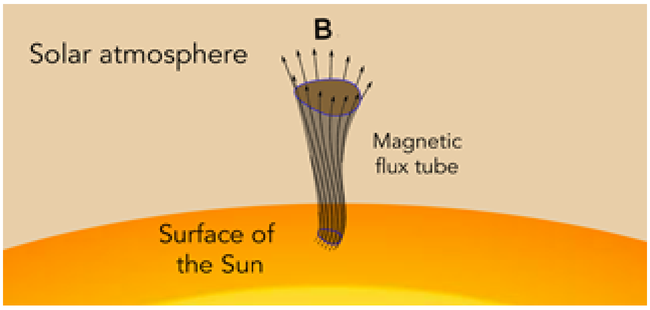
$$P_{mag}(z) = \frac{B^2(z)}{2\mu_0}$$

On the other hand, the gas can be considered to be in hydrostatic equilibrium and hence the gas pressure decays exponentially from an initial pressure value P_0 with increasing *z*. It can be expressed as,

$$P_{gas}(z) = P_0 e^{-z/H}$$

where H is the scale height, i.e. the height at which the pressure falls to $\frac{P_0}{e}$.

Consider one type of feature, a magnetic flux tube rising from the Solar surface up into an unmagnetized environment (see Figure below). Assuming that the total pressure of the material inside the tube and of the material outside it is in equilibrium,



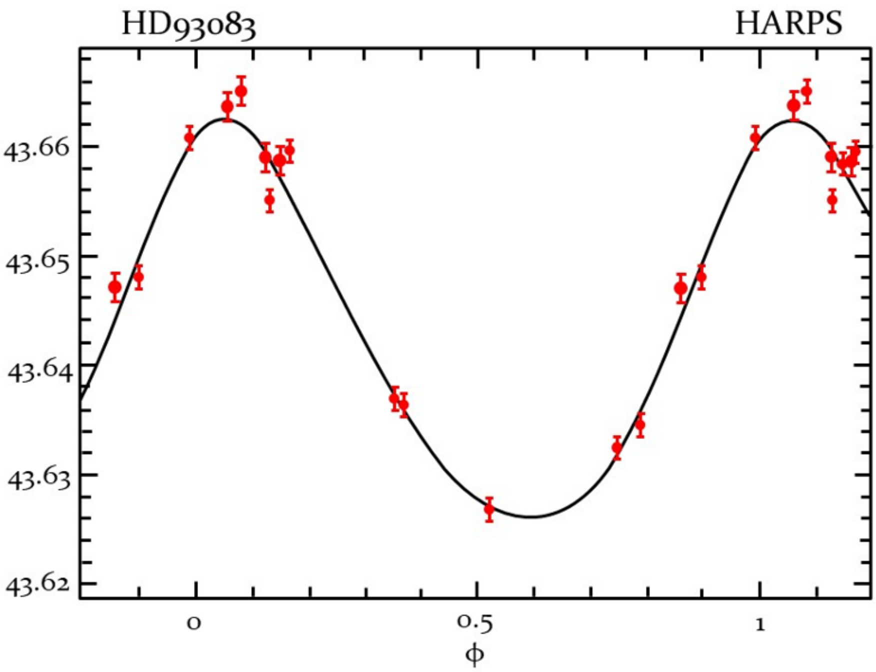
5.1	Find an expression for the magnetic field strength as a function of height z.	7.0pt

5.2	If the magnetic field at the base of a flux tube is $0.3T$, and scale height H in a given solar model is $150\ km$, at what height will the magnetic field be reduced to $0.03T$?	3.0pt

Macondo and Melquiades (12 points).

In 2019, as a part of the NameExoWorlds campaign of the International Astronomical Union, Colombia was granted an opportunity to select a name for the star HD 93083 and its planetary system. HD 93083 is a $K - type$ dwarf star and has one extrasolar planet orbiting it. Today they are officially known as Macondo (star) and Melquiades (planet), from the literary ideas of the Colombian writer Gabriel García Márquez.

This star has an effective temperature of 4995 K and an apparent visual magnitude of 8.3. As per GAIA DR2, the parallax for Macondo is 35.03 milliarcseconds. You may assume the orbit of Melquiades is perfectly circular. In the figure you can see the plot of radial velocity of Macondo with respect to the phase.



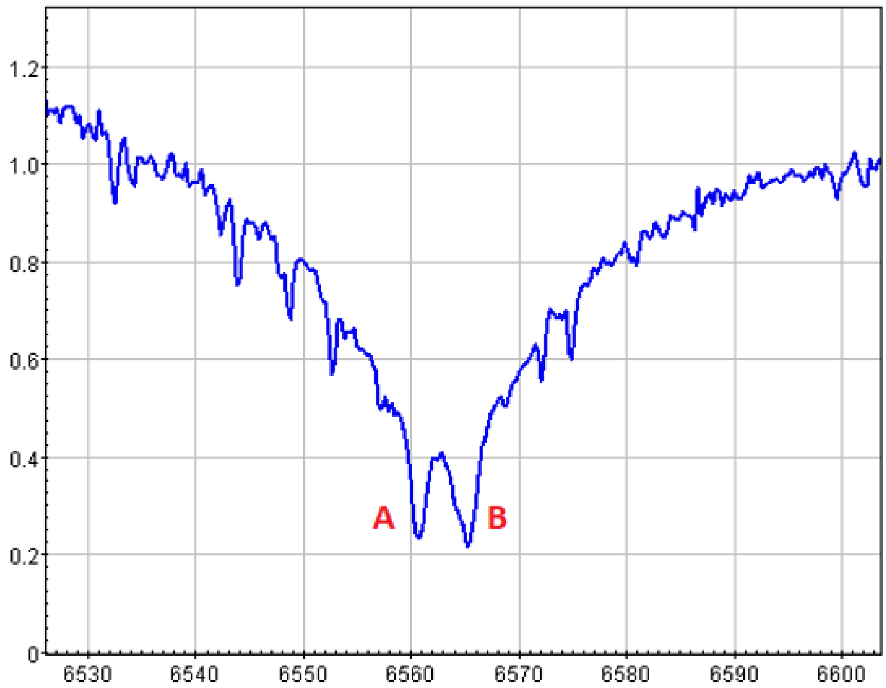
Radial velocity of Macondo (Y-axis in $km\ s^{-1}$) as a function of the phase (X-axis).

6.1	Find the wavelength $(in\, nm)$ of peak emission for Macondo in its rest frame (i.e., ignoring Doppler shifts).	2.0pt
6.2	Find the distance of this system from the Earth (in parsecs) and the absolute visual magnitude (M_V) of the star.	2.0pt
6.3	Calculate the mean radial velocity of Macondo (in $km\ s^{-1}$).	2.0pt

6.4	Calculate the orbital velocity (in km/s) of Melquiades (v_p), if mass of the star (m_s) is $0.7\ M_\odot$ and the mass of exoplanet (m_p) is $7 \times 10^{26} kg$. Assume that the orbital plane of the system is edge-on with respect to our line-of-sight.	2.0pt
6.5	Find the orbital radius of Melquiades (in au) and its orbital period (in days).	4.0pt

Menkalinan (β Aurigae) (13 points).

Almost half of the stars that we see are either binary or multiple star systems. A well-known example of this is Menkalinan (Beta Aurigae), which was initially thought to be a single star, but today recognised as a binary system comprising two stars that we will refer to as Menkalinan A and B. In the following figure, a spectrum of the system (obtained by the observatory of the Universidad de los Andes, in Bogotá) is shown:



Spectrum of Menkalinan binary system in the region of $H\alpha$. Y-axis is for the relative flux, and X-axis measures wavelengths. Menkalinan A is marked as A in the graph, and Menkalinan B is marked as B.

Answer the following questions using the plot and noting that the wavelength of $H\alpha$ line in the laboratory frame is $656.28\,nm$. Assume circular orbits, and assume that the binary system as a whole is at rest with respect to the observer.

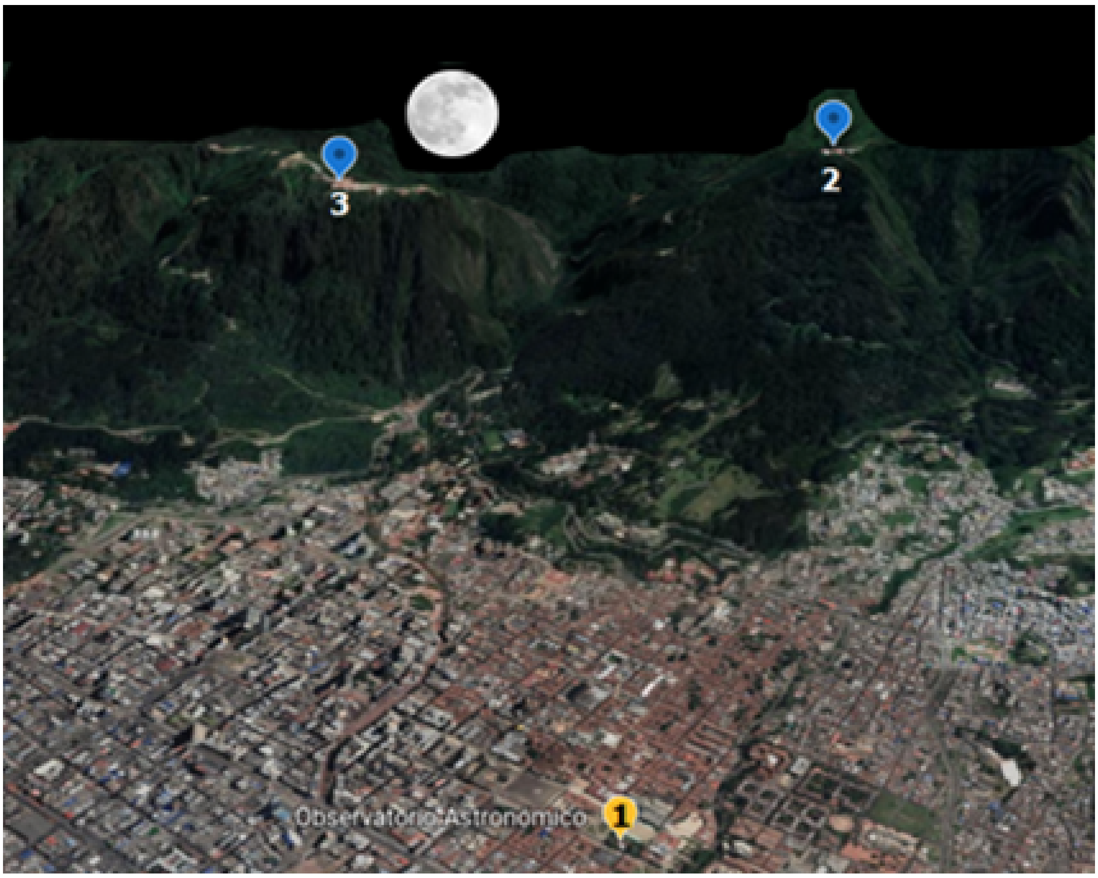
7.1	In the spectrum, we can see the $H\alpha$ line for each star in the system. Calculate the line-of-sight velocity of each star (km/s) and determine, at the time of this observation, which of the two stars is moving towards us.	5.0pt

7.2	The binary system is located 81.1 light years from Earth and has an orbital period of 3.96 days. The semi-major axis for Menkalinan B (smaller star) was measured to be $3.35\,milliarcseconds$. If the mass ratio of the two components is 1.026, find the total mass of the system (in solar masses).	4.0pt

7.3	Calculate the individual masses of Menkalinan A and B in solar masses.	2.0pt
7.4	Since Menkalinan A and B are main sequence stars, use the relation: $$\frac{L}{L_\odot} = (\frac{M}{M_\odot})^{3.5}$$ to estimate the luminosity of each star (in solar luminosity).	2.0pt

IOAA Logo (15 points).

The IOAA2021 logo is formed by the acronym IOAA, where the first letter is represented by the silhouette of the building of the National Astronomical Observatory (OAN) of Colombia, the oldest observatory in America. This observatory is located in Bogota, where it was founded in 1803. The capital city of Colombia is bordered by two famous hills, Monserrate and its neighbor Guadalupe, which are icons of Bogota's cityscape that decorate the logo's background.



Aerial view of Bogota City. Numbers show locations for the quoted places: 1 is for OAN; 2 is for Guadalupe; and 3 is for Monserrate.

Point	Latitude	Longitude	Elevation (m.a.s.l)
1	$4^\circ\ 35'\ 53''\ N$	$74^\circ\ 04'\ 37''\ W$	2607
2	$4^\circ\ 35'\ 30''\ N$	$74^\circ\ 03'\ 15''\ W$	3296
3	$4^\circ\ 36'\ 18''\ N$	$74^\circ\ 03'\ 19''\ W$	3100

8.1	Estimate the distance (in km), between points 2 (Guadalupe) and 3 (Monserrate).	3.0pt

8.2	Estimate the angular separation (in degrees) between Guadalupe (2) and Monserrate (3) as observed from the National Astronomical Observatory of Colombia (1).	6.0pt

8.3	From the OAN, on September 21 at 8:00 p.m. the Moon was observed towards the eastern hills (between Monserrate and Guadalupe). The measured ecliptic coordinates (longitude and latitude) of the Moon are shown in the table. Determine the equatorial coordinates of the Moon at the time of observation.	6.0pt

Local Time: 8:00 p.m.
$Az: +90°42'59''/Alt: +19°01'42''$
$\lambda: +12°20'16''$ / $\beta: -04°24'14''$

Note: Azimuth measured from North to East.

Pluto Satellites (15 points).

9.1	The mass of Charon, the biggest satellite of Pluto, is 1/8th the mass of Pluto. Both bodies move in a circular orbit around a common center of mass. In addition, they both are tidally-locked. The distance between the center of the planet and the center of the satellite is$R = 19\,640\ km$ and radius of the satellite is $r = 593\ km$. Let g_0 be the gravitational acceleration on the surface of Charon due only to its mass.Let A be the point on Charon surface directly facing Pluto, and B the point diametrically opposite. Compute the percentage difference between gravitational acceleration at A and B respect to g_0.	15.0pt

Terrestrial Transit (15 points).

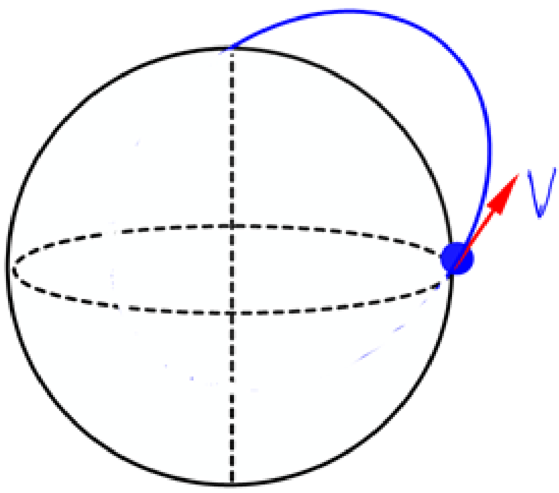
Note: Assume perfect circular orbits in both questions below.

10.1	An alien astronomer from a distant planetary system is observing the Sun. Suddenly, the brightness of the Sun drops due to the transit of the Earth in front of it. What is the maximum duration that this transit may last (in hours)? Assume that the planet where the astronomer observes from, does not move relative to the Sun.	5.0pt
10.2	Imagine that the transit of a given exoplanet as seen from Earth lasts 31 minutes. The host star is a red dwarf, with mass and radius that are 10% of the mass and radius of the Sun. What is the minimum orbital period this exoplanet may have (in days)?	10.0pt

Minimum velocity of a projectile (15 points).

11.1	What is the minimum speed with which a projectile must be launched from the Earth's surface at the equator such that the projectile reaches the north pole?	12.0pt
11.2	Find the eccentricity of the trajectory described by the projectile	3.0pt

You may ignore the rotation of the Earth. Also assume the earth surface is spherical.



Reference Chart

Hodograph (15 points).

In curvilinear motion of a planet around a star, the direction of the velocity vector changes continuously. This can be represented by a so-called "trajectory in velocity space" and is obtained as follows: for each point on the spatial trajectory, the corresponding velocity vector is drawn so that its starting point is at the origin of the velocity space, and its magnitude and direction is the same as the velocity vector at that point. The tip of this variable velocity vector generates a curve in velocity space. (The name 'hodograph' was given to this curve by Hamilton in 1846.)

As an example, see figures 1 and 2 below. For a circular orbit (Figure 1), the magnitude of the velocity is constant and therefore, the hodograph (Figure 2) of the velocity vector for Keplerian circular motion is also a circle, the center of which is located at the origin of the velocity space. The radius of this circle is equal to the constant magnitude of the circular velocity.

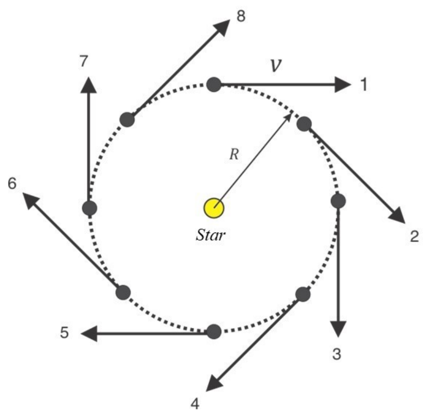


Fig. 1 Spatial trajectory of the Planet with Uniform Circular Motion around the star.

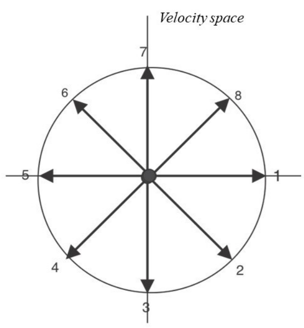


Fig. 2 Corresponding hodograph

12.1	Write an expression for the radius of the hodograph in Fig. 2, as a function of the mass M of the star, and the radius R of the circular orbit of the planet's motion.	1.0pt

12.2 For a planet in a Keplerian trajectory, write the expression for centripetal acceleration vector $(\vec{a})$ and the magnitude of angular momentum (L). For any Keplerian trajectory, it is true that — 4.0pt

$$| \Delta v |= k\Delta\theta \qquad (1)$$

Where k is a constant for each type of Keplerian trajectory. Find the expression for the constant k as a function of the masses M and m of the star and the planet, respectively, and the angular momentum, L.
(Eq.1) allows us to conclude that for any Keplerian trajectory, the hodograph $(v\ as\ a\ function\ of\ \theta)$ is a circle, but except for circular motion, the centre of the hodograph does not coincide with the star. It is not necessary to prove this result, you may simply accept it as a given. For the hodograph of uniform circular motion, the compliance with (eq.1) is completely obvious, as evidenced in Fig. 3

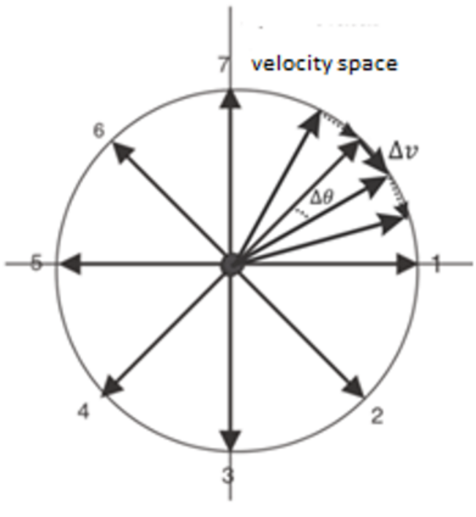


Fig. 3

12.3	Determine the expression of the constant k for the hodograph of circular planetary motion.	2.0pt

12.4 Given that the hodograph of the Keplerian elliptical motion is a circle, determine the radius of this hodograph and the distance between the center of the hodograph and the position of the star, as a function of the velocities at periastron and apoastron. Draw a rough sketch of the hodograph in the answer sheet as per the schematic shown in Fig. 4. The black circle is the star. 4.0pt

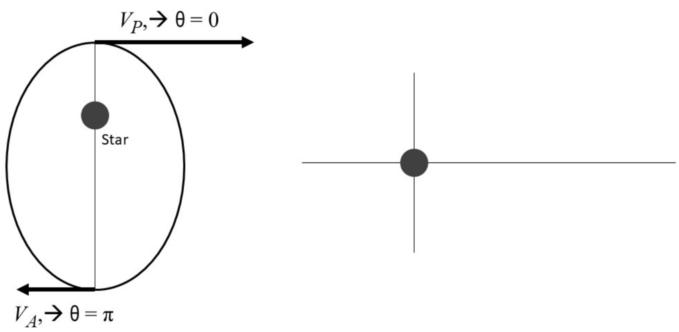


Fig. 4

12.5 Similarly, for the parabolic Keplerian trajectory, determine the radius of the corresponding hodograph and the distance from the center of that hodograph circle to the star. Express the radius as a function of the velocity at periastron. Draw a rough sketch of the hodograph circle in the answer sheet. 4.0pt

Lucy: The First Mission to the Trojan Asteroids (15 points).

CCD cameras on space probes are very sensitive and exposed to space weather conditions. Intense radiation passing through the CCD produces electron-hole pairs in the silicon of the CCD chip. The rate at which these pairs are produced is an important parameter when operating cameras on board spacecraft and can be calculated for radiation of any given energy.

A high energy particle or photon of radiation passing through the CCD will deposit some energy in the chip with each electron-hole pair it creates. The 'stopping power' of silicon for a given type of particle can be measured as the energy per areal density $(areal\ density = mass\ per\ unit\ area)$ that the silicon 'takes away' from the travelling particle.

NASA's Lucy mission will be the first to study the Trojan asteroids and will revolutionize our understanding of the formation of the Solar System. One of the instruments on board is L'LORRI (Lucy LOng Range Reconnaissance Imager), which contains a sensitive CCD in order to produce detailed images of the Trojan asteroids. Unfortunately, the radiation around Jupiter is very intense and it can generate a lot of 'noise' in the pixels of the CCD.

Let us assume that an average charged particle trapped in Jupiter's magnetic field has an energy of $15\ MeV$ and that the flux of such particles in this region is equivalent to about 600 electrons $s^{-1}\ cm^{-2}$. Also assume that for each electron-hole pair which a particle passing through a pixel creates, it deposits exactly the excitation energy of the pair in that pixel. After the pixel crosses a threshold number of electron-hole pairs it is 'excited' and no more pairs can be produced in that pixel. Any remaining energy in the particle is passed to the next pixel (and so on).

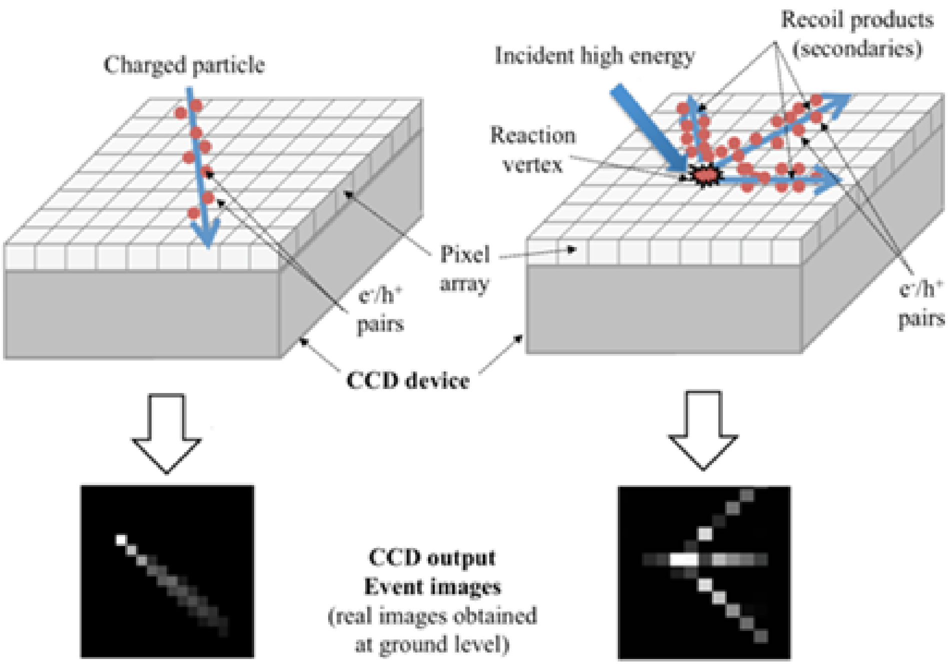
Using the data given below for the CCD chip in the L'LORRI camera, answer the following questions:

13.1	How many pixels will be excited by one such particle of radiation passing through the CCD when the spacecraft is near Jupiter's orbit?	10.0pt

13.2	Given the radiation flux near Jupiter, what percentage of the total number of pixels in an image will be excited?	5.0pt

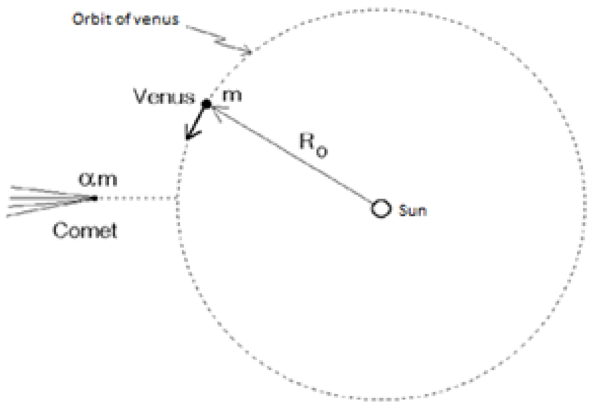
CCD Data:

- **Exposure time of an image = 30 ms**
- **Pixels on the CCD = 1024 x 1024**
- **CCD Area = 13 mm x 13 mm**
- **CCD chip thickness = 0.06 cm**
- **Density of silicon, ρ = 2.34 g cm^{-3}**
- **Excitation energy of single pair = 2.36 eV**
- **Excitation threshold of a single pixel = 250 pairs**
- **Stopping power' of silicon for a 15 MeV electron = 3.012 MeV g^{-1} cm^{2}**



Formation of the Venus-2 (35 points).

A comet of mass αm is heading ("falls") radially towards the Sun. It is known that the total mechanical energy of the comet is zero. The comet crashes into Venus, whose mass is m. We further assume that the orbit of Venus, before the collision, is circular with radius R_0. After the crash, the comet and Venus form a single object, called "Venus-2".



14.1	Find the expression in terms of M_{sun} and R_0 for the orbital speed, v_0 , of Venus before the collision.	1.0pt
14.2	Find an expression for the total mechanical energy of Venus in its orbit before colliding with the comet.	1.0pt
14.3	Find an expression for the radial velocity, v_r , the angular momentum, L , of "Venus-2" immediately after the collision.	10.0pt
14.4	Find an expression for the mechanical energy of the combined object "Venus-2" and express it in terms of energy before the collision, E_i, and α .	5.0pt
14.5	Show that the post-collision orbit of "Venus-2" is elliptical and determine the semi-major axis of the orbit.	5.0pt
14.6	Determine if the year for the inhabitants of "Venus-2" has been shortened or lengthened because of collision with the comet. Write the ratio between the period of Venus-2 and Venus.	3.0pt

14.7	What should be the value of α such that the post-collision orbit of Venus-2 would make it crash in the Sun? We will call this as α_c	5.0pt

14.8	A comet with $\alpha = \alpha_c$ collided with Venus. Calculate the percentage change in the magnitude of Venus' velocity (δv) and the change in the direction of the velocity vector $(\delta\theta)$ immediately after the collision.	5.0pt

Cosmic String (55 points).

Introduction

According to our current understanding, just after the Big Bang, when the Universe was extremely hot, the electromagnetic force, the strong nuclear force as well as the weak nuclear force were unified as one Grand Unified (GUT) force.

When the Universe cooled down to $T_{GUT} = 10^{29}\ K$, the strong nuclear force decoupled from the electroweak force. Later, when the temperature reduced to $T_{EW} = 10^{15}\ K$, the weak force decoupled from the electromagnetic force. These transitions happened in a rapid succession within a small fraction of a second after the Big Bang. It is thought that these phase transitions produced a variety of peculiar objects, called vacuum defects, which may still be observed today.

This question will discuss properties of one such possible type of defect called cosmic strings and their observational effects.

Note 1. Unless otherwise stated use the laws of Newtonian Mechanics

Note 2. You will use the following constants:

- Stefan Boltzmann Constant

$$\sigma = \frac{2\pi^5 {k_B}^4}{15\, h^3 c^2} = \frac{\pi^2 {k_B}^4}{60\, \hbar^3 c^2}$$

- The reduced Planck constant

$$\hbar = \frac{h}{2\pi}$$

- Universal Radiation Constant

$$a = \frac{4\sigma}{c} = 7.5657 \times 10^{-16} J\, m^{-3} K^{-4}$$

- Planck Temperature

$$T_{pl} = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.416784 \times 10^{32} K$$

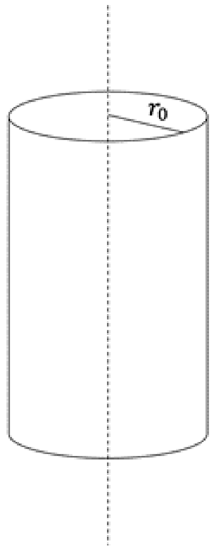
Note 3. Recall that the gravitational field $\vec{g}$ satisfies the Gauss theorem:

$$\vec{g} \cdot \vec{A} = -4\pi G M_{in}$$

Where M_{in} is the mass enclosed by the surface A.

Part A: Gravitational Field of a Cosmic String (22 points).

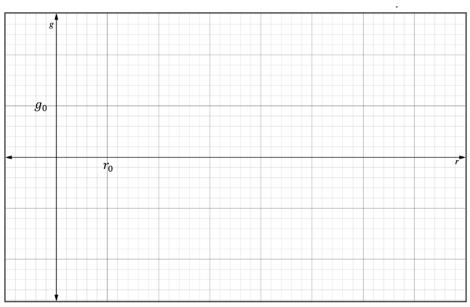
As a first approximation, let us consider a cosmic string as an infinitely long cylinder of radius r_0 and mass per unit length μ .



A.1 Write an expression in terms of the constants G, μ and $r_0$for the gravitational field produced by the string, $\vec{g}(r)$.
Consider the cases $r_0 < r$ and $r_0 > r$ independently — 6.0pt

A.2 Write an expression in terms of the constants G, μ and $r_0$for $g_0 \equiv |\vec{g}(r_0)|$. — 1.0pt

A.3 Let g be defined as $\vec{g}(r) \cdot \hat{r}$. Draw a rough sketch of g vs. r in the figure given in the answer sheet — 3.0pt



A.4	It is possible to define a stable orbit around a Cosmic String. For circular orbits of radius $R > r_0$ and period τ, the following relation is attained $$R = A\tau^{\alpha}$$ where A and α are constants. Find A and α in terms of G and μ	4.0pt

The following three questions refers to a classical newtonian particle moving with speed v when at a distance $r > r_0$ from the string. You will need to use the result below:

$$\int_{x_0}^{x} \frac{dx}{x} = \ln\left(\frac{x}{x_0}\right)$$

A.5	Show that the gravitational potential energy of the particle is $$U = Gm\mu \ln\left(\frac{r}{b}\right)$$ where b is any fixed distance.	3.0pt

A.6	What is the maximum distance, R_{max} , from the string, that the particle can reach?	4.0pt

A.7	Is it possible for the particle to escape the gravitational field? Write YES/NO in the answer sheet.	1.0pt

Part B: Cosmic string as a photon gas (17 points).

Consider now a cosmic string as a photon gas inside a very long cylinder of radius r_0 with adiabatic walls, and in thermal equilibrium at temperature T.

B.1	What is the energy density ρ of the string in terms of $T,\ \hbar, k_B$ and c ?	2.0pt

B.2	The radius r_0 is related to the temperature T vía $$r_0 = \frac{\hbar^{n_1}\ c^{n_2}}{k_B T},$$ where $\hbar$ is the reduced Planck constant, and c is the speed of light in vacuum, k_B is the Boltzmann constant, and n_1 and n_2 are integer numbers. Determine n_1 and n_2	4.0pt

B.3	What is the mass per unit length, μ , of the string in terms of ρ and r_0 ?	2.0pt

B.4	Express the inequality for the weak field condition, defined as $$\frac{2G\mu}{c^2} \ll 1,$$ only in terms of T and T_{pl} .	5.0pt

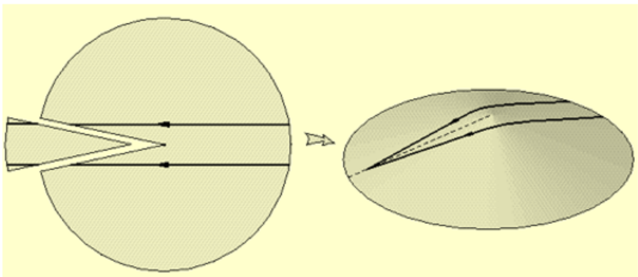
B.5	Calculate $\frac{2G\mu}{c^2}$ for $$\bullet T = T_{EW}$$ $$\bullet T = T_{GUT}$$	3.0pt

B.6	Does the weak field condition hold for T_{EW}? Answer YES or NOT. Does the weak field condition hold for T_{GUT}? Answer YES or NOT.	1.0pt

Part C: Gravitational Lensing from cosmic Strings (16 points).

So far, in part A and B , we have neglected the internal pressure of the photon gas inside the string. If we include it in our analysis, we need to consider the General Theory of Relativity.

After solving the Einstein field equations, one finds that the spacetime around a cosmic string is conical as if a narrow wedge were removed from a flat sheet and the edges connected, as shown below.



http://www.ctc.cam.ac.uk/outreach/origins/cosmic_structures_five.php

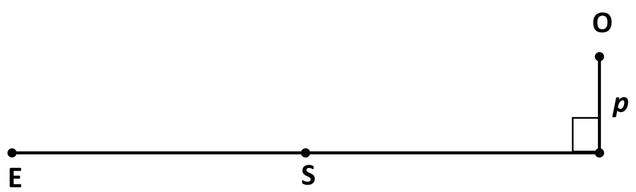
A remarkable result of this model is light deflection by a cosmic string, which leads to the possibility of detection through gravitational lensing.

The angle of deflection (in radians) of a light ray coming from a distant quasar (O in the figure below), as the light passes close to a cosmic string (S in the figure below) and eventually reaching an observer on the Earth, (E in the figure below), is

$$\delta\phi = \frac{4\pi G\mu}{c^2}$$

and is independent of the parameter, p, as shown in the figure below:

In the figure E and O are in a plane perpendicular to the string. The distance between the observer and the string is D_{ES} and the distance between the observer and the source is D_{OE}



C.1	Although the angle of deflection does not depend on parameter p, an Earth-based observer will be able to see more than one image only if the value of p is within a certain range. Find a condition on the value of the parameter p in terms of D_{ES}, D_{OE} , and temperature T , for an Earth-based observer to see more than one image of the object O	6.0pt
C.2	In case the observer sees more than one image, what is the angular separation between each pair? Find an expression in terms of D_{ES}, D_{OE} and $\delta\phi$	6.0pt

C.3	If $D_{OE} = 2D_{ES}$, determine the minimum size of an optical telescope needed to resolve this lensing event produced by GUT string.	4.0pt

Data Analysis 1: Scaling Relations (75 points)

Please read the general instructions in the separate envelope before you start this problem.

Spiral galaxies are disk-like rotating structures, whose dynamical state is fairly grasped by the so-called rotation curves, quantifying the mean rotational velocity of the disk at different distances from the center (see Figure 1, curve B). An interesting feature is the flat region of the curve, which is attributed to the presence of dark matter. Without it, rotation velocities would drop steadily at large radii from the center, as depicted in curve A.

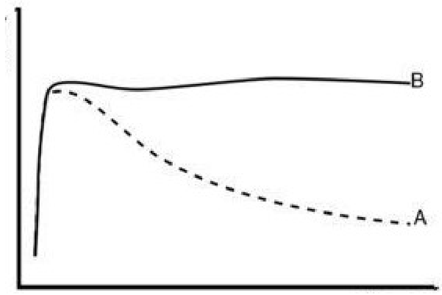


Figure 1: Rotation curves. Circular velocity (Y-axis) vs Radius (X-axis)

In disk galaxies a strong correlation has been observed between the intrinsic luminosity of the whole galaxy and the asymptotic rotational velocity (as given by the rotation curve for the outer edge of the galaxy i.e. R_{max}), a result that is known as the Tully-Fisher relation. This relation also holds if you use the luminosity in a specific band. This is shown on Figure 2 for a number of galaxies in a galaxy cluster. Every dot is a galaxy, and the solid line is the best-fit linear relation between absolute magnitude in K band and $log_{10}(V_{\text{max}})$ for the whole sample.

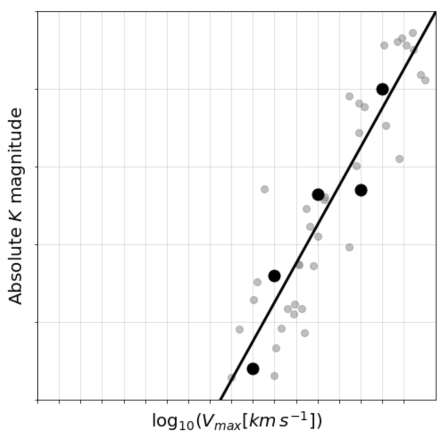


Figure 2: Absolute magnitude in K band vs $\log_{10}(V_{\text{max}}[km s^{-1}])$. Tully-Fisher relation for several galaxies. Every dot represents a galaxy. The dark points are five selected galaxies, for which we will provide some numbers in part 1.2.

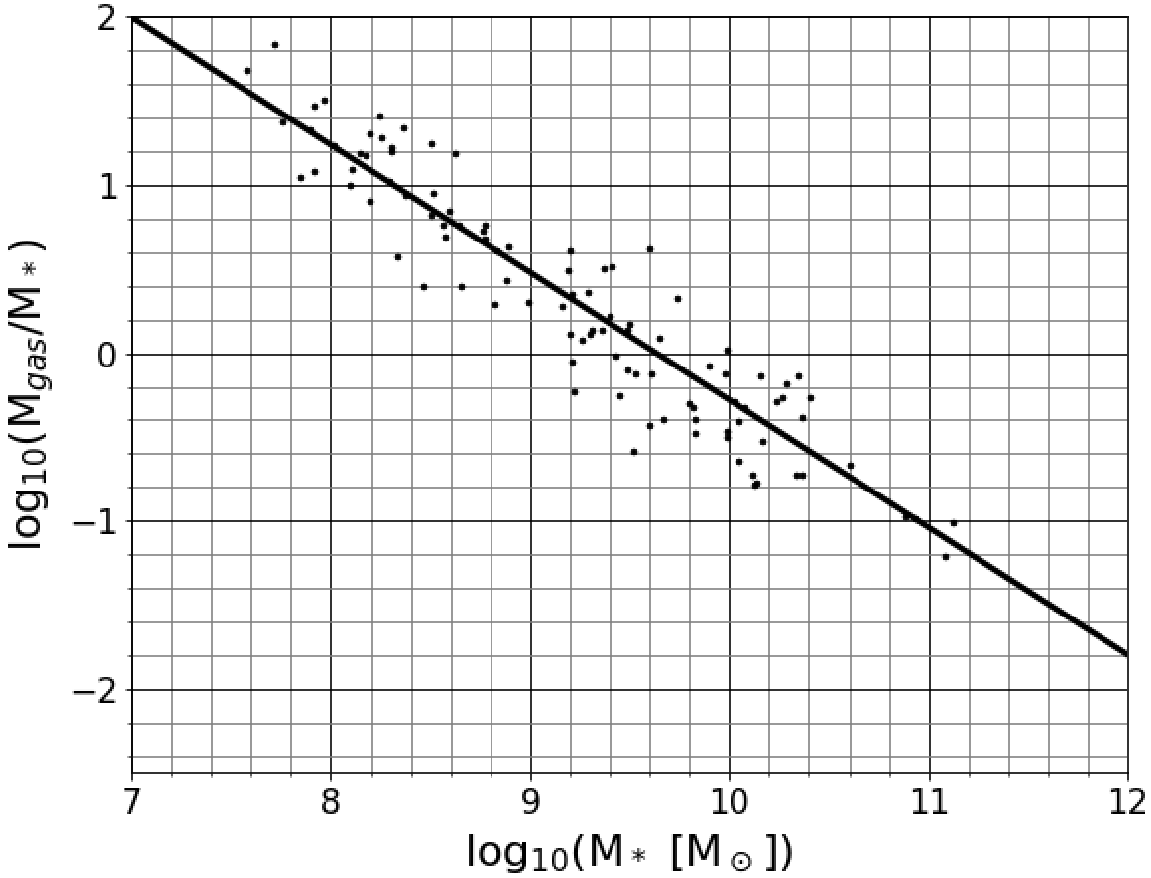


Figure 3: Gas fraction vs stellar mass.

Another interesting trend is shown in Figure 3: disks with larger stellar masses (M_*) tend to have smaller gas fractions (M_{gas}/M_*).

In the following questions you will be asked to extract physical information about the galaxies using the scaling relations just introduced. Consider the following guidelines:

- Assume that V_{max} was measured at the same radius for all galaxies (R_{max}), in the flat part of the rotation curves and well beyond the end of the stellar disk.
- Use M_{dm} for the dark matter mass up to R_{max} and M_{tot} for the sum of all components.(gas, stars and dark matter)
- Assume that all galaxies have identical stellar populations[1], and assume that the gaseous component does not interact with the stellar light. .
- The galaxy cluster is far away. Its distance is much larger than the cluster size.
- In spherically-symmetric mass distributions, to infer the gravitational effect on a particle at distance r from the center, it suffices to consider the total mass enclosed up to that radius $M(\leq r)$ as if it were placed at the very center of the distribution.

[1]The term stellar population refers to the type of stars that are present in a galaxy, and the relative amount of each type with respect to the total number of stars.

Part 1 (20 points).

1.1 From an analysis of Figure 3, find the appropriate constants in the following relation: $M_{gas} = a \times M_*^b$ 5.0pt

$$a = ?$$

$$b = ?$$

1.2 In the plot of the Tully-Fisher relation there are 5 highlighted points. Data for these 5 galaxies is given in the following table. Use this dataset to find the appropriate constants for TF relation presented below the table, by means of a linear fit using the method of least squares. 15.0pt

Note: Treat $log_{10}(V_{\text{max}})$ as the x variable and K as the y variable in the linear fit.

$V_{\text{max}}[km/s]$	$K[mag]$
79.4	-16.8
100.1	-19.2
158.5	-21.3
251.2	-21.4
316.2	-24.0

$$K = c \times log_{10}(V_{max}) + d$$

$$c = ?$$

$$d = ?$$

Part 2 (16 points).

For two galaxies, G1 and G2, in the cluster, the recorded *apparent* magnitudes are:

$$k_1 = 19.2 \qquad ; \qquad k_2 = 25.2$$

Using this information and the relations calibrated in Part 1 find the correct exponents in the following equations:

2.1 6.0pt

$$\frac{M_{*1}}{M_{*2}} = 10^e \qquad ; \qquad e = ?$$

2.2 4.0pt

$$\frac{M_{gas1}}{M_{gas2}} = 10^f \quad ; \quad f = ?$$

2.3 6.0pt

$$\frac{M_{tot1}}{M_{tot2}} = 10^g \quad ; \quad g = ?$$

Part 3 (15 points).

3.1 15.0pt

Galaxy	Apparent magnitude k	$M_{gas}[M_\odot]$	$M_*\ [M_\odot]$	$M_{dm}[M_\odot]$	$M_{tot}\ [M_\odot]$
G_1	19.2				4.39×10^{11}

Fill in the missing values in the table using the fact that for galaxy G_1, the dark-to-baryonic mass ratio up to $R_{\max}$ is 6.82.

Part 4 (24 points).

4.1 Consider a systematic uncertainty of $\sigma_{sys} = \pm 0.2$ in each apparent magnitude due to CCD calibration errors. Then k_1 must be read as $k_1 = 19.2 \pm 0.2$, i.e., the only thing we know is that k_1 most likely lies in the interval $[19.0, 19.4]$. The same goes for k_2. 4.0pt
Recalculate the exponent in the scaling relation $\frac{M_{*1}}{M_{*2}} = 10^e$ (found in 2.1), expressing e as an interval estimated by considering the extreme possible variations in $k_1$and k_2 .

$$e \in [?, ?]$$

4.2 Now we consider that there is always a natural spread of the data around any relation. For instance, for a given value of the K magnitude the TF relation gives a single value of $log_{10}(V_{\text{max}})$, but it would be more realistic to report an interval of plausible values, derived from the natural spread of the data around the mean TF relation. We call this the statistical uncertainty, σ_{stat}. **10.0pt**

Estimate the statistical uncertainty if $log_{10}(V_{\text{max}})$ is inferred from K using the TF relation from question 1.2. For this, consider for each point the difference between the value of $log_{10}(V_{\text{max}})$ estimated from K using your linear fit and the actual measurement of $log_{10}(V_{\text{max}})$, and take σ_{stat} as two times the root mean square (RMS) of these differences†.

$$\sigma_{stat} = ?$$

†The RMS of a set of values is the square root of the arithmetic mean of the squares of those values.

4.3 Recalculate the exponent in the scaling relation $\frac{M_{tot1}}{M_{tot2}} = 10^g$, expressing g as an interval estimated by considering the extreme possible variations arising from both the systematic and statistical uncertainties: **10.0pt**

$$g \in [?, ?]$$

Data Analysis 2: Stars and Exoplanets (75 points)

Please read the general instructions in the separate envelope before you start this problem.

In this problem, we will explore the connection between the physical properties of exoplanets and their host stars and will use the observational data to discover as much as possible about these systems. You may neglect interstellar extinction.

Part 1 (20 points).

Name of planet	Name of star	$T_{eff}\,(K)$	$g\,(ms^{-2})$	m_v (magnitudes)	parallax (milliarcsec)
$Gorgona$	$HD\,209458$	5980	347	7.63	20.67

Table 1: Observational data for exoplanet Gorgona and its parent star *HD 209458*

The effective temperature (T_{eff}) and the gravitational acceleration in the surface of the star (g) can be measured from the shape of the spectrum and its absorption lines. The visual apparent magnitude (m_v) and parallax are measured by doing photometry and astrometry, respectively.

Additionally, it has been observed that every 3.52 days the brightness of the star drops due to the transit of the planet in front of it, as it is represented in this lightcurve:

1.1 20.0pt

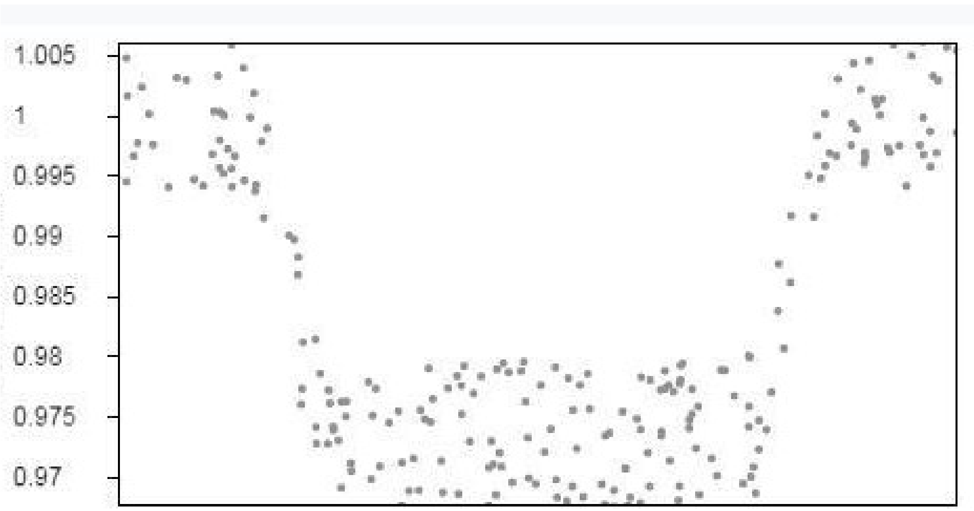


Figure 1: Normalized Flux (y-axis) against time (x-axis) for the parent star *HD 209458*

Use the given information to calculate the following quantities for the *HD 209458* system:

Luminosity of the star	Radius of the star	Mass of the star	Mean planet's orbital radius	Radius of the planet in Jupiter's radius
$L_\star$ $[L_\odot]$	$R_\star$ $[R_\odot]$	$M_\star$ $[M_\odot]$	a $[au]$	R_p $[R_J]$

Note: Assume that the bolometric correction for all F and G type stars is the same.

Part 2 (25 points).

The habitable zone is defined as the region in which a planet may have liquid water on its surface. This is mainly related to the amount of radiation received from the host star, which must be within some limits to ensure a favorable range of planet surface temperatures.

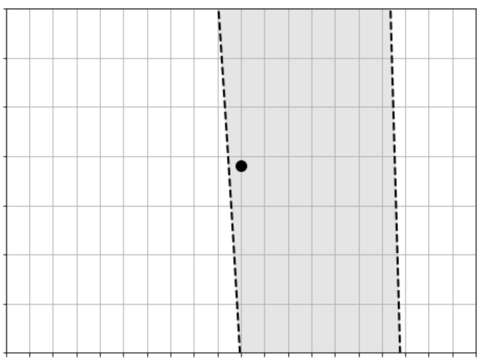
We define the effective flux received by a planet as $S_{eff} = \frac{L}{a^2}$, where L is the star luminosity in solar units, and a is the mean orbital radius in au. The minimum flux in the habitable zone can be approximated by $S_{min} = S_{eff_\odot} + n \cdot T_\star + b \cdot T_\star^2 + c \cdot T_\star^3 + d \cdot T_\star^4$, where $T_\star = (T_{eff} - T_{eff_\odot})$, and $S_{eff_\odot}$ is the equivalent flux for the case of the Sun, which along with the coefficients $n,\ b,\ c,\ d$ is given in the following table. The maximum flux for habitability, S_{max}, is found with the same equation but different constants:

Constant	S_{max}	S_{min}
$S_{eff_\odot}$	1.0512	0.3438
n	1.3242×10^{-4}	5.8942×10^{-5}
b	1.5418×10^{-8}	1.6558×10^{-9}
c	-7.9895×10^{-12}	-3.0045×10^{-12}
d	-1.8328×10^{-15}	-5.2983×10^{-16}

The table below gives real data for 7 different star-planet systems. However planet names have been changed to honour some natural sanctuaries in Colombia:

Stellar Parameters		Planet Parameters	
T_{eff} $[K]$	M_V $[mag]$	Name	$a[au]$
6180	3.68	Tayrona	0.04
5730	3.87	Iguaque	0.04
5980	4.21	Gorgona	0.04
5480	6.04	Amacayacu	0.08
5770	3.48	Malpelo	0.05
6130	3.07	Pisba	0.03
6140	3.85	Tatamá	0.06

2.1 In the following figure, the vertical axis represents the effective temperature of stars, and the horizontal axis represents the effective flux received by orbiting planets. The dot marked on the graph represents planet Earth, and dashed lines mark the limits of the habitable zone. 15.0pt



Put numerical labels at tickmark position on both axes. Draw on the same figure the exact position where Gorgona and Amacayacu would be, as if they were also located at 1 au from their corresponding stars.

2.2 Now considering the real orbital radius given in the table for each planet, indicate with YES or NO which of them are in the habitable zone. Show your quantitative reasoning on the working sheets. 10.0pt

Planet's Name	In habitable zone? YES / NO
Tayrona	
Iguaque	
Gorgona	
Amacayacu	
Malpelo	
Pisba	
Tatamá	

Part 3 (30 points).

In the last page you find a list of 38 exoplanets, and the goal is to find out if low-mass exoplanets (LME) and high-mass exoplanets (HME) tend to orbit around stars with different characteristics.

3.1 To get a robust low-mass subsample one can apply a technique called “iterative sigma-clipping”. The idea is to compute the mean (μ) and the standard deviation (σ) of the masses and to exclude from the sample, those planets with masses above $\mu + \sigma$. Then repeat the same steps with the remaining subsample two more times. We will say that planets in the final subsample are the low-mass ones, and those excluded during the iterations, the high-mass ones. Fill the following table with the numbers you find in the process: 10.0pt

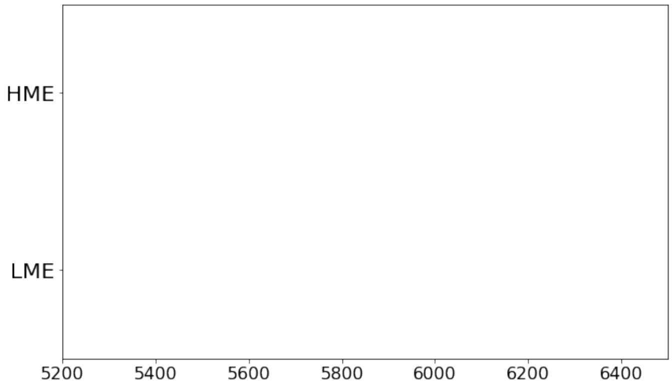
Low-mass sample	Sample size	μ	σ	$\mu+\sigma$	No. of planets to exclude
Full / Original	38				
subsample after 1st iteration					
subsample after 2nd iteration					
subsample after final iteration		---	---	---	---

3.2 Make a plot using the X-axis for the serial number of the planets in the list $(1, 2, 3, \ldots)$, and the Y-axis for the mass of the planets. Mark 3 horizontal lines at the $\mu + \sigma$ thresholds you found in the iterations: 5.0pt

3.3 Let’s investigate the possible difference in the effective temperatures of host stars in both groups, computing some descriptive statistics: 10.0pt

	Min.	1st Quartile	Median 2nd quartile	3rd Quartile	Max
LME					
HME					

3.4 Draw boxplots summarizing the numbers you just computed. Do you see a clear difference in the temperatures of the stars orbited by low-mass and high-mass planets? Write YES or NO. 5.0pt



Planetary type vs host star temperature (K)

No.	Name of Planet	Planet Mass $[M_J]$	T_{eff} of the Host Star
1	KEPLER-37 b	0.01	5520
2	KEPLER-21 b	0.02	6256
3	HD 97658 b	0.02	5468
4	HD 46375 b	0.23	5345
5	HD 219134 h	0.28	5209
6	HD 88133 b	0.30	5582
7	HD33283 b	0.33	5877
8	HD 149026 b	0.36	6096
9	BD-10 3166 b	0.46	5578
10	HD 75289 b	0.47	6196
11	HD 217014 b	0.47	5755
12	HD 2638 b	0.48	5564
13	WASP-13 b	0.49	6025
14	WASP-34 b	0.59	5771
15	HD 209458 b	0.69	5988
16	HAT-P-30 b	0.71	6177
17	WASP-76 b	0.92	6133
18	WASP-74 b	0.97	5727
19	HAT-P-6 b	1.06	6442
20	HD189733 b	1.14	5374
21	WASP-82 b	1.24	6257
22	KELT-7 b	1.29	6460
23	HD 149143 b	1.33	6067
24	KELT-3 b	1.42	6404
25	KELT-2A b	1.49	6164
26	HD86081 b	1.50	6015
27	HAT-P-7 b	1.74	6270
28	HD 118203 b	2.14	5847
29	HAT-P-14 b	2.20	6490
30	WASP-38 b	2.71	6178
31	HD17156 b	3.20	5985
32	KELT-6 c	3.71	6176
33	HD 75732 d	3.86	5548
34	HD 115383 b	4.00	5891
35	HD 120136 b	5.84	6210
36	WASP-14 b	7.34	6195
37	HAT-P-2 b	8.74	6439
38	XO-3 b	11.79	6281

The Solar Slap (75 points)

Please read the general instructions before you start this exam.

Solar Cycle 25 is heating up! It began in December 2019 and will peak in 2025. The start of a new cycle means there will be increasing solar activity until roughly mid-2025. One direct consequence of such activity is the more frequent occurrence of solar flares, which are intense bursts of radiation observed near the Sun's photosphere and low-corona. Solar flares are sometimes accompanied by coronal mass ejections (CMEs), which expel coronal plasma into interplanetary space.

We are living in a golden age for solar astrophysics. In addition to entering a period of high solar activity, we also have new solar telescopes that will allow us to study the Sun as never before. One of these telescopes is the Parker Solar Probe (ParkerSP), the first spacecraft in history to fly into the low solar corona. The ParkerSP has a somewhat eccentric orbit (ε≈0.88) and will approach the Sun as close as 7 million km ($\sim$10 solar radii) on its final orbital perihelion (in 2025).

Just recently, on May 28, 2021, a C-shaped CME was detected by the solar space telescope SOHO (located at a distance of 1.5×10^6 km from Earth, around the Sun-Earth L1 Lagrange point) by means of the onboard LASCO coronographs. The solar eruption generating the CME occurred at 22:19 UTC with an ecliptic traveling angle of 55° (with respect to the Sun-Earth line), heading directly towards the point where ParkerSP was located. Figure 1 shows a sequence of three consecutive images made by NASA, highlighting the evolution of the CME, from the onset to the moment it reaches the ParkerSP.

Assume that all spacecrafts are exactly on the ecliptic plane and the images here show a top view of the ecliptic plane.

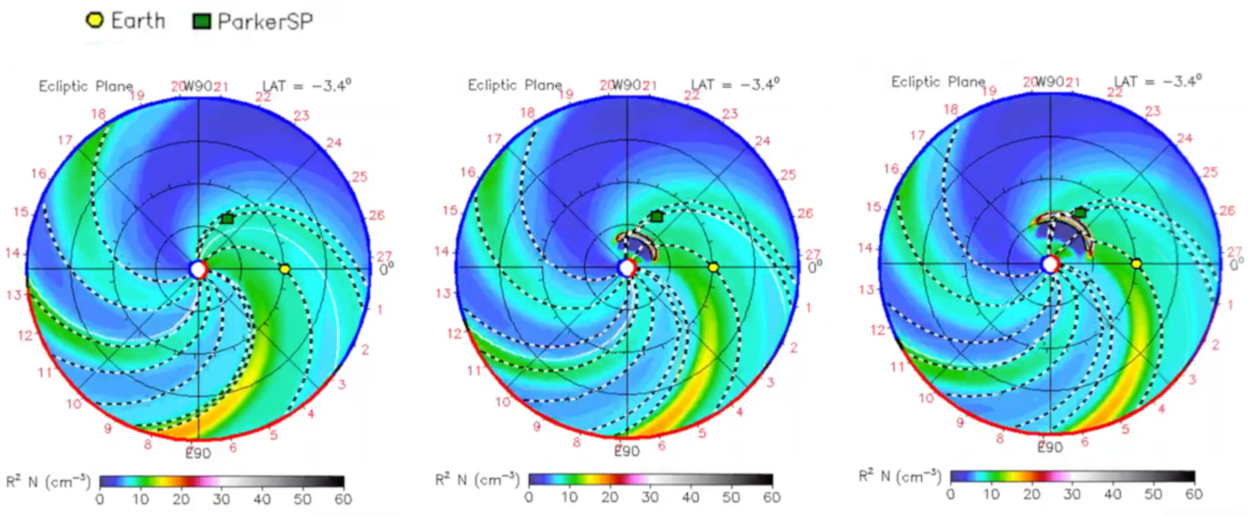


Figure 1: Sequence of images displaying on a heliospheric density map, the evolution of a CME that had its onset on May 28, 2021 at 22:19 UTC. The images show the location of the Sun (center) and of Earth (at 1 AU ≈ 1.5×10^8 km from the Sun) and the spacecraft ParkerSP. Note that the CME front impacts the ParkerSP in the last image of the sequence. The angle formed by Earth-Sun-ParkerSP is 55°.

Part 1 (30 points).

1.1	Using the JHelioviewer software find the CME which occurred on May 28, 2021, by selecting images from the Solar Dynamics Observatory (full disk) and the SOHO-spacecraft coronographs LASCO-C2 (imaging from 2 to 6 solar radii) and LASCO-C3 (imaging from 3.7 to 30 solar radii), as shown in Figure 2. Indicate, in a table, the date and time of each image that you have used.	10.0pt

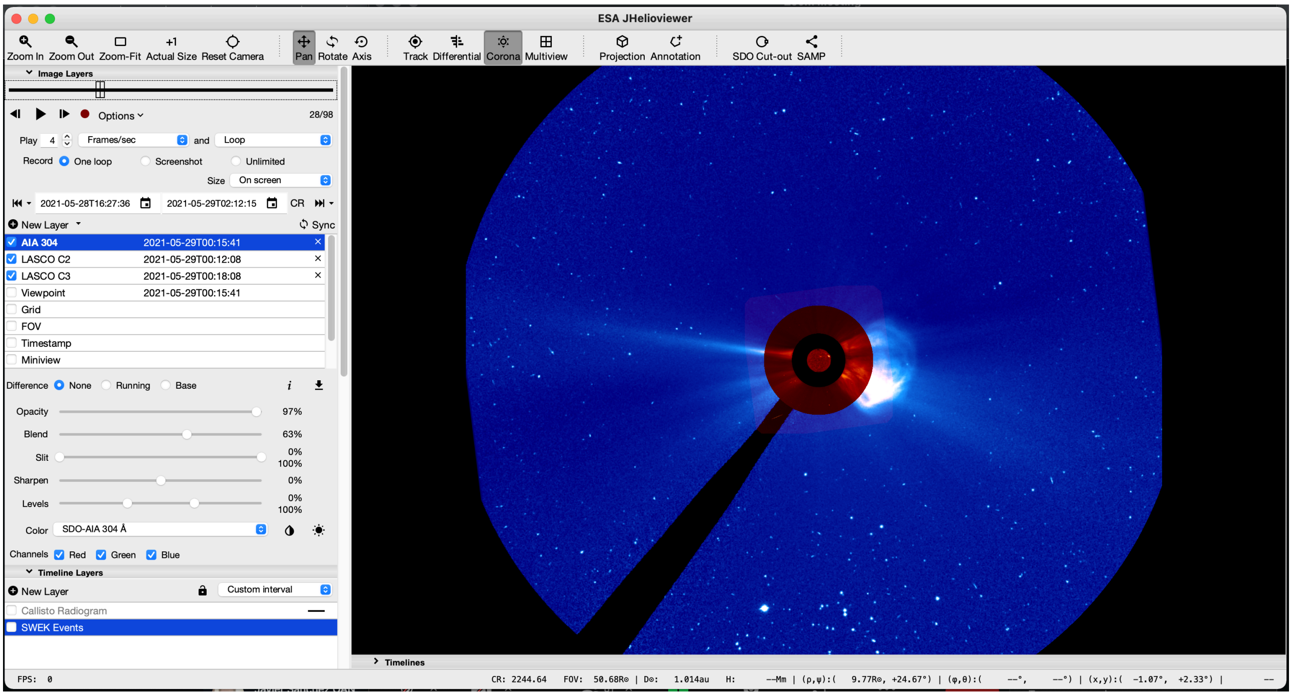


Figure 2: Exploration of solar data for May 28, 2021 with JHelioviewer.

1.2	Use a selection of images to measure the distance of the CME front from the Sun in km.	10.0pt

1.3	Extend the table of data (that you constructed in previous part) to include • Date and time (as reported in 1.1) • Distance of CME front from the Sun in km (as reported in 1.2) • Cumulative velocity in km/s (e.g. if you are at the 4th image, the mean velocity between onset of CME until the time of the 4th image), • Velocity per time interval in km/s (e.g if you are at the 4th image, the mean velocity of CME between the times of the 3rd and 4th image). Make this table in the working sheet.	10.0pt

Note: Both the velocities are to be calculated with respect to the Sun.

Do not forget to label each of the columns of your table accordingly.

Part 2 (15 points).

2.1	Make distance-time and velocity-time graphs (for both cumulative and velocity per time interval) using the measured and calculated data from your table.	15.0pt

Part 3 (10 points).

3.1	Considering that the CME moves at constant speed for distances larger than 30 solar radii, estimate the velocity (in km/s) of the CME front when it impacts the ParkerSP, and the time (in hours) it takes to do so from its onset.	10.0pt

Part 4 (10 points).

From the following statements, mark which are TRUE and which are FALSE.

4.1	If we keep decreasing the time interval between successive images, the precision of measurements of the evolution of the CME and the calculated physical parameters will always keep increasing.	2.0pt

4.2	A more accurate analysis and measurements of the CME evolution should consider the differential rotation of the Sun, and therefore the calculated velocities will be affected.	2.0pt

4.3	Any software (numerical) misalignment among the images when creating the mosaic will have direct effects on the precision of the calculations.	2.0pt

4.4	The different assumptions made in order to construct the model displayed in a heliospheric density map in Figure 1, may affect the estimation of the Sun-ParkerSP distance.	2.0pt

4.5	The interaction of the CME-front with the remnant dust left by the 2019 Borisov comet broadens and diffuses the images. This reduces the contrast in the images, substantially increasing the uncertainty in determining the CME-front and its propagation.	2.0pt

Part 5 (10 points)

5.1	The CME front carries a large number of protons and alpha particles. Calculate the energy (in eV) of a single proton and a single alpha particle as measured by the Solar Wind Electrons Alphas and Protons (SWEAP) instrument on board the ParkerSP. Consider only the mechanical energy of the particles resulting from the propagation of the CME front, neglecting all other forms of energy.	10.0pt

Tools:

The JHelioviewer software (https://www.jhelioviewer.org/download.html) can be used to explore solar data from several solar telescopes as shown in Figure 2. Using the graphic interface, you can select an observing data (Observation Date) and upload multiple solar images by adding layers (AddLayer). Using the option, you can inspect a sequence of images to study the evolution of an eruptive event. By moving the cursor you get the information about the coordinates where you are located (in arcseconds) with respect to the center of the Sun (x:0” y:0”).

Observational Round: Planetarium

A1-1

English (Official)

Simulated Sky (75 points) - 17 questions in 45 minutes.

Please read the general instructions before you start this problem.

On the projection screen in front of you, a series of images will be projected. Look at the image and answer the following questions:

Image 1 (2 minutes).

1.1 (6.0 pt)
Identify the drawn lines on the image.

Line Number:	1	2	3
Celestial Equator			
Ecliptic			
Local Meridian			
Local Vertical			
Zero RA			
Galactic Equator			

Image 2 (2 minutes).

1.2 (3.0 pt)
This image was taken from the town of Fada in Chad. What is the latitude of this place? Select only one choice:

A. 10°

B. 17°

C. 22°

D. 33°

Image 3 (3 minutes).

1.3 (4.0 pt)
Two of the following stars are transiting the meridian in the image. Tick the boxes to those stars.

Arcturus	Mizar	Spica	Regulus

No Image for questions 4, 5 and 6 (8 minutes)

1.4 (2.0 pt)
If the age of the Moon is 27.5 days, in which direction can we observe it and at what time?. Select only one choice.

A. Before sunrise to the east.

B. After sunset to the west.

C. Close to zenith at sunrise.

D. At 9 am rising in the East.

1.5 (5.0 pt)
Which of the Right Ascensions (RA) listed below will approximately coincide with the local meridian at Pune, India $(74°E)$on April 10 at $19:15$ local time?
A. 6 h

B. 7 h

C. 8 h

D. 9 h

E. 10 h

1.6 (4.0 pt)
At what time during this night (April 10) would we find the Regulus $[\alpha\ Leo\ (RA\ 10h\ 08m\ 22.3s,\ Dec\ = +11°58'02")]$ at its maximum altitude as seen from Pune, India? Select only one choice.

A. 18:00h

B. 19:00h

C. 20:00h

D. 21:00h

E. 22:00h

Image 4 for questions 7, 8, 9 and 10

The projected sky now corresponds to -30 **of latitude** (South) with -74 **longitude** (West).

Image 4 (2 minutes).

1.7 (3.0 pt)
Select the constellations pack that crosses the celestial equator on the image.

A. Ori, Tau, Cet, Eri, Psc, Aqr, Aql, Ser

B. Mic, Tau, Per, Cae, Mon, Aql, And, Ser

C. Gru, Tau, Cha, Ara, Aqr, Aql, And, Aps

D. For, Tri, Cet, Psc, Sct, Aql, And, Ser

Image 4 (2 minutes).

1.8 (6.0 pt)
Choose the name of stars indicated by the arrows (tick in the correct cell).

Star Number	1	2	3
Achernar			
Alnair			
Altair			
Alnath			
Ankaa			
Canopus			

Image 4 (3 minutes).

1.9 (6.0 pt)
Are the following Messier objects presented in the projected sky?. Select (X) YES or NO depending on the case.

Object	YES	NO
M15 (Peg)		
M16 (Aql)		
M27 (Vul)		
M6 (Sco)		
M15 (Cas)		
M27 (CMi)		

Image 4 (3 minutes).

Choose the name of the selected stars (tick in the correct cell). If the same object is listed more than once, choose only the option with the correct constellation:

1.10 (3.0 pt)

Star Number:	1	2	3
Miaplacidus (Car)			
Miaplacidus (Lac)			
Sham (Sge)			
Sham (For)			
The Persian (Gru)			
The Persian (Ind)			

Image 5 for questions 11 and 12 (4 minutes).

1.11 (4.0 pt)
From the list below select the option that contain the constellations that are along the line of the ecilptic:

A. Psc, Cap, Sgr, Oph

B. Lib, Sco, Oph, Sgr

C. Cap, Sge, Lib, Vir

D. Sgr, Oph, Sco, Vir

E. CrA, Sco, Oph, Leo

1.12 (3.0 pt)
Which constellation or part of contellation is encircled in the image?

A. Sagittarius (Sgr)

B. Corvus (Crv)

C. Telescopium (Tel)

D. Corona Australis (CrA)

Image 6 (2 minutes).

1.13 (3.0 pt)
You are making an observation through a telescope pointed towards the centre of the galaxy. You find an interesting object as shown in the image 6. What is this object?

A. M42 - Orion Nebula

B. M31 - Andromeda Galaxy

C. NGC 2024 - Flame Nebula

D. M8 - Lagoon Nebula

E. M20 - Trifid Nebula

Image 7 (2 minutes).

1.14 (5.0 pt)
In the Messier Astronomical Catalogue there's a mysterious object: a galaxy. This object in the image cannot be easily identified. Some astronomers had said that it was a duplicate of another Messier Object, but here We have it as an original and awesome galaxy. Wich object is this?

A. M31 - Andromeda Galaxy

B. M33 - Triangulum Galaxy

C. M51 - Whirlpool Galaxy

D. M101 - Pinwheel Galaxy

E. M102 - Spindle Galaxy

Image 8 (2 minutes).

1.15 (2.0 pt)
Estimate the magnitude of Shaula in the constellation Scorpio.

A. 1

B. 1.5

C. 2

D. 2.5

C. 3

Image 9 (4 minutes).

1.16 (8.0 pt)
Several stars have been indicated in image 9. Tick the stars indicated in the table below.

Star Number:	1	2	3	4
Acubens				
Adhara				
Aludra				
Alzirr				
Arneb				
Gomeisa				
α Mon				
Mebsuta				
Mirzam				
Wasat				

Image 10 (4 minutes).

1.17 (8.0 pt)
A Messier object has been indicated in image 10. Tick the correct cell below to indicate what it is.

Object Number:	1	2	3	4
M 41				
M 42				
M 46				
M 45				
M 48				
M 50				
M 64				
M 93				

Group Radio Astronomy (115 points)

Please read the general instructions in the separate envelope before you start this problem.

Measuring the Perseus arm using 21 cm HI line data

Context

Our goal here is to kinematically estimate the distance of (part of) the Perseus Arm of the Milky Way (Figure 1), from the center of the Milky Way, based on the line-of-sight velocity of neutral hydrogen gas via its 21 cm emission line.

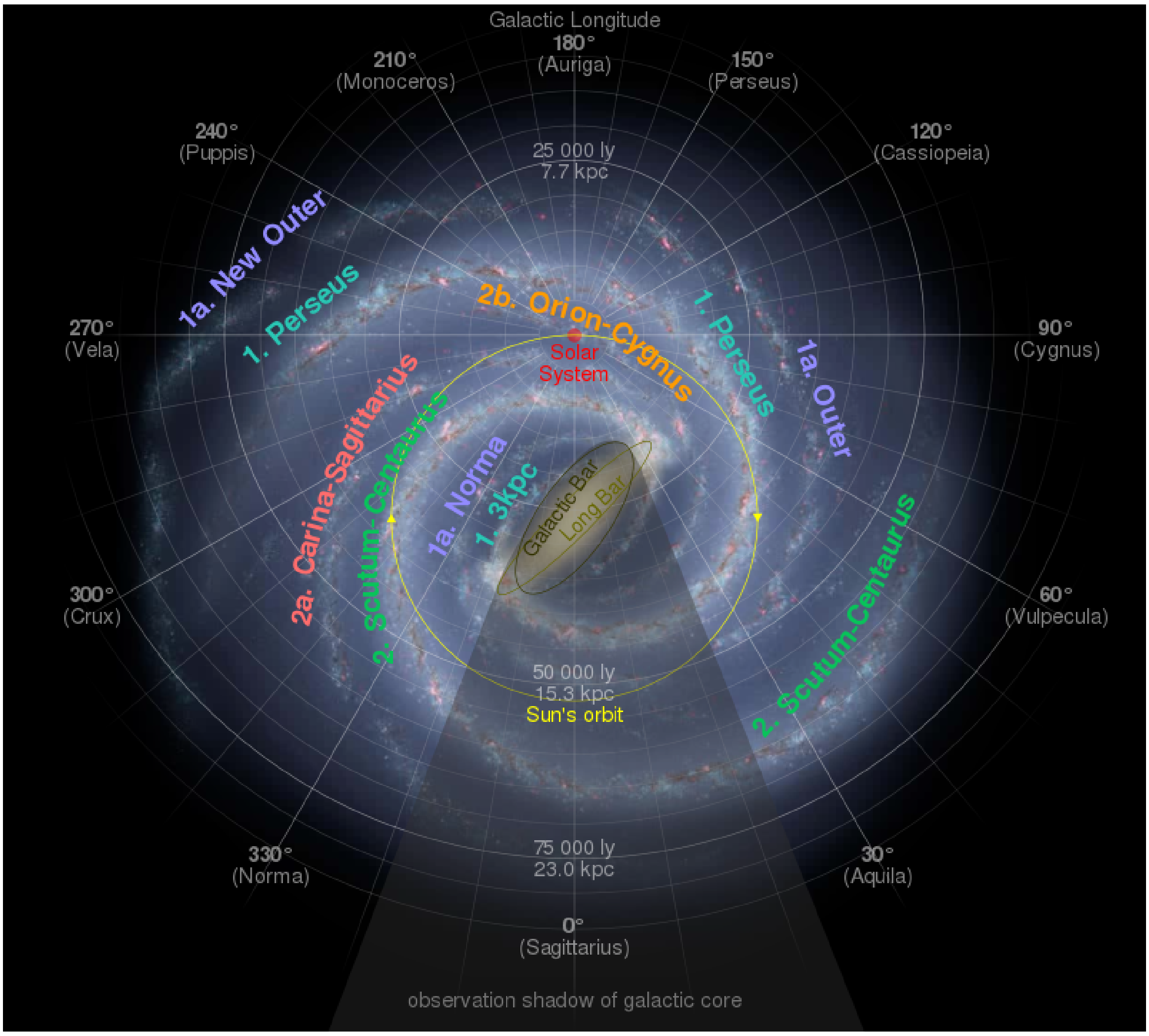


Figure 1: Distance-galactic longitude map of the Milky Way arms https://en.wikipedia.org/wiki/Perseus_Arm#/media/File:Milky_Way_Arms_ssc2008-10.svg

For this problem we will use a subset of the Canadian Galactic Plane Survey (CGPS, Figure 2), in which individual radio telescope pointings can each yield the 21 cm line spectrum emitted by all the galactic neutral hydrogen along the line of sight of the radio telescope.

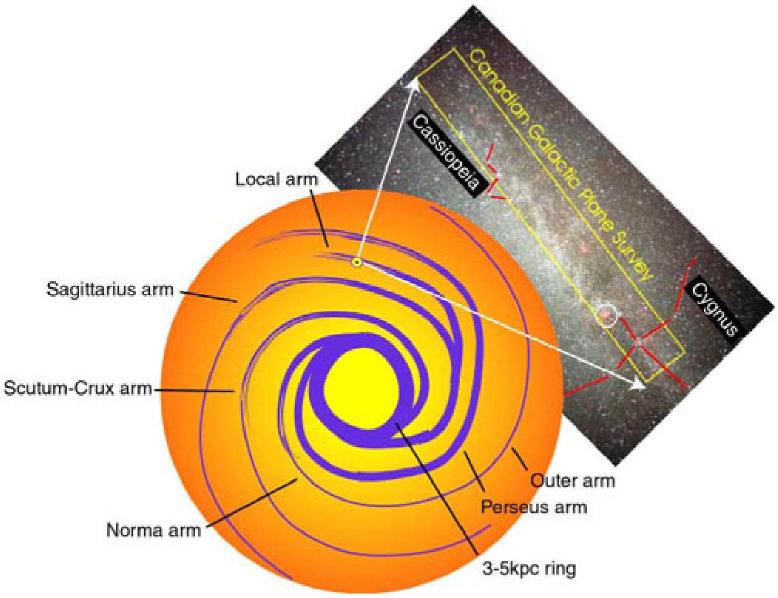


Figure 2: Canadian galactic plane survey http://www.ras.ucalgary.ca/CGPS

By translating the Doppler wavelength shift of the 21 cm emission to a line-of-sight velocity, it is then possible to identify individual emission components that correspond to distinct galactic arms. This identification allows for a reconstruction of the shape of each arm with respect to the Galactic Center.

In the spectrum corresponding to a radio telescope pointing, the Perseus arm can be readily identified because it is often the brightest feature along each line of sight.

The frame of reference of the radio telescope observations can be taken to be the Sun, located at a distance R_0 from the Galactic Center (GC). The telescope has a pointing along a Line of Sight (LOS) defined by a galactic longitude l and a fixed galactic latitude $b = 0$. Along this LOS, the telescope picks up the emission of a parcel of neutral H gas from the Perseus arm that is located at a distance r from the Sun. This same parcel of gas is located at a distance R from the Galactic Center. Let us assume that both the Sun and the gas parcel are in exact circular orbits around the GC. Additionally, it can be assumed that both the Sun and the gas parcel are in the region where the rotation curve of the Milky Way is flat. The measured (Doppler) velocity is denoted as $\mathrm{v}_{\mathrm{LOS}}$, which equals to the velocity of the gas parcel along the line of sight.

Data set

For this problem we attach a .csv file (21cmsurvey_full.csv, Excel and other spreadsheet software-readable) which contain 21 cm HI line brightness temperature (T_b) data vs. line-of-sight velocity (V_{LOS}) for a range of galactic longitudes (for galactic latitude $= 0$).

Row **1**: Line-of-sight velocities v_{LOS} (173 values, units: kms^{-1}).

Column **1** (after row 1): Galactic Longitude l (1024 values, units: $^\circ$).

Rows **2-1025**: 21 cm HI Brightness Temperature T_b (units: K). Each row yields the spectrum for the pointing defined by l (row name - column 1). There are thus 1024 spectra. Each spectrum has 173 T_b measurements, one for each v_{LOS}.

	A	B	C	D	E	F	G
1	longitude	17.499242	16.674782	15.850322	15.025862	14.201402	13.376942
2	142.195	7.6806355	-3.6773872	10.236036	12.072731	2.6496887	-5.4096527
3	142.2	-2.3566856	-17.443382	10.948601	15.752264	-5.6430779	-4.0766678
4	142.205	-7.2586327	-16.816818	11.409309	14.382421	-8.1247673	-2.1908302
5	142.21	-4.8997993	-1.3861237	8.1782017	0.1741447	-6.5460701	2.8831139
6	142.215	1.4211311	17.361675	3.865963	-19.79607	-5.4956512	10.672174
7	142.22	10.801174	29.229548	6.5995045	-28.279266	-6.2942162	17.140533
8	142.225	15.174841	25.408731	12.852865	-18.843937	-8.4810486	11.249598
9	142.23	11.863876	11.36631	13.676001	-3.8985252	-8.6407623	-3.4193878
10	142.235	1.5808449	-5.765934	4.6522408	3.5158234	-6.70578	-18.493797
11	142.24	-3.855526	-13.573421	-5.8457909	0.7269974	-4.1995239	-23.408031
12	142.245	-1.1465569	-7.5473442	-7.0313492	-3.400959	-1.7116928	-18.352516
13	142.25	5.9913673	8.6634827	2.0968399	-1.6011238	4.3635292	-6.9637794
14	142.255	9.1303349	24.567169	13.166147	4.2713852	13.448717	4.9778061

Part 1 (50 points).

1.1	Make a spectral plot of v_{LOS} vs. T_b for an adequate number of different values (at least 20 plots) of galactic longitudes covering the full range of observations. Identify the peak line of sight velocity of the Perseus gas parcel at each of the plotted longitudes. Make sure to evenly sample the data set. **Note:** Use the plot of the first or the last longitude as a guide to identify correct peaks in the plots at the intermediate longitudes.	45.0pt

1.2	Why does the emission near $v_{LOS} = 0$ (which we associate with our local arm) have a lower brightness temperature than the emission from the Perseus arm?	5.0pt

Part 2 (20 points).

2.1	Derive an expression to calculate R from v_{LOS}, $v_{\odot}$, and l. You can assume: • That both the Solar System and and the Perseus arm gas parcel along the line of sight have a purely tangential velocity, with a negligible radial component. • A flat galactic rotation curve, i.e. $$\lvert \mathrm{v} \rvert = \lvert \mathrm{v}_{\odot} \rvert$$ where v is the velocity of the gas parcel.	20.0pt

Part 3 (20 points).

3.1	Using the v_{LOS} values you found earlier, make a plot of galactic longitude l vs. R (radius with respect to the Galactic Center, in kpc) for the Perseus arm. Find the average distance of the Perseus arm for the given longitude range. Also report the standard deviation in your result. Use the values: $$v_{\odot} \approx 225\ kms^{-1}$$ $$R_0 \approx 8\ \mathrm{kpc}$$	20.0pt

Part 4 (25 points).

4.1	The data also shows 21cm emission from the Norma arm of the Milky Way, which is its outer arm. This emission is most clearly seen around the galactic longitude of 145^o. Repeat the exercise for the Norma arm to find its distance from GC. Use at least 5 data points to determine the distance of the Norma arm from the Galactic Centre (at these galactic longitudes).	25.0pt